

# Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.9-trig<sup>m</sup>-a+b-sin<sup>n</sup>+c-sin<sup>-2-n</sup>-<sup>p</sup>

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 19 ]. This is test number [ 81 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$  functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 19 )	% 0.00 ( 0 )
Mathematica	% 100.00 ( 19 )	% 0.00 ( 0 )
Maple	% 100.00 ( 19 )	% 0.00 ( 0 )
Maxima	% 26.32 ( 5 )	% 73.68 ( 14 )
Fricas	% 84.21 ( 16 )	% 15.79 ( 3 )
Sympy	% 31.58 ( 6 )	% 68.42 ( 13 )
Giac	% 47.37 ( 9 )	% 52.63 ( 10 )
Mupad	% 100.00 ( 19 )	% 0.00 ( 0 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

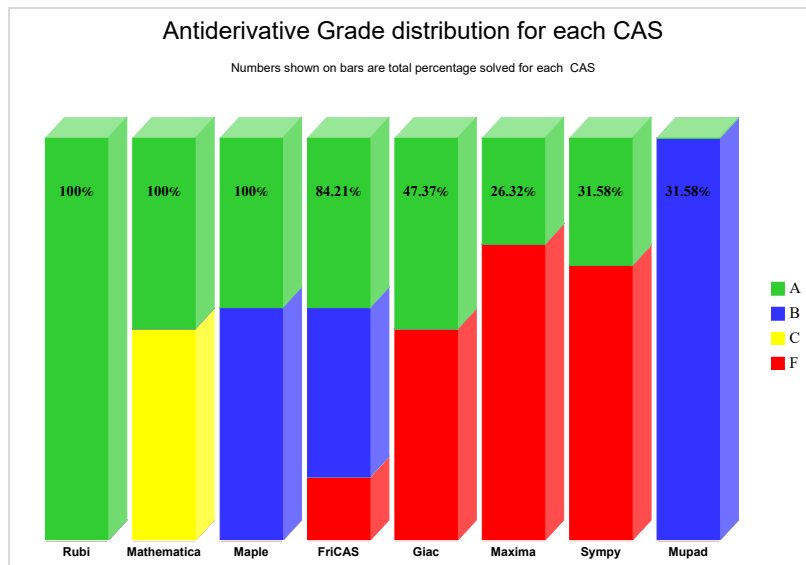
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

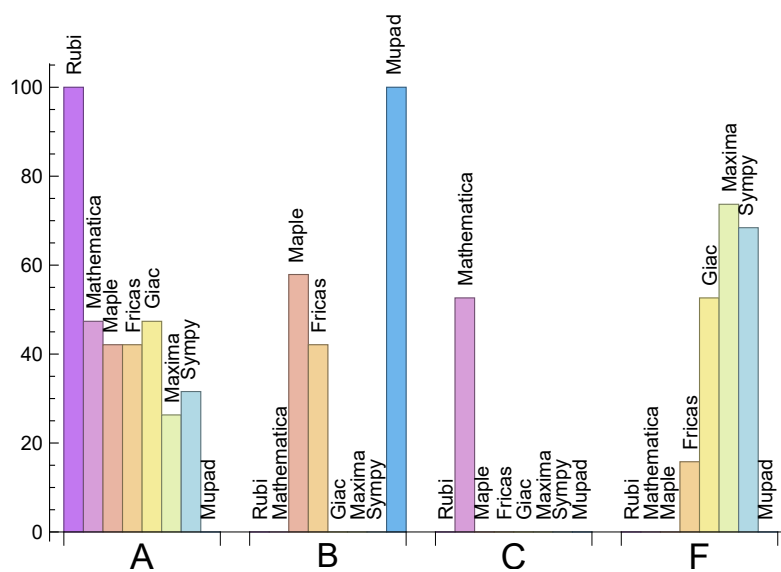
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	47.37	0.00	52.63	0.00
Maple	42.11	57.89	0.00	0.00
Maxima	26.32	0.00	0.00	73.68
Fricas	42.11	42.11	0.00	15.79
Sympy	31.58	0.00	0.00	68.42
Giac	47.37	0.00	0.00	52.63
Mupad	0.00	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	14	14.29 %	57.14 %	28.57 %
Fricas	3	0.00 %	100.00 %	0.00 %
Sympy	13	46.15 %	53.85 %	0.00 %
Giac	10	0.00 %	100.00 %	0.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

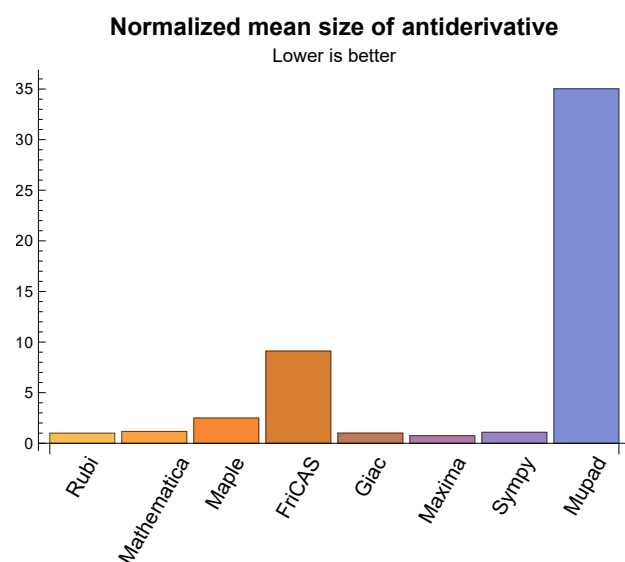
## 1.3 Performance

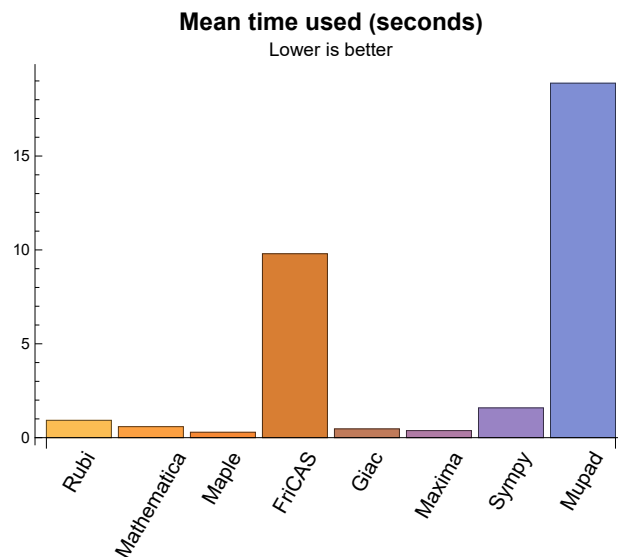
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.92	170.47	1.00	221.00	1.00
Mathematica	0.59	209.95	1.17	233.00	1.20
Maple	0.29	580.53	2.50	549.00	2.52
Maxima	0.38	10.60	0.75	13.00	0.71
Fricas	9.80	2198.00	9.11	726.50	4.10
Sympy	1.59	25.17	1.09	13.50	0.71
Giac	0.47	75.78	1.01	17.00	1.00
Mupad	18.88	9923.79	35.03	5048.00	22.34

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.



**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

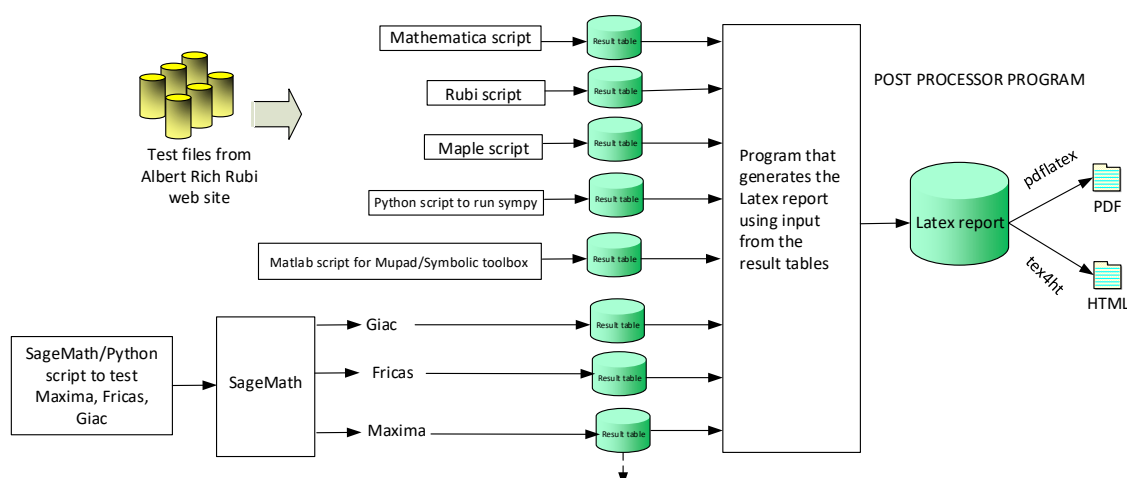
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 9, 11, 12, 14, 15, 16, 17, 18, 19 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 13 }

F grade: { }

#### 2.1.3 Maple

A grade: { 4, 11, 12, 15, 16, 17, 18, 19 }

B grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 14 }

C grade: { }

F grade: { }

#### 2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 }

#### 2.1.5 FriCAS

A grade: { 9, 11, 12, 15, 16, 17, 18, 19 }

B grade: { 1, 2, 3, 4, 5, 6, 10, 14 }

C grade: { }

F grade: { 7, 8, 13 }

## 2.1.6 Sympy

A grade: { 11, 15, 16, 17, 18, 19 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14 }

## 2.1.7 Giac

A grade: { 9, 11, 12, 14, 15, 16, 17, 18, 19 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 13 }

## 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 }

C grade: { }

F grade: { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	410	1181	0	8169	0	0	39682
normalized size	1	1.00	1.27	3.66	0.00	25.29	0.00	0.00	122.85
time (sec)	N/A	3.129	1.287	0.342	0.000	4.543	0.000	0.000	26.408
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	358	890	0	6531	0	0	21407
normalized size	1	1.00	1.20	2.99	0.00	21.92	0.00	0.00	71.84
time (sec)	N/A	3.741	0.963	0.289	0.000	2.268	0.000	0.000	25.032
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	310	638	0	4985	0	0	15461
normalized size	1	1.00	1.23	2.52	0.00	19.70	0.00	0.00	61.11
time (sec)	N/A	1.038	0.639	0.279	0.000	1.203	0.000	0.000	27.750
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	268	216	0	3519	0	0	5048
normalized size	1	1.00	1.19	0.96	0.00	15.57	0.00	0.00	22.34
time (sec)	N/A	0.548	0.715	0.276	0.000	0.786	0.000	0.000	24.992
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	233	610	0	3495	0	0	5064
normalized size	1	1.00	1.05	2.76	0.00	15.81	0.00	0.00	22.91
time (sec)	N/A	0.397	0.600	0.281	0.000	0.808	0.000	0.000	25.733

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	306	849	0	5296	0	0	11540
normalized size	1	1.00	1.25	3.48	0.00	21.70	0.00	0.00	47.30
time (sec)	N/A	0.762	1.296	0.355	0.000	131.811	0.000	0.000	26.517

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	388	1087	0	0	0	0	16102
normalized size	1	1.00	1.43	4.01	0.00	0.00	0.00	0.00	59.42
time (sec)	N/A	0.895	1.274	0.373	0.000	0.000	0.000	0.000	25.681

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	481	1369	0	0	0	0	21909
normalized size	1	1.00	1.45	4.14	0.00	0.00	0.00	0.00	66.19
time (sec)	N/A	3.210	1.624	0.391	0.000	0.000	0.000	0.000	24.322

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	143	0	276	0	78	229
normalized size	1	1.00	0.96	1.88	0.00	3.63	0.00	1.03	3.01
time (sec)	N/A	0.142	0.116	0.289	0.000	1.043	0.000	0.290	0.215

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	314	1246	0	971	0	0	11164
normalized size	1	1.00	1.37	5.42	0.00	4.22	0.00	0.00	48.54
time (sec)	N/A	0.585	0.479	0.345	0.000	0.887	0.000	0.000	26.303

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	36	0	139	99	35	47
normalized size	1	1.00	1.00	1.03	0.00	3.97	2.83	1.00	1.34
time (sec)	N/A	0.045	0.014	0.209	0.000	0.715	7.229	0.129	15.066



Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	119	224	0	482	0	131	1001
normalized size	1	1.00	0.93	1.75	0.00	3.77	0.00	1.02	7.82
time (sec)	N/A	0.175	0.235	0.291	0.000	1.291	0.000	1.226	17.350
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	407	1934	0	0	0	0	37118
normalized size	1	1.00	1.26	5.97	0.00	0.00	0.00	0.00	114.56
time (sec)	N/A	2.268	1.008	0.375	0.000	0.000	0.000	0.000	27.595
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	202	549	0	1244	0	377	2743
normalized size	1	1.00	0.98	2.67	0.00	6.04	0.00	1.83	13.32
time (sec)	N/A	0.500	0.766	0.339	0.000	9.167	0.000	1.845	35.307
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	16	15	17	15	17	9
normalized size	1	1.00	0.71	0.76	0.71	0.81	0.71	0.81	0.43
time (sec)	N/A	0.026	0.009	0.200	0.323	0.453	0.453	0.186	0.155
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	26	14	13	17	12	17	9
normalized size	1	1.00	1.53	0.82	0.76	1.00	0.71	1.00	0.53
time (sec)	N/A	0.027	0.056	0.209	0.344	0.441	0.423	0.147	15.145
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	30	16	15	17	15	17	9
normalized size	1	1.00	1.43	0.76	0.71	0.81	0.71	0.81	0.43
time (sec)	N/A	0.028	0.049	0.218	0.326	0.451	0.463	0.139	0.118

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	5	5	5
normalized size	1	1.00	1.00	0.67	0.56	0.56	0.56	0.56	0.56
time (sec)	N/A	0.029	0.010	0.215	0.455	0.440	0.499	0.119	0.116

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.026	0.010	0.202	0.435	0.430	0.452	0.137	14.926

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [8] had the largest ratio of [.4737]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	8	1.00	19	0.421
2	A	10	6	1.00	19	0.316
3	A	9	5	1.00	19	0.263
4	A	8	4	1.00	17	0.235
5	A	7	4	1.00	14	0.286
6	A	10	6	1.00	17	0.353
7	A	12	8	1.00	19	0.421
8	A	14	9	1.00	19	0.474
9	A	7	6	1.00	19	0.316
10	A	9	5	1.00	19	0.263
11	A	3	3	1.00	17	0.176
12	A	9	8	1.00	17	0.471
13	A	11	6	1.00	19	0.316
14	A	10	9	1.00	19	0.474

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	4	3	1.00	13	0.231
16	A	4	3	1.00	15	0.200
17	A	4	3	1.00	15	0.200
18	A	3	3	1.00	15	0.200
19	A	3	3	1.00	15	0.200



# Chapter 3

## Listing of integrals

**3.1** 
$$\int \frac{\sin^4(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

**Optimal.** Leaf size=323

$$\frac{\sqrt{2} \left( -\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^3\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{\sqrt{2} \left( \frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b+\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^3\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out]  $\frac{1}{2}x/c + (-a*c + b^2)*x/c^3 + b*\cos(x)/c^2 - 1/2*\cos(x)*\sin(x)/c - \arctan(1/2*(2*c + (b - (-4*a*c + b^2)^(1/2))*\tan(1/2*x))*2^(1/2)/(b^2 - 2*c*(a+c) - b*(-4*a*c + b^2)^(1/2))^(1/2))*2^(1/2)*(b^3 - 2*a*b*c + (-2*a^2*c^2 + 4*a*b^2*c - b^4)/(-4*a*c + b^2)^(1/2))/c^3 / (b^2 - 2*c*(a+c) - b*(-4*a*c + b^2)^(1/2))^(1/2) - \arctan(1/2*(2*c + (b + (-4*a*c + b^2)^(1/2))*\tan(1/2*x))*2^(1/2)/(b^2 - 2*c*(a+c) + b*(-4*a*c + b^2)^(1/2))^(1/2))*2^(1/2)*(b^3 - 2*a*b*c + (2*a^2*c^2 - 4*a*b^2*c + b^4)/(-4*a*c + b^2)^(1/2))/c^3 / (b^2 - 2*c*(a+c) + b*(-4*a*c + b^2)^(1/2))^(1/2)}$

**Rubi [A]** time = 3.13, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3256, 2638, 2635, 8, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} \left( -\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^3\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{\sqrt{2} \left( \frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b+\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^3\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^4/(a + b\*SIN[x] + c\*SIN[x]^2), x]

[Out]  $x/(2*c) + ((b^2 - a*c)*x)/c^3 - (\text{Sqrt}[2]*(b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(2*c + (b - \text{Sqrt}[b^2 - 4*a*c]))*\text{Tan}[x/2]]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a+c) - b*\text{Sqrt}[b^2 - 4*a*c]])]/(c^3*\text{Sqrt}[b^2 - 2*c*(a+c) - b*\text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[2]*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(2*c + (b + \text{Sqrt}[b^2 - 4*a*c]))*\text{Tan}[x/2]]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a+c) + b*\text{Sqrt}[b^2 - 4*a*c]])]/(c^3*\text{Sqrt}[b^2 - 2*c*(a+c) + b*\text{Sqrt}[b^2 - 4*a*c]]) + (b*\text{Cos}[x])/c^2 - (\text{Cos}[x]*\text{Sin}[x])/(2*c)$

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] + (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3256

```
Int[sin[(d_) + (e_)*(x_)]^(m_)*((a_) + (b_)*sin[(d_) + (e_)*(x_)]^(n_) + (c_)*sin[(d_) + (e_)*(x_)]^(n2_))^(p_), x_Symbol] := Int[ExpandTrig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[m, n, p]
```

Rule 3292

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])/((a_) + (b_)*sin[(d_) + (e_)*(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2), x_Symbol] := Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( \frac{b^2 - ac}{c^3} - \frac{b \sin(x)}{c^2} + \frac{\sin^2(x)}{c} + \frac{-ab^2 \left(1 - \frac{ac}{b^2}\right) - b^3 \left(1 - \frac{2ac}{b^2}\right) \sin(x)}{c^3 (a + b \sin(x) + c \sin^2(x))} \right) dx \\
&= \frac{(b^2 - ac)x}{c^3} + \frac{\int \frac{-ab^2 \left(1 - \frac{ac}{b^2}\right) - b^3 \left(1 - \frac{2ac}{b^2}\right) \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{c^3} - \frac{b \int \sin(x) dx}{c^2} + \frac{\int \sin^2(x) dx}{c} \\
&= \frac{(b^2 - ac)x}{c^3} + \frac{b \cos(x)}{c^2} - \frac{\cos(x) \sin(x)}{2c} + \frac{\int 1 dx}{2c} - \frac{\left(b^3 - 2abc - \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}}\right)}{c^3} \\
&= \frac{x}{2c} + \frac{(b^2 - ac)x}{c^3} + \frac{b \cos(x)}{c^2} - \frac{\cos(x) \sin(x)}{2c} - \frac{\left(2 \left(b^3 - 2abc - \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}}\right)\right)}{c^3} \\
&= \frac{x}{2c} + \frac{(b^2 - ac)x}{c^3} + \frac{b \cos(x)}{c^2} - \frac{\cos(x) \sin(x)}{2c} + \frac{\left(4 \left(b^3 - 2abc - \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}}\right)\right)}{c^3} \\
&= \frac{x}{2c} + \frac{(b^2 - ac)x}{c^3} - \frac{\sqrt{2} \left(b^3 - 2abc - \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}}\right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{c^3 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}}
\end{aligned}$$

**Mathematica [C]** time = 1.29, size = 410, normalized size = 1.27

$$\frac{4 \left( 2ia^2c^2 - 4iab^2c - 2abc\sqrt{4ac - b^2} + b^3\sqrt{4ac - b^2} + ib^4 \right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right)(b - i\sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} - \frac{4 \left( -2ia^2c^2 + 4iab^2c - 2abc\sqrt{4ac - b^2} + b^3\sqrt{4ac - b^2} - ib^4 \right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{4c^3 \sqrt{2ac - \frac{b^2}{2}} \sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^4/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out]  $(4*b^2*x + 2*c*(-2*a + c)*x - (4*(I*b^4 - (4*I)*a*b^2*c + (2*I)*a^2*c^2 + b^3*\text{Sqrt}[-b^2 + 4*a*c] - 2*a*b*c*\text{Sqrt}[-b^2 + 4*a*c])* \text{ArcTan}[(2*c + (b - I*\text{Sqrt}[-b^2 + 4*a*c])* \text{Tan}[x/2])]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) - I*b*\text{Sqrt}[-b^2 + 4*a*c]])))/(\text{Sqrt}[-1/2*b^2 + 2*a*c]*\text{Sqrt}[b^2 - 2*c*(a + c) - I*b*\text{Sqrt}[-b^2 + 4*a*c]]) - (4*((-I)*b^4 + (4*I)*a*b^2*c - (2*I)*a^2*c^2 + b^3*\text{Sqrt}[-b^2 + 4*a*c] - 2*a*b*c*\text{Sqrt}[-b^2 + 4*a*c])* \text{ArcTan}[(2*c + (b + I*\text{Sqrt}[-b^2 + 4*a*c])* \text{Tan}[x/2])]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) + I*b*\text{Sqrt}[-b^2 + 4*a*c]])))/(\text{Sqrt}[-1/2*b^2 + 2*a*c]*\text{Sqrt}[b^2 - 2*c*(a + c) + I*b*\text{Sqrt}[-b^2 + 4*a*c]]) + 4*b*c*\text{Cos}[x] - c^2*\text{Sin}[2*x])/ (4*c^3)$

**fricas [B]** time = 4.54, size = 8169, normalized size = 25.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="fricas")

[Out]  $-1/4*(\text{sqrt}(2)*c^3*\text{sqrt}((a^2*b^6 - b^8 - 2*a^4*c^4 - 2*(a^5 - 8*a^3*b^2)*c^3 + (9*a^4*b^2 - 20*a^2*b^4)*c^2 - 2*(3*a^3*b^4 - 4*a*b^6)*c + (4*a*c^9 + (8*a^2 - b^2)*c^8 + 2*(2*a^3 - 3*a*b^2)*c^7 - (a^2*b^2 - b^4)*c^6))*\text{sqrt}(-(a^4*b^10 - 2*a^2*b^12 + b^14 + 16*a^6*b^2*c^6 + 8*(3*a^7*b^2 - 10*a^5*b^4)*c^5$

$$\begin{aligned}
& + (9a^8b^2 - 92a^6b^4 + 148a^4b^6) * c^4 - 4 * (6a^7b^4 - 31a^5b^6 + 32a^3b^8) * c^3 + 2 * (11a^6b^6 - 37a^4b^8 + 28a^2b^{10}) * c^2 - 4 * (2a^5b^8 - 5a^3b^{10} + 3a * b^{12}) * c / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3b^2 + 2a * b^4) * c^{13} - (a^4b^2 - 2a^2b^4 + b^6) * c^{12})) / (4a * c^9 + (8a^2 - b^2) * c^8 + 2 * (2a^3 - 3a * b^2) * c^7 - (a^2b^2 - b^4) * c^6)) * \log(16a^7 * b * c^4 + 4 * (3a^8 * b - 10a^6 * b^3) * c^3 - 8 * (2a^7 * b^3 - 3a^5 * b^5) * c^2 + 2 * (4a^5 * c^9 + (8a^6 - a^4 * b^2) * c^8 + 2 * (2a^7 - 3a^5 * b^2) * c^7 - (a^6 * b^2 - a^4 * b^4) * c^6)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8 * (3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4 * (6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2 * (11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4 * (2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) * \sin(x) + 4 * (a^6 * b^5 - a^4 * b^7) * c - \sqrt{2} * ((8a^3 * c^{12} + 6 * (4a^4 - 3a^2 * b^2) * c^{11} + 2 * (12a^5 - 25a^3 * b^2 + 4a * b^4) * c^{10} + (8a^6 - 38a^4 * b^2 + 35a^2 * b^4 - b^6) * c^9 - 2 * (3a^5 * b^2 - 8a^3 * b^4 + 5a * b^6) * c^8 + (a^4 * b^4 - 2a^2 * b^6 + b^8) * c^7)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8 * (3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4 * (6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2 * (11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4 * (2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) * \cos(x) - (32a^5 * b^2 * c^6 + 8 * (5a^6 * b^2 - 13a^4 * b^4) * c^5 + 2 * (6a^7 * b^2 - 47a^5 * b^4 + 56a^3 * b^6) * c^4 - (19a^6 * b^4 - 69a^4 * b^6 + 54a^2 * b^8) * c^3 + 4 * (2a^5 * b^6 - 5a^3 * b^8 + 3a * b^{10}) * c^2 - (a^4 * b^8 - 2a^2 * b^{10} + b^{12}) * c) * \cos(x)) * \sqrt{((a^2 * b^6 - b^8 - 2a^4 * c^4 - 2 * (a^5 - 8a^3 * b^2) * c^3 + (9a^4 * b^2 - 20a^2 * b^4) * c^2 - 2 * (3a^3 * b^4 - 4a * b^6) * c + (4a * c^9 + (8a^2 - b^2) * c^8 + 2 * (2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8 * (3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4 * (6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2 * (11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4 * (2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) / (4a * c^9 + (8a^2 - b^2) * c^8 + 2 * (2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) + 2 * (a^6 * b^6 - a^4 * b^8 + 4 * a^7 * b^2 * c^3 + (3a^8 * b^2 - 10a^6 * b^4) * c^2 - 2 * (2a^7 * b^4 - 3a^5 * b^6) * c) * \sin(x)) - \sqrt{2} * c^3 * \sqrt{((a^2 * b^6 - b^8 - 2a^4 * c^4 - 2 * (a^5 - 8a^3 * b^2) * c^3 + (9a^4 * b^2 - 20a^2 * b^4) * c^2 - 2 * (3a^3 * b^4 - 4a * b^6) * c - (4a * c^9 + (8a^2 - b^2) * c^8 + 2 * (2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8 * (3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4 * (6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2 * (11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4 * (2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) / (4a * c^9 + (8a^2 - b^2) * c^8 + 2 * (2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \log(16a^7 * b * c^4 + 4 * (3a^8 * b - 10a^6 * b^3) * c^3 - 8 * (2a^7 * b^3 - 3a^5 * b^5) * c^2 - 2 * (4a^5 * c^9 + (8a^6 - a^4 * b^2) * c^8 + 2 * (2a^7 - 3a^5 * b^2) * c^7 - (a^6 * b^2 - a^4 * b^4) * c^6)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8 * (3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4 * (6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2 * (11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4 * (2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12 * (2a^3 - a * b^2) * c^{15} + 2 * (8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4 * (a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) * \sin(x) + 4 * (a^6 * b^5 - a^4 * b^7) * c - \sqrt{2} * ((8a^3 * c^{12} + 6 * (4a^4 - 3a^2 * b^2) * c^{11} + 2 * (12a^5 - 25a^3 * b^2 + 4a * b^4) * c^{10} + (8a^6 - 38a^4 * b^2 + 35a^2 * b^4 - b^6) * c^9 - 2 * (3a^5 * b^2 - 8a^3 * b^4 + 5a * b^6) * c^8 + (a^4 * b^4 - 2a^2 * b^6 + b^8) * c^7)) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8
\end{aligned}$$



$$\begin{aligned}
& * (3a^7b^2 - 10a^5b^4) * c^5 + (9a^8b^2 - 92a^6b^4 + 148a^4b^6) * c^4 \\
& - 4(6a^7b^4 - 31a^5b^6 + 32a^3b^8) * c^3 + 2(11a^6b^6 - 37a^4b^8 \\
& + 28a^2b^{10}) * c^2 - 4(2a^5b^8 - 5a^3b^{10} + 3a * b^{12}) * c / (4a * c^{17} + ( \\
& 16a^2 - b^2) * c^{16} + 12(2a^3 - a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + b^4) \\
& * c^{14} + 4(a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^ \\
& 12)) * \cos(x) + (32a^5 * b^2 * c^6 + 8(5a^6 * b^2 - 13a^4 * b^4) * c^5 + 2(6a^7 * b \\
& ^2 - 47a^5 * b^4 + 56a^3 * b^6) * c^4 - (19a^6 * b^4 - 69a^4 * b^6 + 54a^2 * b^8) * \\
& c^3 + 4(2a^5 * b^6 - 5a^3 * b^8 + 3a * b^{10}) * c^2 - (a^4 * b^8 - 2a^2 * b^{10} + b \\
& ^{12}) * c) * \cos(x) * \sqrt{(a^2 * b^6 - b^8 - 2a^4 * c^4 - 2(a^5 - 8a^3 * b^2) * c^3 + \\
& (9a^4 * b^2 - 20a^2 * b^4) * c^2 - 2(3a^3 * b^4 - 4a * b^6) * c - (4a * c^9 + (8a^ \\
& 2 - b^2) * c^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \sqrt{-(a^4 * b^ \\
& 10 - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8(3a^7 * b^2 - 10a^5 * b^4) * c^5 + \\
& (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4(6a^7 * b^4 - 31a^5 * b^6 + 32 \\
& a^3 * b^8) * c^3 + 2(11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4(2a^5 * b^ \\
& 8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12(2a^3 - \\
& a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4(a^5 - 3a^3 * b^2 + 2a \\
& * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) / (4a * c^9 + (8a^2 - b^2) * c \\
& ^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) + 2(a^6 * b^6 - a^4 * b^8 \\
& + 4a^7 * b^2 * c^3 + (3a^8 * b^2 - 10a^6 * b^4) * c^2 - 2(2a^7 * b^4 - 3a^5 * b^6) \\
& * c) * \sin(x)) + \sqrt{2} * c^3 * \sqrt{(a^2 * b^6 - b^8 - 2a^4 * c^4 - 2(a^5 - 8a^3 * \\
& b^2) * c^3 + (9a^4 * b^2 - 20a^2 * b^4) * c^2 - 2(3a^3 * b^4 - 4a * b^6) * c - (4a * \\
& c^9 + (8a^2 - b^2) * c^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \sqrt{ \\
& -(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8(3a^7 * b^2 - 10a^5 * \\
& b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4(6a^7 * b^4 - 31a^ \\
& 5 * b^6 + 32a^3 * b^8) * c^3 + 2(11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - \\
& 4(2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + \\
& 12(2a^3 - a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4(a^5 - 3a^ \\
& 3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) / (4a * c^9 + (8a \\
& ^2 - b^2) * c^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \log(-16a^7 \\
& * b * c^4 - 4(3a^8 * b - 10a^6 * b^3) * c^3 + 8(2a^7 * b^3 - 3a^5 * b^5) * c^2 + 2( \\
& 4a^5 * c^9 + (8a^6 - a^4 * b^2) * c^8 + 2(2a^7 - 3a^5 * b^2) * c^7 - (a^6 * b^2 - \\
& a^4 * b^4) * c^6) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8(3a^ \\
& 7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4( \\
& 6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2(11a^6 * b^6 - 37a^4 * b^8 + 28a^ \\
& 2 * b^{10}) * c^2 - 4(2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^ \\
& 2 - b^2) * c^{16} + 12(2a^3 - a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + b^4) * c^{14} \\
& + 4(a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) * \\
& \sin(x) - 4(a^6 * b^5 - a^4 * b^7) * c - \sqrt{2} * ((8a^3 * c^{12} + 6(4a^4 - 3a^2 * \\
& b^2) * c^{11} + 2(12a^5 - 25a^3 * b^2 + 4a * b^4) * c^{10} + (8a^6 - 38a^4 * b^2 + \\
& 35a^2 * b^4 - b^6) * c^9 - 2(3a^5 * b^2 - 8a^3 * b^4 + 5a * b^6) * c^8 + (a^4 * b^4 \\
& - 2a^2 * b^6 + b^8) * c^7) * \sqrt{-(a^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^ \\
& 6 + 8(3a^7 * b^2 - 10a^5 * b^4) * c^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) \\
& * c^4 - 4(6a^7 * b^4 - 31a^5 * b^6 + 32a^3 * b^8) * c^3 + 2(11a^6 * b^6 - 37a^4 \\
& * b^8 + 28a^2 * b^{10}) * c^2 - 4(2a^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{1 \\
& 7} + (16a^2 - b^2) * c^{16} + 12(2a^3 - a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + \\
& b^4) * c^{14} + 4(a^5 - 3a^3 * b^2 + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^ \\
& 6) * c^{12})) * \cos(x) + (32a^5 * b^2 * c^6 + 8(5a^6 * b^2 - 13a^4 * b^4) * c^5 + 2(6a^ \\
& 7 * b^2 - 47a^5 * b^4 + 56a^3 * b^6) * c^4 - (19a^6 * b^4 - 69a^4 * b^6 + 54a^2 * \\
& b^8) * c^3 + 4(2a^5 * b^6 - 5a^3 * b^8 + 3a * b^{10}) * c^2 - (a^4 * b^8 - 2a^2 * b^{10} \\
& + b^{12}) * c) * \cos(x) * \sqrt{(a^2 * b^6 - b^8 - 2a^4 * c^4 - 2(a^5 - 8a^3 * b^2) * c^ \\
& 3 + (9a^4 * b^2 - 20a^2 * b^4) * c^2 - 2(3a^3 * b^4 - 4a * b^6) * c - (4a * c^9 + \\
& (8a^2 - b^2) * c^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) * \sqrt{-(a \\
& ^4 * b^{10} - 2a^2 * b^{12} + b^{14} + 16a^6 * b^2 * c^6 + 8(3a^7 * b^2 - 10a^5 * b^4) * c \\
& ^5 + (9a^8 * b^2 - 92a^6 * b^4 + 148a^4 * b^6) * c^4 - 4(6a^7 * b^4 - 31a^5 * b^6 \\
& + 32a^3 * b^8) * c^3 + 2(11a^6 * b^6 - 37a^4 * b^8 + 28a^2 * b^{10}) * c^2 - 4(2a \\
& ^5 * b^8 - 5a^3 * b^{10} + 3a * b^{12}) * c) / (4a * c^{17} + (16a^2 - b^2) * c^{16} + 12(2 \\
& a^3 - a * b^2) * c^{15} + 2(8a^4 - 11a^2 * b^2 + b^4) * c^{14} + 4(a^5 - 3a^3 * b^2 \\
& + 2a * b^4) * c^{13} - (a^4 * b^2 - 2a^2 * b^4 + b^6) * c^{12})) / (4a * c^9 + (8a^2 - b \\
& ^2) * c^8 + 2(2a^3 - 3a * b^2) * c^7 - (a^2 * b^2 - b^4) * c^6)) - 2(a^6 * b^6 - a^
\end{aligned}$$

$$\begin{aligned}
& 4*b^8 + 4*a^7*b^2*c^3 + (3*a^8*b^2 - 10*a^6*b^4)*c^2 - 2*(2*a^7*b^4 - 3*a^5*b^6)*c*\sin(x) - \sqrt{2}*c^3*\sqrt{(a^2*b^6 - b^8 - 2*a^4*c^4 - 2*(a^5 - 8*a^3*b^2)*c^3 + (9*a^4*b^2 - 20*a^2*b^4)*c^2 - 2*(3*a^3*b^4 - 4*a*b^6)*c + (4*a*c^9 + (8*a^2 - b^2)*c^8 + 2*(2*a^3 - 3*a*b^2)*c^7 - (a^2*b^2 - b^4)*c^6)*\sqrt{-(a^4*b^10 - 2*a^2*b^12 + b^14 + 16*a^6*b^2*c^6 + 8*(3*a^7*b^2 - 10*a^5*b^4)*c^5 + (9*a^8*b^2 - 92*a^6*b^4 + 148*a^4*b^6)*c^4 - 4*(6*a^7*b^4 - 31*a^5*b^6 + 32*a^3*b^8)*c^3 + 2*(11*a^6*b^6 - 37*a^4*b^8 + 28*a^2*b^10)*c^2 - 4*(2*a^5*b^8 - 5*a^3*b^10 + 3*a*b^12)*c)/(4*a*c^17 + (16*a^2 - b^2)*c^16 + 12*(2*a^3 - a*b^2)*c^15 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^14 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^13 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^12)})))/(4*a*c^9 + (8*a^2 - b^2)*c^8 + 2*(2*a^3 - 3*a*b^2)*c^7 - (a^2*b^2 - b^4)*c^6))*\log(-16*a^7*b*c^4 - 4*(3*a^8*b - 10*a^6*b^3)*c^3 + 8*(2*a^7*b^3 - 3*a^5*b^5)*c^2 - 2*(4*a^5*c^9 + (8*a^6 - a^4*b^2)*c^8 + 2*(2*a^7 - 3*a^5*b^2)*c^7 - (a^6*b^2 - a^4*b^4)*c^6)*\sqrt{-(a^4*b^10 - 2*a^2*b^12 + b^14 + 16*a^6*b^2*c^6 + 8*(3*a^7*b^2 - 10*a^5*b^4)*c^5 + (9*a^8*b^2 - 92*a^6*b^4 + 148*a^4*b^6)*c^4 - 4*(6*a^7*b^4 - 31*a^5*b^6 + 32*a^3*b^8)*c^3 + 2*(11*a^6*b^6 - 37*a^4*b^8 + 28*a^2*b^10)*c^2 - 4*(2*a^5*b^8 - 5*a^3*b^10 + 3*a*b^12)*c)/(4*a*c^17 + (16*a^2 - b^2)*c^16 + 12*(2*a^3 - a*b^2)*c^15 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^14 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^13 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^12))*\sin(x) - 4*(a^6*b^5 - a^4*b^7)*c - \sqrt{2}*((8*a^3*c^12 + 6*(4*a^4 - 3*a^2*b^2)*c^11 + 2*(12*a^5 - 25*a^3*b^2 + 4*a*b^4)*c^10 + (8*a^6 - 38*a^4*b^2 + 35*a^2*b^4 - b^6)*c^9 - 2*(3*a^5*b^2 - 8*a^3*b^4 + 5*a*b^6)*c^8 + (a^4*b^4 - 2*a^2*b^6 + b^8)*c^7)*\sqrt{-(a^4*b^10 - 2*a^2*b^12 + b^14 + 16*a^6*b^2*c^6 + 8*(3*a^7*b^2 - 10*a^5*b^4)*c^5 + (9*a^8*b^2 - 92*a^6*b^4 + 148*a^4*b^6)*c^4 - 4*(6*a^7*b^4 - 31*a^5*b^6 + 32*a^3*b^8)*c^3 + 2*(11*a^6*b^6 - 37*a^4*b^8 + 28*a^2*b^10)*c^2 - 4*(2*a^5*b^8 - 5*a^3*b^10 + 3*a*b^12)*c)/(4*a*c^17 + (16*a^2 - b^2)*c^16 + 12*(2*a^3 - a*b^2)*c^15 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^14 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^13 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^12))*\cos(x) - (32*a^5*b^2*c^6 + 8*(5*a^6*b^2 - 13*a^4*b^4)*c^5 + 2*(6*a^7*b^2 - 47*a^5*b^4 + 56*a^3*b^6)*c^4 - (19*a^6*b^4 - 69*a^4*b^6 + 54*a^2*b^8)*c^3 + 4*(2*a^5*b^6 - 5*a^3*b^8 + 3*a*b^10)*c^2 - (a^4*b^8 - 2*a^2*b^10 + b^12)*c)*\cos(x))*\sqrt{(a^2*b^6 - b^8 - 2*a^4*c^4 - 2*(a^5 - 8*a^3*b^2)*c^3 + (9*a^4*b^2 - 20*a^2*b^4)*c^2 - 2*(3*a^3*b^4 - 4*a*b^6)*c + (4*a*c^9 + (8*a^2 - b^2)*c^8 + 2*(2*a^3 - 3*a*b^2)*c^7 - (a^2*b^2 - b^4)*c^6)*\sqrt{-(a^4*b^10 - 2*a^2*b^12 + b^14 + 16*a^6*b^2*c^6 + 8*(3*a^7*b^2 - 10*a^5*b^4)*c^5 + (9*a^8*b^2 - 92*a^6*b^4 + 148*a^4*b^6)*c^4 - 4*(6*a^7*b^4 - 31*a^5*b^6 + 32*a^3*b^8)*c^3 + 2*(11*a^6*b^6 - 37*a^4*b^8 + 28*a^2*b^10)*c^2 - 4*(2*a^5*b^8 - 5*a^3*b^10 + 3*a*b^12)*c)/(4*a*c^17 + (16*a^2 - b^2)*c^16 + 12*(2*a^3 - a*b^2)*c^15 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^14 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^13 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^12)})) - 2*(a^6*b^6 - a^4*b^8 + 4*a^7*b^2*c^3 + (3*a^8*b^2 - 10*a^6*b^4)*c^2 - 2*(2*a^7*b^4 - 3*a^5*b^6)*c)*\sin(x) + 2*c^2*\cos(x)*\sin(x) - 4*b*c*\cos(x) - 2*(2*b^2 - 2*a*c + c^2)*x)/c^3
\end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.34, size = 1181, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x)+c*sin(x)^2),x)
```

```
[Out] 12/c^2*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b-4/c^3*a/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3-16/c*a^3/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))+20/c^2*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*b^2-4/c^3*a/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*b^4+12/c^2*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b-4/c^3*a/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3+16/c*a^3/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))-20/c^2*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2+4/c^3*a/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*b^4+1/c/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/c^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2*b-1/c/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/c^2/(tan(1/2*x)^2+1)^2*b-2/c^2*arctan(tan(1/2*x))*a+2/c^3*arctan(tan(1/2*x))*b^2+1/2*x/c
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [B] time = 26.41, size = 39682, normalized size = 122.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + c*sin(x)^2 + b*sin(x)),x)
```

```
[Out] ((2*b)/c^2 - tan(x/2)/c + tan(x/2)^3/c + (2*b*tan(x/2)^2)/c^2)/(2*tan(x/2)^2 + tan(x/2)^4 + 1) - atan((((2048*(44*a^5*c^9 - 16*a^4*c^10 - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^10*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 - 128*a^2*b^8*c^4 - 8*a^2*b^10*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6*c^3 + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5))/c^8 - ((a^2*b^8 - b^10 + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-4*a*c - b^2)^3)^(1/2) - 10*a^3*b^6*c + a^2*b^5*(-4*a*c - b^2)^3)^(1/2) - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-4*a*c - b^2)^3)^(1/2) - 4*a^3*b^3*c*(-4*a*c - b^2)^3)^(1/2) + 3*a^4*b*c^2*(-4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^2*(-4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c*(-4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^10 + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c
```

$$\begin{aligned}
& \left( a^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7 \right)^{1/2} \left( (2048(4a^3b^3c^{11} + 13a^4b^5c^9 + 4a^5b^7c^7 - 12a^6b^9c^5 - 16a^2b^3c^{12} + 44a^3b^4c^{11} + 4a^4b^5c^{10} + 80a^5b^6c^9 + 12a^6b^7c^8 - 63a^2b^3c^{10} - 16a^2b^5c^8 + 76a^2b^7c^6 - a^3b^3c^9 - 104a^3b^5c^7 + 12a^3b^7c^5 - 56a^4b^3c^8 - 60a^4b^5c^6 + 48a^5b^3c^7)) / c^8 - \left( (2048(12a^5b^5c^{11} - 16a^6b^3c^{13} + 64a^2b^3c^{14} + 80a^3b^3c^{13} + 48a^4b^3c^{12} - 68a^2b^3c^{12} - 12a^3b^3c^{11})) / c^8 + (2048 \tan(x/2) (256a^2c^{15} + 576a^3c^{14} + 416a^4c^{13} + 96a^5c^{12} - 64a^6b^2c^{14} + 68a^6b^4c^{12} - 8a^6b^6c^{10} - 416a^2b^2c^{13} + 72a^2b^4c^{11} - 264a^3b^2c^{12} + 8a^3b^4c^{10} - 56a^4b^2c^{11})) / c^8 \right) \left( -(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-4ac - b^2)^3 \right)^{1/2} \\
& - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12a^6b^8c + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} + 3a^4b^3c^2(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6a^2b^5c^2(-4ac - b^2)^3)^{1/2} \left( (2*(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a^2b^2c^9 + 10a^2b^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)) \right)^{1/2} \\
& - (2048(32a^3c^{13} + 64a^4c^{12} - 16a^5c^{11} - 48a^6c^{10} + 2a^2b^4c^{11} - 14a^2b^6c^9 - 16a^2b^2c^{12} + 96a^2b^4c^{10} + 8a^2b^6c^8 - 176a^3b^2c^{11} - 46a^3b^4c^9 + 60a^4b^2c^{10} - 8a^4b^4c^8 + 44a^5b^2c^9)) / c^8 + (2048 \tan(x/2) (32a^5b^5c^{10} - 16a^6b^7c^8 + 256a^3b^3c^{12} + 320a^4b^3c^{11} + 128a^5b^3c^{10} - 192a^2b^3c^{11} + 128a^2b^5c^9 - 336a^3b^3c^{10} + 16a^3b^5c^8 - 96a^4b^3c^9)) / c^8 \right) \left( -(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-4ac - b^2)^3 \right)^{1/2} \\
& - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12a^6b^8c + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} + 3a^4b^3c^2(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6a^2b^5c^2(-4ac - b^2)^3)^{1/2} \left( (2*(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a^2b^2c^9 + 10a^2b^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)) \right)^{1/2} \\
& + (2048 \tan(x/2) (128a^3c^{12} - 64a^2c^{13} + 184a^4c^{11} - 296a^5c^{10} - 352a^6c^9 - 72a^7c^8 + 16a^2b^2c^{12} + 48a^2b^4c^{10} + a^2b^6c^8 - 92a^2b^8c^6 + 8a^2b^{10}c^4 - 224a^2b^2c^{11} + 56a^2b^4c^9 + 732a^2b^6c^7 - 88a^2b^8c^5 - 286a^3b^2c^{10} - 1817a^3b^4c^8 + 440a^3b^6c^6 - 8a^3b^8c^4 + 1502a^4b^2c^9 - 1140a^4b^4c^7 + 72a^4b^6c^5 + 1208a^5b^2c^8 - 220a^5b^4c^6 + 256a^6b^2c^7)) / c^8 + (2048 \tan(x/2) (8a^5b^5c^8 + 28a^6b^7c^6 + 16a^6b^9c^4 - 16a^6b^{11}c^2 + 64a^3b^3c^{10} - 176a^4b^3c^9 - 32a^5b^3c^8 + 128a^6b^3c^7 + 112a^7b^3c^6 - 48a^2b^3c^9 - 192a^2b^5c^7 - 112a^2b^7c^5 + 160a^2b^9c^3 + 364a^3b^3c^8 + 212a^3b^5c^6 - 592a^3b^7c^4 + 16a^3b^9c^2 - 72a^4b^3c^7 + 1008a^4b^5c^5 - 128a^4b^7c^3 - 720a^5b^3c^6 + 336a^5b^5c^4 - 352a^6b^3c^5)) / c^8 \right) \left( -(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-4ac - b^2)^3 \right)^{1/2} \\
& - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12a^6b^8c + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} + 3a^4b^3c^2(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6a^2b^5c^2(-4ac - b^2)^3)^{1/2} \left( (2*(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a^2b^2c^9 + 10a^2b^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)) \right)^{1/2} \\
& + (2048(16a^2b^{11} - 12a^4b^9 - 144a^3b^9c - 28a^5b^3c^7 + 84a^5b^7c + 97a^6b^3c^6 - 52a^7b^3c^5 - 60a^8b^3c^4 + 4a^2b^7c^4 + 16a^2b^9c^2 - 28a^3b^5c^5 - 128a^3b^7c^3 + 56a^4b^3c^6 + 333a^4b^5c^4 + 452a^4b^7c^2 - 321a^5b^3c^5 - 600a^5b^5c^3 + 328a^6b^3c^4 - 192a^6b^5c^2 + 180a^7b^3c^3)) / c^8 + (2048 \tan(x/2) (32a^6b^{12} - 32a^3b^{10} + 4a^5b^8 + 16a^5c^8 - 48a^6c^7 + 2a^7c^6 + 56a^8c^5 + 12a^9c^4 + 8a^6b^8c^4 + 32a^6b^{10}c^2 - 320a^2b^{10}c + 256a^4b^8c - 24a^6b^6c - 64a^2b^6c^5 - 288a^2b^8c^3 + 160a^3b^4c^6 + 888a^3b^6c^4 + 1152a^3b^8c^2 - 128a^4b^2c^7 - 1104a^4b^4c^5 - 1824a^4b^6c^3 + 504a^5b^2c^6 + 1249a^5b^4c^4 - 700a^5b^6c^2
\end{aligned}$$

$$\begin{aligned}
& - 292a^6b^2c^5 + 812a^6b^4c^3 - 392a^7b^2c^4 + 44a^7b^4c^2 - 32 \\
& *a^8b^2c^3)/c^8)*(-a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7*(-4a* \\
& c - b^2)^3)^{(1/2)} - 10a^3b^6c + a^2b^5*(-(4a*c - b^2)^3)^{(1/2)} - 52a^ \\
& 2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c \\
& ^3 + 12a*b^8c + 4a^3b*c^3*(-(4a*c - b^2)^3)^{(1/2)} - 4a^3b^3c*(-(4a \\
& *c - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4a*c - b^2)^3)^{(1/2)} - 10a^2b^3c^2* \\
& (-4a*c - b^2)^3)^{(1/2)} + 6a*b^5c*(-(4a*c - b^2)^3)^{(1/2))}/(2*(16a^2c \\
& ^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a*b^2c^9 + 10a*b^4* \\
& c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)))^{(1/2)}*i + ((2048*(16 \\
& a^2b^{11} - 12a^4b^9 - 144a^3b^9c - 28a^5b*c^7 + 84a^5b^7c + 97a \\
& ^6b*c^6 - 52a^7b*c^5 - 60a^8b*c^4 + 4a^2b^7c^4 + 16a^2b^9c^2 - 2 \\
& 8a^3b^5c^5 - 128a^3b^7c^3 + 56a^4b^3c^6 + 333a^4b^5c^4 + 452a^ \\
& 4b^7c^2 - 321a^5b^3c^5 - 600a^5b^5c^3 + 328a^6b^3c^4 - 192a^6b \\
& ^5c^2 + 180a^7b^3c^3))/c^8 - ((2048*(44a^5c^9 - 16a^4c^{10} - 4a^6c \\
& ^8 - 64a^7c^7 + 12a^8c^6 + 4a*b^6c^7 + 15a*b^8c^5 + 14a*b^{10}c^3 - \\
& 28a^2b^4c^8 - 119a^2b^6c^6 - 128a^2b^8c^4 - 8a^2b^{10}c^2 + 52a \\
& ^3b^2c^9 + 290a^3b^4c^7 + 397a^3b^6c^5 + 62a^3b^8c^3 - 227a^4b \\
& ^2c^8 - 491a^4b^4c^6 - 148a^4b^6c^4 + 8a^4b^8c^2 + 221a^5b^2c^ \\
& 7 + 102a^5b^4c^5 - 60a^5b^6c^3 + 68a^6b^2c^6 + 136a^6b^4c^4 - 1 \\
& 00a^7b^2c^5))/c^8 + (-a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7*(-4 \\
& *a*c - b^2)^3)^{(1/2)} - 10a^3b^6c + a^2b^5*(-(4a*c - b^2)^3)^{(1/2)} - 52 \\
& a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2 \\
& c^3 + 12a*b^8c + 4a^3b*c^3*(-(4a*c - b^2)^3)^{(1/2)} - 4a^3b^3c*(-( \\
& 4a*c - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4a*c - b^2)^3)^{(1/2)} - 10a^2b^3c \\
& ^2*(-(4a*c - b^2)^3)^{(1/2)} + 6a*b^5c*(-(4a*c - b^2)^3)^{(1/2))}/(2*(16a^ \\
& 2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a*b^2c^9 + 10a*b \\
& ^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)))^{(1/2)}*((2048*(4a* \\
& b^3c^{11} + 13a*b^5c^9 + 4a*b^7c^7 - 12a*b^9c^5 - 16a^2b*c^{12} + 44a \\
& ^3b*c^{11} + 4a^4b*c^{10} + 80a^5b*c^9 + 12a^6b*c^8 - 63a^2b^3c^{10} - \\
& 16a^2b^5c^8 + 76a^2b^7c^6 - a^3b^3c^9 - 104a^3b^5c^7 + 12a^3b^ \\
& 7c^5 - 56a^4b^3c^8 - 60a^4b^5c^6 + 48a^5b^3c^7))/c^8 - ((2048*(32 \\
& a^3c^{13} + 64a^4c^{12} - 16a^5c^{11} - 48a^6c^{10} + 2a*b^4c^{11} - 14a*b \\
& ^6c^9 - 16a^2b^2c^{12} + 96a^2b^4c^{10} + 8a^2b^6c^8 - 176a^3b^2c^ \\
& 11 - 46a^3b^4c^9 + 60a^4b^2c^{10} - 8a^4b^4c^8 + 44a^5b^2c^9))/c^ \\
& 8 + ((2048*(12a*b^5c^{11} - 16a*b^3c^{13} + 64a^2b*c^{14} + 80a^3b*c^{13} + \\
& 48a^4b*c^{12} - 68a^2b^3c^{12} - 12a^3b^3c^{11}))/c^8 + (2048*\tan(x/2)* \\
& (256a^2c^{15} + 576a^3c^{14} + 416a^4c^{13} + 96a^5c^{12} - 64a*b^2c^{14} + \\
& 68a*b^4c^{12} - 8a*b^6c^{10} - 416a^2b^2c^{13} + 72a^2b^4c^{11} - 264a^3 \\
& *b^2c^{12} + 8a^3b^4c^{10} - 56a^4b^2c^{11}))/c^8)*(-a^2b^8 - b^{10} + 8a \\
& ^5c^5 + 8a^6c^4 - b^7*(-(4a*c - b^2)^3)^{(1/2)} - 10a^3b^6c + a^2b^5* \\
& (-4a*c - b^2)^3)^{(1/2)} - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 \\
& + 33a^4b^4c^2 - 38a^5b^2c^3 + 12a*b^8c + 4a^3b*c^3*(-(4a*c - b^ \\
& 2)^3)^{(1/2)} - 4a^3b^3c*(-(4a*c - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4a*c - \\
& b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4a*c - b^2)^3)^{(1/2)} + 6a*b^5c*(-(4a \\
& *c - b^2)^3)^{(1/2))}/(2*(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b \\
& ^6c^6 - 8a*b^2c^9 + 10a*b^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3* \\
& b^2c^7)))^{(1/2)} - (2048*\tan(x/2)*(32a*b^5c^{10} - 16a*b^7c^8 + 256a^3b \\
& *c^{12} + 320a^4b*c^{11} + 128a^5b*c^{10} - 192a^2b^3c^{11} + 128a^2b^5c^ \\
& 9 - 336a^3b^3c^{10} + 16a^3b^5c^8 - 96a^4b^3c^9))/c^8)*(-a^2b^8 - \\
& b^{10} + 8a^5c^5 + 8a^6c^4 - b^7*(-(4a*c - b^2)^3)^{(1/2)} - 10a^3b^6c \\
& + a^2b^5*(-(4a*c - b^2)^3)^{(1/2)} - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a \\
& ^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12a*b^8c + 4a^3b*c^3*(-( \\
& 4a*c - b^2)^3)^{(1/2)} - 4a^3b^3c*(-(4a*c - b^2)^3)^{(1/2)} + 3a^4b*c^2* \\
& (-4a*c - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4a*c - b^2)^3)^{(1/2)} + 6a*b^ \\
& 5c*(-(4a*c - b^2)^3)^{(1/2))}/(2*(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b \\
& ^4c^8 - b^6c^6 - 8a*b^2c^9 + 10a*b^4c^7 - 32a^2b^2c^8 + a^2b^4c^ \\
& 6 - 8a^3b^2c^7)))^{(1/2)} + (2048*\tan(x/2)*(128a^3c^{12} - 64a^2c^{13} + 1 \\
& 84a^4c^{11} - 296a^5c^{10} - 352a^6c^9 - 72a^7c^8 + 16a*b^2c^{12} + 48* \\
& a*b^4c^{10} + a*b^6c^8 - 92a*b^8c^6 + 8a*b^{10}c^4 - 224a^2b^2c^{11} + 5
\end{aligned}$$

$$\begin{aligned}
 & 6*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8*c^5 - 286*a^3*b^2*c^{10} - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 + 1502*a^4*b^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 - 220*a^5*b^4*c^6 + 256*a^6*b^2*c^7)/c^8) + (2048*\tan(x/2)*(8*a*b^5*c^8 + 28*a*b^7*c^6 + 16*a*b^9*c^4 - 16*a*b^{11}*c^2 + 64*a^3*b*c^{10} - 176*a^4*b*c^9 - 32*a^5*b*c^8 + 128*a^6*b*c^7 + 112*a^7*b*c^6 - 48*a^2*b^3*c^9 - 192*a^2*b^5*c^7 - 112*a^2*b^7*c^5 + 160*a^2*b^9*c^3 + 364*a^3*b^3*c^8 + 212*a^3*b^5*c^6 - 592*a^3*b^7*c^4 + 16*a^3*b^9*c^2 - 72*a^4*b^3*c^7 + 1008*a^4*b^5*c^5 - 128*a^4*b^7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 352*a^6*b^3*c^5))/c^8)*(-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} + (2048*\tan(x/2)*(32*a*b^{12} - 32*a^3*b^{10} + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^{10}*c^2 - 320*a^2*b^{10}*c + 256*a^4*b^8*c - 24*a^6*b^6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3))/c^8)*(-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)}*i)/((4096*(16*a^6*b^6 - 4*a^8*b^4 - 4*a^7*c^5 + 15*a^8*c^4 - 14*a^9*c^3 - 48*a^7*b^4*c + 4*a^9*b^2*c + 4*a^6*b^2*c^4 + 16*a^6*b^4*c^2 - 32*a^7*b^2*c^3 + 44*a^8*b^2*c^2))/c^8 + (((2048*(44*a^5*c^9 - 16*a^4*c^{10} - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^{10}*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 - 128*a^2*b^8*c^4 - 8*a^2*b^{10}*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6*c^3 + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5))/c^8 - (-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)}*((2048*(4*a*b^3*c^{11} + 13*a*b^5*c^9 + 4*a*b^7*c^7 - 12*a*b^9*c^5 - 16*a^2*b*c^{12} + 44*a^3*b*c^{11} + 4*a^4*b*c^{10} + 80*a^5*b*c^9 + 12*a^6*b*c^8 - 63*a^2*b^3*c^{10} - 16*a^2*b^5*c^8 + 76*a^2*b^7*c^6 - a^3*b^3*c^9 - 104*a^3*b^5*c^7 + 12*a^3*b^7*c^5 - 56*a^4*b^3*c^8 - 60*a^4*b^5*c^6 + 48*a^5*b^3*c^7))/c^8 - (((2048*(12*a*b^5*c^{11} - 16*a*b^3*c^{13} + 64*a^2*b*c^{14} + 80*a^3*b*c^{13} + 48*a^4*b*c^{12} - 68*a^2*b^3*c^{12} - 12*a^3*b^3*c^{11}))/c^8 + (2048*\tan(x/2)*(256*a^2*c^{15} + 576*a^3*c^{14} + 416*a^4*c^{13} + 96*a^5*c^{12} - 64*a*b^2*c^{14} + 68*a*b^4*c^{12} - 8*a*b^6*c^{10} - 416*a^2*b^2*c^{13} + 72*a^2*b^4*c^{11} - 264*a^3*b^2*c^{12} + 8*a^3*b^4*c^{10} - 56*a^4*b^2*c^{11}))/c^8)*(-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a
 \end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(- \\
& (4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 \\
& - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8* \\
& a^3*b^2*c^7)))^{(1/2)} - (2048*(32*a^3*c^{13} + 64*a^4*c^{12} - 16*a^5*c^{11} - 48* \\
& a^6*c^{10} + 2*a*b^4*c^{11} - 14*a*b^6*c^9 - 16*a^2*b^2*c^{12} + 96*a^2*b^4*c^{10} \\
& + 8*a^2*b^6*c^8 - 176*a^3*b^2*c^{11} - 46*a^3*b^4*c^9 + 60*a^4*b^2*c^{10} - 8*a \\
& ^4*b^4*c^8 + 44*a^5*b^2*c^9))/c^8 + (2048*\tan(x/2)*(32*a*b^5*c^{10} - 16*a*b^ \\
& 7*c^8 + 256*a^3*b*c^{12} + 320*a^4*b*c^{11} + 128*a^5*b*c^{10} - 192*a^2*b^3*c^{11} \\
& + 128*a^2*b^5*c^9 - 336*a^3*b^3*c^{10} + 16*a^3*b^5*c^8 - 96*a^4*b^3*c^9))/c \\
& ^8)*(-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} \\
& ) - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a \\
& ^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c \\
& + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 \\
& + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2 \\
& *c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} + (2048*\tan(x/2)*(128*a^3*c^{12} \\
& - 64*a^2*c^{13} + 184*a^4*c^{11} - 296*a^5*c^{10} - 352*a^6*c^9 - 72*a^7*c^8 + 16 \\
& *a*b^2*c^{12} + 48*a*b^4*c^{10} + a*b^6*c^8 - 92*a*b^8*c^6 + 8*a*b^{10}*c^4 - 224 \\
& *a^2*b^2*c^{11} + 56*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8*c^5 - 286*a^3 \\
& *b^2*c^{10} - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 + 1502*a^4*b \\
& ^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 - 220*a^5*b^4 \\
& *c^6 + 256*a^6*b^2*c^7))/c^8) + (2048*\tan(x/2)*(8*a*b^5*c^8 + 28*a*b^7*c^6 \\
& + 16*a*b^9*c^4 - 16*a*b^{11}*c^2 + 64*a^3*b*c^{10} - 176*a^4*b*c^9 - 32*a^5*b*c \\
& ^8 + 128*a^6*b*c^7 + 112*a^7*b*c^6 - 48*a^2*b^3*c^9 - 192*a^2*b^5*c^7 - 112 \\
& *a^2*b^7*c^5 + 160*a^2*b^9*c^3 + 364*a^3*b^3*c^8 + 212*a^3*b^5*c^6 - 592*a^ \\
& 3*b^7*c^4 + 16*a^3*b^9*c^2 - 72*a^4*b^3*c^7 + 1008*a^4*b^5*c^5 - 128*a^4*b^ \\
& 7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 352*a^6*b^3*c^5))/c^8)*(-(a^2*b \\
& ^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b \\
& ^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - \\
& 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^ \\
& 3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b \\
& *c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6 \\
& *a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^ \\
& 8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b \\
& ^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} + (2048*(16*a^2*b^{11} - 12*a^4*b^9 - 144*a^3 \\
& *b^9*c - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8 \\
& *b*c^4 + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 \\
& + 56*a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 60 \\
& 0*a^5*b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3))/c^8 + \\
& (2048*\tan(x/2)*(32*a*b^{12} - 32*a^3*b^{10} + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6* \\
& c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^{10}*c^2 - 3 \\
& 20*a^2*b^{10}*c + 256*a^4*b^8*c - 24*a^6*b^6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8 \\
& *c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c \\
& ^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c \\
& ^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 \\
& + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3))/c^8)*(-(a^2*b^8 - b^{10} + 8*a^5*c^5 + 8* \\
& a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b \\
& ^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3 \\
& )^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8* \\
& a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{( \\
& 1/2)} - ((2048*(16*a^2*b^{11} - 12*a^4*b^9 - 144*a^3*b^9*c - 28*a^5*b*c^7 + 8 \\
& 4*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8*b*c^4 + 4*a^2*b^7*c^4 + \\
& 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 + 56*a^4*b^3*c^6 + 333*a^ \\
& 4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 600*a^5*b^5*c^3 + 328*a^6*b \\
& ^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3))/c^8 - ((2048*(44*a^5*c^9 - 16*
\end{aligned}$$

$$\begin{aligned}
& a^4c^{10} - 4a^6c^8 - 64a^7c^7 + 12a^8c^6 + 4ab^6c^7 + 15ab^8c^5 \\
& + 14ab^{10}c^3 - 28a^2b^4c^8 - 119a^2b^6c^6 - 128a^2b^8c^4 - 8a^2b^{10}c^2 + 52a^3b^2c^9 + 290a^3b^4c^7 + 397a^3b^6c^5 + 62a^3b^8c^3 \\
& - 227a^4b^2c^8 - 491a^4b^4c^6 - 148a^4b^6c^4 + 8a^4b^8c^2 + 221a^5b^2c^7 + 102a^5b^4c^5 - 60a^5b^6c^3 + 68a^6b^2c^6 + 136a^6b^4c^4 \\
& - 100a^7b^2c^5)/c^8 + (-(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-(4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-(4ac - b^2)^3)^{1/2} \\
& - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12ab^8c + 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} \\
& - 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} + 3a^4b^2c^2(-(4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-(4ac - b^2)^3)^{1/2} + 6ab^5c^3(-(4ac - b^2)^3)^{1/2} \\
& )/(2(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8ab^2c^9 + 10ab^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)))^{1/2} \\
& *((2048(4ab^3c^{11} + 13ab^5c^9 + 4ab^7c^7 - 12ab^9c^5 - 16a^2b^3c^{12} + 44a^3b^3c^{11} + 4a^4b^3c^{10} + 80a^5b^3c^9 + 12a^6b^3c^8 - 63a^2b^3c^{10} \\
& - 16a^2b^5c^8 + 76a^2b^7c^6 - a^3b^3c^9 - 104a^3b^5c^7 + 12a^3b^7c^5 - 56a^4b^3c^8 - 60a^4b^5c^6 + 48a^5b^3c^7)/c^8 - ((2048(32a^3c^{13} + 64a^4c^{12} - 16a^5c^{11} - 48a^6c^{10} + 2ab^4c^{11} \\
& - 14ab^6c^9 - 16a^2b^2c^{12} + 96a^2b^4c^{10} + 8a^2b^6c^8 - 176a^3b^2c^{11} - 46a^3b^4c^9 + 60a^4b^2c^{10} - 8a^4b^4c^8 + 44a^5b^2c^9))/c^8 + ((2048(12ab^5c^{11} - 16ab^3c^{13} + 64a^2b^3c^{14} \\
& + 80a^3b^3c^{13} + 48a^4b^3c^{12} - 68a^2b^3c^{12} - 12a^3b^3c^{11}))/c^8 + (2048\tan(x/2)(256a^2c^{15} + 576a^3c^{14} + 416a^4c^{13} + 96a^5c^{12} - 64ab^2c^{14} \\
& + 68ab^4c^{12} - 8ab^6c^{10} - 416a^2b^2c^{13} + 72a^2b^4c^{11} - 264a^3b^2c^{12} + 8a^3b^4c^{10} - 56a^4b^2c^{11}))/c^8)*(-(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-(4ac - b^2)^3)^{1/2} - 10a^3b^6c \\
& + a^2b^5(-(4ac - b^2)^3)^{1/2} - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12ab^8c + 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} \\
& + 3a^4b^2c^2(-(4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-(4ac - b^2)^3)^{1/2} + 6ab^5c^3(-(4ac - b^2)^3)^{1/2}))/2(16a^2c^{10} + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8ab^2c^9 + 10ab^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 \\
& - 8a^3b^2c^7)))^{1/2} - (2048\tan(x/2)(32ab^5c^{10} - 16ab^7c^8 + 256a^3b^3c^{12} + 320a^4b^3c^{11} + 128a^5b^3c^{10} - 192a^2b^3c^{11} + 128a^2b^5c^9 - 336a^3b^3c^{10} + 16a^3b^5c^8 - 96a^4b^3c^9))/c^8 \\
& *(-(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-(4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-(4ac - b^2)^3)^{1/2} - 52a^2b^6c^2 + 96a^3b^4c^3 - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12ab^8c \\
& + 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} + 3a^4b^2c^2(-(4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-(4ac - b^2)^3)^{1/2} + 6ab^5c^3(-(4ac - b^2)^3)^{1/2}))/2(16a^2c^{10} + 32a^3c^9 \\
& + 16a^4c^8 + b^4c^8 - b^6c^6 - 8ab^2c^9 + 10ab^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)))^{1/2} + (2048\tan(x/2)(128a^3c^{12} - 64a^2c^{13} + 184a^4c^{11} - 296a^5c^{10} - 352a^6c^9 - 72a^7c^8 + 16ab^2c^{12} \\
& + 48ab^4c^{10} + ab^6c^8 - 92ab^8c^6 + 8ab^{10}c^4 - 224a^2b^2c^{11} + 56a^2b^4c^9 + 732a^2b^6c^7 - 88a^2b^8c^5 - 286a^3b^2c^{10} - 1817a^3b^4c^8 + 440a^3b^6c^6 - 8a^3b^8c^4 + 1502a^4b^2c^9 \\
& - 1140a^4b^4c^7 + 72a^4b^6c^5 + 1208a^5b^2c^8 - 220a^5b^4c^6 + 256a^6b^2c^7))/c^8) + (2048\tan(x/2)(8ab^5c^8 + 28ab^7c^6 + 16ab^9c^4 - 16ab^{11}c^2 + 64a^3b^3c^{10} - 176a^4b^3c^9 - 32a^5b^3c^8 \\
& + 128a^6b^3c^7 + 112a^7b^3c^6 - 48a^2b^3c^9 - 192a^2b^5c^7 - 112a^2b^7c^5 + 160a^2b^9c^3 + 364a^3b^3c^8 + 212a^3b^5c^6 - 592a^3b^7c^4 + 16a^3b^9c^2 - 72a^4b^3c^7 + 1008a^4b^5c^5 - 128a^4b^7c^3 \\
& - 720a^5b^3c^6 + 336a^5b^5c^4 - 352a^6b^3c^5))/c^8)*(-(a^2b^8 - b^{10} + 8a^5c^5 + 8a^6c^4 - b^7(-(4ac - b^2)^3)^{1/2} - 10a^3b^6c + a^2b^5(-(4ac - b^2)^3)^{1/2} - 52a^2b^6c^2 + 96a^3b^4c^3 \\
& - 66a^4b^2c^4 + 33a^4b^4c^2 - 38a^5b^2c^3 + 12ab^8c + 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} - 4a^3b^3c^3(-(4ac - b^2)^3)^{1/2} + 3a^4b^2c^2(-(4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-(4ac - b^2)^3)^{1/2} +
\end{aligned}$$



$$\begin{aligned}
& 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 \\
& + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} + (2048*\tan(x/2)*(32*a*b^{12} - 32*a^3*b^{10} \\
& + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^{10}*c^2 - 320*a^2*b^{10}*c + 256*a^4*b^8*c - 24*a^6*b^6*c \\
& - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 \\
& + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3))/c^8)* \\
& (-a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^6*c^2 + 96*a^3*b^4*c^3 \\
& - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 1 \\
& 6*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} + (4096*\tan(x/2)*(32*a^5*b^7 - 16*a^7*b^5 \\
& - 16*a^6*b*c^5 - 128*a^6*b^5*c + 60*a^7*b*c^4 - 48*a^8*b*c^3 + 32*a^8*b^3*c - 16*a^9*b*c^2 + 8*a^5*b^3*c^4 + 32*a^5*b^5*c^2 - 96*a^6*b^3*c^3 + \\
& 144*a^7*b^3*c^2))/c^8))*(-a^2*b^8 - b^{10} + 8*a^5*c^5 + 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 5 \\
& 2*a^2*b^6*c^2 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + 33*a^4*b^4*c^2 - 38*a^5*b^2*c^3 + 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4 \\
& a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a \\
& ^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)}*2i - \operatorname{atan}((( \\
& ((2048*(44*a^5*c^9 - 16*a^4*c^{10} - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^{10}*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 \\
& - 128*a^2*b^8*c^4 - 8*a^2*b^{10}*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6 \\
& *c^3 + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5))/c^8 - ((2048*(4*a*b^3*c^{11} + 13*a*b^5*c^9 + 4*a*b^7*c^7 - 12*a*b^9*c^5 - 16*a^2*b*c^{12} + 4 \\
& 4*a^3*b*c^{11} + 4*a^4*b*c^{10} + 80*a^5*b*c^9 + 12*a^6*b*c^8 - 63*a^2*b^3*c^{10} - 16*a^2*b^5*c^8 + 76*a^2*b^7*c^6 - a^3*b^3*c^9 - 104*a^3*b^5*c^7 + 12*a^3 \\
& *b^7*c^5 - 56*a^4*b^3*c^8 - 60*a^4*b^5*c^6 + 48*a^5*b^3*c^7))/c^8 - (((2048 \\
& *(12*a*b^5*c^{11} - 16*a*b^3*c^{13} + 64*a^2*b*c^{14} + 80*a^3*b*c^{13} + 48*a^4*b*c^{12} - 68*a^2*b^3*c^{12} - 12*a^3*b^3*c^{11}))/c^8 + (2048*\tan(x/2)*(256*a^2*c^{15} \\
& + 576*a^3*c^{14} + 416*a^4*c^{13} + 96*a^5*c^{12} - 64*a*b^2*c^{14} + 68*a*b^4*c^{12} - 8*a*b^6*c^{10} - 416*a^2*b^2*c^{13} + 72*a^2*b^4*c^{11} - 264*a^3*b^2*c^{12} \\
& + 8*a^3*b^4*c^{10} - 56*a^4*b^2*c^{11}))/c^8)*((b^{10} - a^2*b^8 - 8*a^5*c^5 - 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c + a^2*b^5*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3 \\
& )^{(1/2)})/(2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7)))^{(1/2)} - (2048*(32*a^3*c^{13} + 64*a^4*c^{12} - 16*a^5*c^{11} - 48*a^6*c^{10} + 2*a*b^4*c^{11} - 14*a*b^6*c^9 - 16*a^2*b^2*c^{12} + 96*a^2*b^4*c^{10} + 8*a^2*b^6*c^8 \\
& - 176*a^3*b^2*c^{11} - 46*a^3*b^4*c^9 + 60*a^4*b^2*c^{10} - 8*a^4*b^4*c^8 + 44*a^5*b^2*c^9))/c^8 + (2048*\tan(x/2)*(32*a*b^5*c^{10} - 16*a*b^7*c^8 + 256*a^3 \\
& *b*c^{12} + 320*a^4*b*c^{11} + 128*a^5*b*c^{10} - 192*a^2*b^3*c^{11} + 128*a^2*b^5*c^9 - 336*a^3*b^3*c^{10} + 16*a^3*b^5*c^8 - 96*a^4*b^3*c^9))/c^8)*((b^{10} - a^2*b^8 - 8*a^5*c^5 - 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c \\
& + a^2*b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(-(4 \\
& a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2
\end{aligned}$$

$$\begin{aligned}
& *(- (4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(- (4*a*c - b^2)^3)^{(1/2)} / (2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7))^{(1/2)} + (2048*\tan(x/2)*(128*a^3*c^{12} - 64*a^2*c^{13} + 184*a^4*c^{11} - 296*a^5*c^{10} - 352*a^6*c^9 - 72*a^7*c^8 + 16*a*b^2*c^{12} + 48*a*b^4*c^{10} + a*b^6*c^8 - 92*a*b^8*c^6 + 8*a*b^{10}*c^4 - 224*a^2*b^2*c^{11} + 56*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8*c^5 - 286*a^3*b^2*c^{10} - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 + 1502*a^4*b^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 - 220*a^5*b^4*c^6 + 256*a^6*b^2*c^7)) / c^8 * ((b^{10} - a^2*b^8 - 8*a^5*c^5 - 8*a^6*c^4 - b^7*(- (4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c + a^2*b^5*(- (4*a*c - b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7))^{(1/2)} + (2048*\tan(x/2)*(8*a*b^5*c^8 + 28*a*b^7*c^6 + 16*a*b^9*c^4 - 16*a*b^{11}*c^2 + 64*a^3*b*c^{10} - 176*a^4*b*c^9 - 32*a^5*b*c^8 + 128*a^6*b*c^7 + 112*a^7*b*c^6 - 48*a^2*b^3*c^9 - 192*a^2*b^5*c^7 - 112*a^2*b^7*c^5 + 160*a^2*b^9*c^3 + 364*a^3*b^3*c^8 + 212*a^3*b^5*c^6 - 592*a^3*b^7*c^4 + 16*a^3*b^9*c^2 - 72*a^4*b^3*c^7 + 1008*a^4*b^5*c^5 - 128*a^4*b^7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 352*a^6*b^3*c^5)) / c^8 * ((b^{10} - a^2*b^8 - 8*a^5*c^5 - 8*a^6*c^4 - b^7*(- (4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c + a^2*b^5*(- (4*a*c - b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7))^{(1/2)} + (2048*(16*a^2*b^{11} - 12*a^4*b^9 - 144*a^3*b^9*c - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8*b*c^4 + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 + 56*a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 600*a^5*b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3)) / c^8 + (2048*\tan(x/2)*(32*a*b^{12} - 32*a^3*b^{10} + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^{10}*c^2 - 320*a^2*b^{10}*c + 256*a^4*b^8*c - 24*a^6*b^6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3)) / c^8 * ((b^{10} - a^2*b^8 - 8*a^5*c^5 - 8*a^6*c^4 - b^7*(- (4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c + a^2*b^5*(- (4*a*c - b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^{10} + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6*c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^2*c^7))^{(1/2)} * i + ((2048*(16*a^2*b^{11} - 12*a^4*b^9 - 144*a^3*b^9*c - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8*b*c^4 + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 + 56*a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 600*a^5*b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3)) / c^8 - ((2048*(44*a^5*c^9 - 16*a^4*c^{10} - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^{10}*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 - 128*a^2*b^8*c^4 - 8*a^2*b^{10}*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6*c^3 + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5)) / c^8 + ((2048*(4*
\end{aligned}$$

$$\begin{aligned}
& a^3b^3c^{11} + 13a^4b^5c^9 + 4a^5b^7c^7 - 12a^6b^9c^5 - 16a^7b^{11}c^3 + 44a^8b^{13}c \\
& - 16a^9b^{15}c^2 + 4a^{10}b^{17}c - 4a^{11}b^{19}c^2 + 12a^{12}b^{21}c^3 - 16a^{13}b^{23}c^4 + 12a^{14}b^{25}c^5 \\
& - 8a^{15}b^{27}c^6 + 4a^{16}b^{29}c^7 - 2a^{17}b^{31}c^8 + 2a^{18}b^{33}c^9 - 2a^{19}b^{35}c^{10} + 2a^{20}b^{37}c^{11} \\
& - 2a^{21}b^{39}c^{12} + 2a^{22}b^{41}c^{13} - 2a^{23}b^{43}c^{14} + 2a^{24}b^{45}c^{15} - 2a^{25}b^{47}c^{16} + 2a^{26}b^{49}c^{17} \\
& - 2a^{27}b^{51}c^{18} + 2a^{28}b^{53}c^{19} - 2a^{29}b^{55}c^{20} + 2a^{30}b^{57}c^{21} - 2a^{31}b^{59}c^{22} + 2a^{32}b^{61}c^{23} \\
& - 2a^{33}b^{63}c^{24} + 2a^{34}b^{65}c^{25} - 2a^{35}b^{67}c^{26} + 2a^{36}b^{69}c^{27} - 2a^{37}b^{71}c^{28} + 2a^{38}b^{73}c^{29} \\
& - 2a^{39}b^{75}c^{30} + 2a^{40}b^{77}c^{31} - 2a^{41}b^{79}c^{32} + 2a^{42}b^{81}c^{33} - 2a^{43}b^{83}c^{34} + 2a^{44}b^{85}c^{35} \\
& - 2a^{45}b^{87}c^{36} + 2a^{46}b^{89}c^{37} - 2a^{47}b^{91}c^{38} + 2a^{48}b^{93}c^{39} - 2a^{49}b^{95}c^{40} + 2a^{50}b^{97}c^{41} \\
& - 2a^{51}b^{99}c^{42} + 2a^{52}b^{101}c^{43} - 2a^{53}b^{103}c^{44} + 2a^{54}b^{105}c^{45} - 2a^{55}b^{107}c^{46} + 2a^{56}b^{109}c^{47} \\
& - 2a^{57}b^{111}c^{48} + 2a^{58}b^{113}c^{49} - 2a^{59}b^{115}c^{50} + 2a^{60}b^{117}c^{51} - 2a^{61}b^{119}c^{52} + 2a^{62}b^{121}c^{53} \\
& - 2a^{63}b^{123}c^{54} + 2a^{64}b^{125}c^{55} - 2a^{65}b^{127}c^{56} + 2a^{66}b^{129}c^{57} - 2a^{67}b^{131}c^{58} + 2a^{68}b^{133}c^{59} \\
& - 2a^{69}b^{135}c^{60} + 2a^{70}b^{137}c^{61} - 2a^{71}b^{139}c^{62} + 2a^{72}b^{141}c^{63} - 2a^{73}b^{143}c^{64} + 2a^{74}b^{145}c^{65} \\
& - 2a^{75}b^{147}c^{66} + 2a^{76}b^{149}c^{67} - 2a^{77}b^{151}c^{68} + 2a^{78}b^{153}c^{69} - 2a^{79}b^{155}c^{70} + 2a^{80}b^{157}c^{71} \\
& - 2a^{81}b^{159}c^{72} + 2a^{82}b^{161}c^{73} - 2a^{83}b^{163}c^{74} + 2a^{84}b^{165}c^{75} - 2a^{85}b^{167}c^{76} + 2a^{86}b^{169}c^{77} \\
& - 2a^{87}b^{171}c^{78} + 2a^{88}b^{173}c^{79} - 2a^{89}b^{175}c^{80} + 2a^{90}b^{177}c^{81} - 2a^{91}b^{179}c^{82} + 2a^{92}b^{181}c^{83} \\
& - 2a^{93}b^{183}c^{84} + 2a^{94}b^{185}c^{85} - 2a^{95}b^{187}c^{86} + 2a^{96}b^{189}c^{87} - 2a^{97}b^{191}c^{88} + 2a^{98}b^{193}c^{89} \\
& - 2a^{99}b^{195}c^{90} + 2a^{100}b^{197}c^{91} - 2a^{101}b^{199}c^{92} + 2a^{102}b^{201}c^{93} - 2a^{103}b^{203}c^{94} + 2a^{104}b^{205}c^{95} \\
& - 2a^{105}b^{207}c^{96} + 2a^{106}b^{209}c^{97} - 2a^{107}b^{211}c^{98} + 2a^{108}b^{213}c^{99} - 2a^{109}b^{215}c^{100} + 2a^{110}b^{217}c^{101} \\
& - 2a^{111}b^{219}c^{102} + 2a^{112}b^{221}c^{103} - 2a^{113}b^{223}c^{104} + 2a^{114}b^{225}c^{105} - 2a^{115}b^{227}c^{106} + 2a^{116}b^{229}c^{107} \\
& - 2a^{117}b^{231}c^{108} + 2a^{118}b^{233}c^{109} - 2a^{119}b^{235}c^{110} + 2a^{120}b^{237}c^{111} - 2a^{121}b^{239}c^{112} + 2a^{122}b^{241}c^{113} \\
& - 2a^{123}b^{243}c^{114} + 2a^{124}b^{245}c^{115} - 2a^{125}b^{247}c^{116} + 2a^{126}b^{249}c^{117} - 2a^{127}b^{251}c^{118} + 2a^{128}b^{253}c^{119} \\
& - 2a^{129}b^{255}c^{120} + 2a^{130}b^{257}c^{121} - 2a^{131}b^{259}c^{122} + 2a^{132}b^{261}c^{123} - 2a^{133}b^{263}c^{124} + 2a^{134}b^{265}c^{125} \\
& - 2a^{135}b^{267}c^{126} + 2a^{136}b^{269}c^{127} - 2a^{137}b^{271}c^{128} + 2a^{138}b^{273}c^{129} - 2a^{139}b^{275}c^{130} + 2a^{140}b^{277}c^{131} \\
& - 2a^{141}b^{279}c^{132} + 2a^{142}b^{281}c^{133} - 2a^{143}b^{283}c^{134} + 2a^{144}b^{285}c^{135} - 2a^{145}b^{287}c^{136} + 2a^{146}b^{289}c^{137} \\
& - 2a^{147}b^{291}c^{138} + 2a^{148}b^{293}c^{139} - 2a^{149}b^{295}c^{140} + 2a^{150}b^{297}c^{141} - 2a^{151}b^{299}c^{142} + 2a^{152}b^{301}c^{143} \\
& - 2a^{153}b^{303}c^{144} + 2a^{154}b^{305}c^{145} - 2a^{155}b^{307}c^{146} + 2a^{156}b^{309}c^{147} - 2a^{157}b^{311}c^{148} + 2a^{158}b^{313}c^{149} \\
& - 2a^{159}b^{315}c^{150} + 2a^{160}b^{317}c^{151} - 2a^{161}b^{319}c^{152} + 2a^{162}b^{321}c^{153} - 2a^{163}b^{323}c^{154} + 2a^{164}b^{325}c^{155} \\
& - 2a^{165}b^{327}c^{156} + 2a^{166}b^{329}c^{157} - 2a^{167}b^{331}c^{158} + 2a^{168}b^{333}c^{159} - 2a^{169}b^{335}c^{160} + 2a^{170}b^{337}c^{161} \\
& - 2a^{171}b^{339}c^{162} + 2a^{172}b^{341}c^{163} - 2a^{173}b^{343}c^{164} + 2a^{174}b^{345}c^{165} - 2a^{175}b^{347}c^{166} + 2a^{176}b^{349}c^{167} \\
& - 2a^{177}b^{351}c^{168} + 2a^{178}b^{353}c^{169} - 2a^{179}b^{355}c^{170} + 2a^{180}b^{357}c^{171} - 2a^{181}b^{359}c^{172} + 2a^{182}b^{361}c^{173} \\
& - 2a^{183}b^{363}c^{174} + 2a^{184}b^{365}c^{175} - 2a^{185}b^{367}c^{176} + 2a^{186}b^{369}c^{177} - 2a^{187}b^{371}c^{178} + 2a^{188}b^{373}c^{179} \\
& - 2a^{189}b^{375}c^{180} + 2a^{190}b^{377}c^{181} - 2a^{191}b^{379}c^{182} + 2a^{192}b^{381}c^{183} - 2a^{193}b^{383}c^{184} + 2a^{194}b^{385}c^{185} \\
& - 2a^{195}b^{387}c^{186} + 2a^{196}b^{389}c^{187} - 2a^{197}b^{391}c^{188} + 2a^{198}b^{393}c^{189} - 2a^{199}b^{395}c^{190} + 2a^{200}b^{397}c^{191} \\
& - 2a^{201}b^{399}c^{192} + 2a^{202}b^{401}c^{193} - 2a^{203}b^{403}c^{194} + 2a^{204}b^{405}c^{195} - 2a^{205}b^{407}c^{196} + 2a^{206}b^{409}c^{197} \\
& - 2a^{207}b^{411}c^{198} + 2a^{208}b^{413}c^{199} - 2a^{209}b^{415}c^{200} + 2a^{210}b^{417}c^{201} - 2a^{211}b^{419}c^{202} + 2a^{212}b^{421}c^{203} \\
& - 2a^{213}b^{423}c^{204} + 2a^{214}b^{425}c^{205} - 2a^{215}b^{427}c^{206} + 2a^{216}b^{429}c^{207} - 2a^{217}b^{431}c^{208} + 2a^{218}b^{433}c^{209} \\
& - 2a^{219}b^{435}c^{210} + 2a^{220}b^{437}c^{211} - 2a^{221}b^{439}c^{212} + 2a^{222}b^{441}c^{213} - 2a^{223}b^{443}c^{214} + 2a^{224}b^{445}c^{215} \\
& - 2a^{225}b^{447}c^{216} + 2a^{226}b^{449}c^{217} - 2a^{227}b^{451}c^{218} + 2a^{228}b^{453}c^{219} - 2a^{229}b^{455}c^{220} + 2a^{230}b^{457}c^{221} \\
& - 2a^{231}b^{459}c^{222} + 2a^{232}b^{461}c^{223} - 2a^{233}b^{463}c^{224} + 2a^{234}b^{465}c^{225} - 2a^{235}b^{467}c^{226} + 2a^{236}b^{469}c^{227} \\
& - 2a^{237}b^{471}c^{228} + 2a^{238}b^{473}c^{229} - 2a^{239}b^{475}c^{230} + 2a^{240}b^{477}c^{231} - 2a^{241}b^{479}c^{232} + 2a^{242}b^{481}c^{233} \\
& - 2a^{243}b^{483}c^{234} + 2a^{244}b^{485}c^{235} - 2a^{245}b^{487}c^{236} + 2a^{246}b^{489}c^{237} - 2a^{247}b^{491}c^{238} + 2a^{248}b^{493}c^{239} \\
& - 2a^{249}b^{495}c^{240} + 2a^{250}b^{497}c^{241} - 2a^{251}b^{499}c^{242} + 2a^{252}b^{501}c^{243} - 2a^{253}b^{503}c^{244} + 2a^{254}b^{505}c^{245} \\
& - 2a^{255}b^{507}c^{246} + 2a^{256}b^{509}c^{247} - 2a^{257}b^{511}c^{248} + 2a^{258}b^{513}c^{249} - 2a^{259}b^{515}c^{250} + 2a^{260}b^{517}c^{251} \\
& - 2a^{261}b^{519}c^{252} + 2a^{262}b^{521}c^{253} - 2a^{263}b^{523}c^{254} + 2a^{264}b^{525}c^{255} - 2a^{265}b^{527}c^{256} + 2a^{266}b^{529}c^{257} \\
& - 2a^{267}b^{531}c^{258} + 2a^{268}b^{533}c^{259} - 2a^{269}b^{535}c^{260} + 2a^{270}b^{537}c^{261} - 2a^{271}b^{539}c^{262} + 2a^{272}b^{541}c^{263} \\
& - 2a^{273}b^{543}c^{264} + 2a^{274}b^{545}c^{265} - 2a^{275}b^{547}c^{266} + 2a^{276}b^{549}c^{267} - 2a^{277}b^{551}c^{268} + 2a^{278}b^{553}c^{269} \\
& - 2a^{279}b^{555}c^{270} + 2a^{280}b^{557}c^{271} - 2a^{281}b^{559}c^{272} + 2a^{282}b^{561}c^{273} - 2a^{283}b^{563}c^{274} + 2a^{284}b^{565}c^{275} \\
& - 2a^{285}b^{567}c^{276} + 2a^{286}b^{569}c^{277} - 2a^{287}b^{571}c^{278} + 2a^{288}b^{573}c^{279} - 2a^{289}b^{575}c^{280} + 2a^{290}b^{577}c^{281} \\
& - 2a^{291}b^{579}c^{282} + 2a^{292}b^{581}c^{283} - 2a^{293}b^{583}c^{284} + 2a^{294}b^{585}c^{285} - 2a^{295}b^{587}c^{286} + 2a^{296}b^{589}c^{287} \\
& - 2a^{297}b^{591}c^{288} + 2a^{298}b^{593}c^{289} - 2a^{299}b^{595}c^{290} + 2a^{300}b^{597}c^{291} - 2a^{301}b^{599}c^{292} + 2a^{302}b^{601}c^{293} \\
& - 2a^{303}b^{603}c^{294} + 2a^{304}b^{605}c^{295} - 2a^{305}b^{607}c^{296} + 2a^{306}b^{609}c^{297} - 2a^{307}b^{611}c^{298} + 2a^{308}b^{613}c^{299} \\
& - 2a^{309}b^{615}c^{300} + 2a^{310}b^{617}c^{301} - 2a^{311}b^{619}c^{302} + 2a^{312}b^{621}c^{303} - 2a^{313}b^{623}c^{304} + 2a^{314}b^{625}c^{305} \\
& - 2a^{315}b^{627}c^{306} + 2a^{316}b^{629}c^{307} - 2a^{317}b^{631}c^{308} + 2a^{318}b^{633}c^{309} - 2a^{319}b^{635}c^{310} + 2a^{320}b^{637}c^{311} \\
& - 2a^{321}b^{639}c^{312} + 2a^{322}b^{641}c^{313} - 2a^{323}b^{643}c^{314} + 2a^{324}b^{645}c^{315} - 2a^{325}b^{647}c^{316} + 2a^{326}b^{649}c^{317} \\
& - 2a^{327}b^{651}c^{318} + 2a^{328}b^{653}c^{319} - 2a^{329}b^{655}c^{320} + 2a^{330}b^{657}c^{321} - 2a^{331}b^{659}c^{322} + 2a^{332}b^{661}c^{323} \\
& - 2a^{333}b^{663}c^{324} + 2a^{334}b^{665}c^{325} - 2a^{335}b^{667}c^{326} + 2a^{336}b^{669}c^{327} - 2a^{337}b^{671}c^{328} + 2a^{338}b^{673}c^{329} \\
& - 2a^{339}b^{675}c^{330} + 2a^{340}b^{677}c^{331} - 2a^{341}b^{679}c^{332} + 2a^{342}b^{681}c^{333} - 2a^{343}b^{683}c^{334} + 2a^{344}b^{685}c^{335} \\
& - 2a^{345}b^{687}c^{336} + 2a^{346}b^{689}c^{337} - 2a^{347}b^{691}c^{338} + 2a^{348}b^{693}c^{339} - 2a^{349}b^{695}c^{340} + 2a^{350}b^{697}c^{341} \\
& - 2a^{351}b^{699}c^{342} + 2a^{352}b^{701}c^{343} - 2a^{353}b^{703}c^{344} + 2a^{354}b^{705}c^{345} - 2a^{355}b^{707}c^{346} + 2a^{356}b^{709}c^{347} \\
& - 2a^{357}b^{711}c^{348} + 2a^{358}b^{713}c^{349} - 2a^{359}b^{715}c^{350} + 2a^{360}b^{717}c^{351} - 2a^{361}b^{719}c^{352} + 2a^{362}b^{721}c^{353} \\
& - 2a^{363}b^{723}c^{354} + 2a^{364}b^{725}c^{355} - 2a^{365}b^{727}c^{356} + 2a^{366}b^{729}c^{357} - 2a^{367}b^{731}c^{358} + 2a^{368}b^{733}c^{359} \\
& - 2a^{369}b^{735}c^{360} + 2a^{370}b^{737}c^{361} - 2a^{371}b^{739}c^{362} + 2a^{372}b^{741}c^{363} - 2a^{373}b^{743}c^{364} + 2a^{374}b^{745}c^{365} \\
& - 2a^{375}b^{747}c^{366} + 2a^{376}b^{749}c^{367} - 2a^{377}b^{751}c^{368} + 2a^{378}b^{753}c^{369} - 2a^{379}b^{755}c^{370} + 2a^{380}b^{757}c^{371} \\
& - 2a^{381}b^{759}c^{372} + 2a^{382}b^{761}c^{373} - 2a^{383}b^{763}c^{374} + 2a^{384}b^{765}c^{375} - 2a^{385}b^{767}c^{376} + 2a^{386}b^{769}c^{377} \\
& - 2a^{387}b^{771}c^{378} + 2a^{388}b^{773}c^{379} - 2a^{389}b^{775}c^{380} + 2a^{390}b^{777}c^{381} - 2a^{391}b^{779}c^{382} + 2a^{392}b^{781}c^{383} \\
& - 2a^{393}b^{783}c^{384} + 2a^{394}b^{785}c^{385} - 2a^{395}b^{787}c^{386} + 2a^{396}b^{789}c^{387} - 2a^{397}b^{791}c^{388} + 2a^{398}b^{793}c^{389} \\
& - 2a^{399}b^{795}c^{390} + 2a^{400}b^{797}c^{391} - 2a^{401}b^{799}c^{392} + 2a^{402}b^{801}c^{393} - 2a^{403}b^{803}c^{394} + 2a^{404}b^{805}c^{395} \\
& - 2a^{405}b^{807}c^{396} + 2a^{406}b^{809}c^{397} - 2a^{407}b^{811}c^{398} + 2a^{408}b^{813}c^{399} - 2a^{409}b^{815}c^{400} + 2a^{410}b^{817}c^{401} \\
& - 2a^{411}b^{819}c^{402} + 2a^{412}b^{821}c^{403} - 2a^{413}b^{823}c^{404} + 2a^{414}b^{825}c^{405} - 2a^{415}b^{827}c^{406} + 2a^{416}b^{829}c^{407} \\
& - 2a^{417}b^{831}c^{408} + 2a^{418}b^{833}c^{409} - 2a^{419}b^{835}c^{410} + 2a^{420}b^{837}c^{411} - 2a^{421}b^{839}c^{412} + 2a^{422}b^{841}c^{413} \\
& - 2a^{423}b^{843}c^{414} + 2a^{424}b^{845}c^{415} - 2a^{425}b^{847}c^{416} + 2a^{426}b^{849}c^{417} - 2a^{427}b^{851}c^{418} + 2a^{428}b^{853}c^{419} \\
& - 2a^{429}b^{855}c^{420} + 2a^{430}b^{857}c^{421} - 2a^{431}b^{859}c^{422} + 2a^{432}b^{861}c^{423} - 2a^{433}b^{863}c^{424} + 2a^{434}b^{865}c^{425} \\
& - 2a^{435}b^{867}c^{426} + 2a^{436}b^{869}c^{427} - 2a^{437}b^{871}c^{428} + 2a^{438}b^{873}c^{429} - 2a^{439}b^{875}c^{430} + 2a^{440}b^{877}c^{431} \\
& - 2a^{441}b^{879}c^{432} + 2a^{442}b^{881}c^{433} - 2a^{443}b^{883}c^{434} + 2a^{444}b^{885}c^{435} - 2a^{445}b^{887}c^{436} + 2a^{446}b^{889}c^{437} \\
& - 2a^{447}b^{891}c^{438} + 2a^{448}b^{893}c^{439} - 2a^{449}b^{895}c^{440} + 2a^{450}b^{897}c^{441} - 2a^{451}b^{899}c^{442} + 2a^{452}b^{901}c^{443} \\
& - 2a^{453}b^{903}c^{444} + 2a^{454}b^{905}c^{445} - 2a^{455}b^{907}c^{446} + 2a^{456}b^{909}c^{447} - 2a^{457}b^{911}c^{448} + 2a^{458}b^{913}c^{449} \\
& - 2a^{459}b^{915}c^{450} + 2a^{460}b^{917}c^{451} - 2a^{461}b^{919}c^{452} + 2a^{462}b^{921}c^{453} - 2a^{463}b^{923}c^{454} + 2a^{464}b^{925}c^{455} \\
& - 2a^{465}b^{927}c^{456} + 2a^{466}b^{929}c^{457} - 2a^{467}b^{931}c^{458} + 2a^{468}b^{933}c^{459} - 2a^{469}b^{935}c^{460} + 2a^{470}b^{937}c^{461} \\
& - 2a^{471}b^{939}c^{462} + 2a^{472}b^{941}c^{463} - 2a^{473}b^{943}c^{464} + 2a^{474}b^{945}c^{465} - 2a^{475}b^{947}c^{466} + 2a^{476}b^{949}c^{467} \\
& - 2a^{477}b^{951}c^{468} + 2a^{478}b^{953}c^{469} - 2a^{479}b^{955}c^{470} + 2a^{480}b^{957}c^{471} - 2a^{481}b^{959}c^{472} + 2a^{482}b^{961}c^{473} \\
& - 2a^{483}b^{963}c^{474} + 2a^{484}b^{965}c^{475} - 2a^{485}b^{967}c^{476} + 2a^{486}b^{969}c^{477} - 2a^{487}b^{971}c^{478} + 2a^{488}b^{973}c^{479} \\
& - 2a^{489}b^{975}c^{480} + 2a^{490}b^{977}c^{481} - 2a^{491}b^{979}c^{482} + 2a^{492}b^{981}c^{483} - 2a^{493}b^{983}c^{484} + 2a^{494}b^{985}c^{485} \\
& - 2a^{495}b^{987}c^{486} + 2a^{496}b^{989}c^{487} - 2a^{497}b^{991}c^{488} + 2a^{498}b^{993}c^{489} - 2a^{499}b^{995}c^{490} + 2a^{500}b^{997}c^{491} \\
& - 2a^{501}b^{999}c^{492} + 2a^{502}b^{1001}c^{493} - 2a^{503}b^{999}c^{494} + 2a^{504}b^{1001}c^{495} - 2a^{505}b^{1003}c^{496} + 2a^{506}b^{1005}c^{497} \\
& - 2a^{507}b^{1007}c^{498} + 2a^{508}b^{1009}c^{499} - 2a^{509}b^{1011}c^{500} + 2a^{510}b^{1013}c^{501} - 2a^{511}b^{1015}c^{502} + 2a^{512}b^{1017}c^{503} \\
& - 2a^{513}b^{1019}c^{504} + 2a^{514}b^{1021}c^{505} - 2a^{515}b^{1023}c^{506} + 2a^{516}b^{1025}c^{507} - 2a^{517}b^{1027}c^{508} + 2a^{518}b^{1029}c^{509} \\
& - 2a^{519}b^{1031}c^{510} + 2a^{520}b^{1033}c^{511} - 2a^{521}b^{1035}c^{512} + 2a^{522}b^{1037}c^{513} - 2a^{523}b^{1039}c^{514} + 2a^{524}b^{1041}c^{515} \\
& - 2a^{525}b^{1043}c^{516} + 2a^{526}b^{1045}c^{517} - 2a^{527}b^{1047}c^{518} + 2a^{528}b^{1049}c^{519} - 2a^{529}b^{1051}c^{520} + 2a^{530}b^{1053}c^{521} \\
& - 2a^{531}b^{1055}c^{522} + 2a^{532}b^{1057}c^{523} - 2a^{533}b^{1059}c^{524} + 2a^{534}b^{1061}c^{525} - 2a^{535}b^{1063}c^{526} + 2a^{536}b^{1065}c^{527} \\
& - 2a^{537}b^{1067}c^{528} + 2a^{538}b^{1069}c^{529} - 2a^{539}b^{1071}c^{530} + 2a^{540}b^{1073}c^{531} - 2a^{541}b^{1075}c^{532} + 2a^{542}b^{1077}c^{533} \\
& - 2a^{543}b^{1079}c^{534} + 2a^{544}b^{1081}c^{535} - 2a^{545}b^{1083}c^{536} + 2a^{546}b^{1085}c^{537} - 2a^{547}b^{1087}c^{538} + 2a^{548}b^{1089}c^{539} \\
& - 2a^{549}b^{1091}c^{540} + 2a^{550}b^{1093}c^{541} - 2a^{551}b^{1095}c^{542} + 2a^{552}b^{1097}c^{543} - 2a^{553}b^{1099}c^{544} + 2a^{554}b^{1101}c^{545} \\
& - 2a^{555}b^{1103}c^{546} + 2a^{556}b^{1105}c^{547} - 2a^{557}b^{1107}c^{548} + 2a^{558}b^{1109}c^{549} - 2a^{559}b^{1111}c^{550} + 2a^{560}b^{1113}c^{551} \\
& - 2a^{561}b^{1115}c^{552} + 2a^{562}b^{1117}c^{553} - 2a^{563}b^{1119}c^{554} + 2a^{564}b^{1121}c^{555} - 2a^{565}b^{1123}c^{556} + 2a^{566}b^{1125}c^{557} \\
& - 2a^{567}b^{1127}c^{558} + 2a^{568}b^{1129}c^{559} - 2a^{569}b^{1131}c^{560} + 2a^{570}b^{1133}c^{561} - 2a^{571}b^{1135}c^{562} + 2a^{572}b^{1137}c^{563} \\
& - 2a^{573}b^{1139}c^{564} + 2a^{574}b^{1141}c^{565} - 2a^{575}b^{1143}c^{566} + 2a^{576}b^{1145}c^{567} - 2a^{577}b^{1147}c^{568} + 2a^{578}b^{1149}c^{569} \\
& - 2a^{579}b^{1151}c^{570} + 2a^{580}b^{1153}c^{571} - 2a^{58$$

$$\begin{aligned}
& 6 + 888a^3b^6c^4 + 1152a^3b^8c^2 - 128a^4b^2c^7 - 1104a^4b^4c^5 \\
& - 1824a^4b^6c^3 + 504a^5b^2c^6 + 1249a^5b^4c^4 - 700a^5b^6c^2 \\
& - 292a^6b^2c^5 + 812a^6b^4c^3 - 392a^7b^2c^4 + 44a^7b^4c^2 - 32 \\
& *a^8b^2c^3)/c^8)*((b^{10} - a^2b^8 - 8a^5c^5 - 8a^6c^4 - b^7*(-(4ac \\
& - b^2)^3)^{(1/2)} + 10a^3b^6c + a^2b^5*(-(4ac - b^2)^3)^{(1/2)} + 52a^2 \\
& *b^6c^2 - 96a^3b^4c^3 + 66a^4b^2c^4 - 33a^4b^4c^2 + 38a^5b^2c^3 \\
& - 12a*b^8c + 4a^3b*c^3*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^3c*(-(4ac \\
& - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*( \\
& -(4ac - b^2)^3)^{(1/2)} + 6a*b^5c*(-(4ac - b^2)^3)^{(1/2)})/(2*(16a^2c^ \\
& 10 + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b^6c^6 - 8a*b^2c^9 + 10a*b^4c \\
& ^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3b^2c^7)))^{(1/2)}*1i)/((4096*(16a \\
& ^6b^6 - 4a^8b^4 - 4a^7c^5 + 15a^8c^4 - 14a^9c^3 - 48a^7b^4c + 4 \\
& *a^9b^2c + 4a^6b^2c^4 + 16a^6b^4c^2 - 32a^7b^2c^3 + 44a^8b^2c \\
& ^2))/c^8 + (((2048*(44a^5c^9 - 16a^4c^10 - 4a^6c^8 - 64a^7c^7 + 12* \\
& a^8c^6 + 4a*b^6c^7 + 15a*b^8c^5 + 14a*b^10c^3 - 28a^2b^4c^8 - 119 \\
& *a^2b^6c^6 - 128a^2b^8c^4 - 8a^2b^10c^2 + 52a^3b^2c^9 + 290a^3* \\
& b^4c^7 + 397a^3b^6c^5 + 62a^3b^8c^3 - 227a^4b^2c^8 - 491a^4b^4* \\
& c^6 - 148a^4b^6c^4 + 8a^4b^8c^2 + 221a^5b^2c^7 + 102a^5b^4c^5 - \\
& 60a^5b^6c^3 + 68a^6b^2c^6 + 136a^6b^4c^4 - 100a^7b^2c^5))/c^8 \\
& - ((2048*(4a*b^3c^11 + 13a*b^5c^9 + 4a*b^7c^7 - 12a*b^9c^5 - 16a^2 \\
& *b*c^12 + 44a^3b*c^11 + 4a^4b*c^10 + 80a^5b*c^9 + 12a^6b*c^8 - 63a \\
& ^2b^3c^10 - 16a^2b^5c^8 + 76a^2b^7c^6 - a^3b^3c^9 - 104a^3b^5c \\
& ^7 + 12a^3b^7c^5 - 56a^4b^3c^8 - 60a^4b^5c^6 + 48a^5b^3c^7))/c^ \\
& 8 - (((2048*(12a*b^5c^11 - 16a*b^3c^13 + 64a^2b*c^14 + 80a^3b*c^13 \\
& + 48a^4b*c^12 - 68a^2b^3c^12 - 12a^3b^3c^11))/c^8 + (2048*tan(x/2)* \\
& (256a^2c^15 + 576a^3c^14 + 416a^4c^13 + 96a^5c^12 - 64a*b^2c^14 + \\
& 68a*b^4c^12 - 8a*b^6c^10 - 416a^2b^2c^13 + 72a^2b^4c^11 - 264a^ \\
& 3b^2c^12 + 8a^3b^4c^10 - 56a^4b^2c^11))/c^8)*((b^{10} - a^2b^8 - 8a \\
& ^5c^5 - 8a^6c^4 - b^7*(-(4ac - b^2)^3)^{(1/2)} + 10a^3b^6c + a^2b^5* \\
& -(4ac - b^2)^3)^{(1/2)} + 52a^2b^6c^2 - 96a^3b^4c^3 + 66a^4b^2c^4 \\
& - 33a^4b^4c^2 + 38a^5b^2c^3 - 12a*b^8c + 4a^3b*c^3*(-(4ac - b^ \\
& 2)^3)^{(1/2)} - 4a^3b^3c*(-(4ac - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4ac - \\
& b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1/2)} + 6a*b^5c*(-(4a \\
& c - b^2)^3)^{(1/2)})/(2*(16a^2c^10 + 32a^3c^9 + 16a^4c^8 + b^4c^8 - b \\
& ^6c^6 - 8a*b^2c^9 + 10a*b^4c^7 - 32a^2b^2c^8 + a^2b^4c^6 - 8a^3* \\
& b^2c^7)))^{(1/2)} - (2048*(32a^3c^13 + 64a^4c^12 - 16a^5c^11 - 48a^6* \\
& c^10 + 2a*b^4c^11 - 14a*b^6c^9 - 16a^2b^2c^12 + 96a^2b^4c^10 + 8* \\
& a^2b^6c^8 - 176a^3b^2c^11 - 46a^3b^4c^9 + 60a^4b^2c^10 - 8a^4b \\
& ^4c^8 + 44a^5b^2c^9))/c^8 + (2048*tan(x/2)*(32a*b^5c^10 - 16a*b^7c^ \\
& 8 + 256a^3b*c^12 + 320a^4b*c^11 + 128a^5b*c^10 - 192a^2b^3c^11 + 1 \\
& 28a^2b^5c^9 - 336a^3b^3c^10 + 16a^3b^5c^8 - 96a^4b^3c^9))/c^8)* \\
& ((b^{10} - a^2b^8 - 8a^5c^5 - 8a^6c^4 - b^7*(-(4ac - b^2)^3)^{(1/2)} + 1 \\
& 0a^3b^6c + a^2b^5*(-(4ac - b^2)^3)^{(1/2)} + 52a^2b^6c^2 - 96a^3b^ \\
& 4c^3 + 66a^4b^2c^4 - 33a^4b^4c^2 + 38a^5b^2c^3 - 12a*b^8c + 4a \\
& ^3b*c^3*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^3c*(-(4ac - b^2)^3)^{(1/2)} + \\
& 3a^4b*c^2*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1 \\
& /2)} + 6a*b^5c*(-(4ac - b^2)^3)^{(1/2)})/(2*(16a^2c^10 + 32a^3c^9 + 16 \\
& *a^4c^8 + b^4c^8 - b^6c^6 - 8a*b^2c^9 + 10a*b^4c^7 - 32a^2b^2c^8 \\
& + a^2b^4c^6 - 8a^3b^2c^7)))^{(1/2)} + (2048*tan(x/2)*(128a^3c^12 - 64* \\
& a^2c^13 + 184a^4c^11 - 296a^5c^10 - 352a^6c^9 - 72a^7c^8 + 16a*b^ \\
& 2c^12 + 48a*b^4c^10 + a*b^6c^8 - 92a*b^8c^6 + 8a*b^10c^4 - 224a^2* \\
& b^2c^11 + 56a^2b^4c^9 + 732a^2b^6c^7 - 88a^2b^8c^5 - 286a^3b^2* \\
& c^10 - 1817a^3b^4c^8 + 440a^3b^6c^6 - 8a^3b^8c^4 + 1502a^4b^2c^ \\
& 9 - 1140a^4b^4c^7 + 72a^4b^6c^5 + 1208a^5b^2c^8 - 220a^5b^4c^6 \\
& + 256a^6b^2c^7))/c^8)*((b^{10} - a^2b^8 - 8a^5c^5 - 8a^6c^4 - b^7*(-( \\
& 4ac - b^2)^3)^{(1/2)} + 10a^3b^6c + a^2b^5*(-(4ac - b^2)^3)^{(1/2)} + 5 \\
& 2a^2b^6c^2 - 96a^3b^4c^3 + 66a^4b^2c^4 - 33a^4b^4c^2 + 38a^5b \\
& ^2c^3 - 12a*b^8c + 4a^3b*c^3*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^3c*(- \\
& (4ac - b^2)^3)^{(1/2)} + 3a^4b*c^2*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3*
\end{aligned}$$

$$\begin{aligned}
& c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (- (4 * a * c - b^2)^3)^{(1/2)} / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7))^{(1/2)} + (2048 * \tan(x/2) * (8 * a * b^5 * c^8 + 28 * a * b^7 * c^6 + 16 * a * b^9 * c^4 - 16 * a * b^{11} * c^2 + 64 * a^3 * b * c^{10} - 176 * a^4 * b * c^9 - 32 * a^5 * b * c^8 + 128 * a^6 * b * c^7 + 112 * a^7 * b * c^6 - 48 * a^2 * b^3 * c^9 - 192 * a^2 * b^5 * c^7 - 112 * a^2 * b^7 * c^5 + 160 * a^2 * b^9 * c^3 + 364 * a^3 * b^3 * c^8 + 212 * a^3 * b^5 * c^6 - 592 * a^3 * b^7 * c^4 + 16 * a^3 * b^9 * c^2 - 72 * a^4 * b^3 * c^7 + 1008 * a^4 * b^5 * c^5 - 128 * a^4 * b^7 * c^3 - 720 * a^5 * b^3 * c^6 + 336 * a^5 * b^5 * c^4 - 352 * a^6 * b^3 * c^5)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (- (4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (- (4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7))^{(1/2)} + (2048 * (16 * a^2 * b^{11} - 12 * a^4 * b^9 - 144 * a^3 * b^9 * c - 28 * a^5 * b * c^7 + 84 * a^5 * b^7 * c + 97 * a^6 * b * c^6 - 52 * a^7 * b * c^5 - 60 * a^8 * b * c^4 + 4 * a^2 * b^7 * c^4 + 16 * a^2 * b^9 * c^2 - 28 * a^3 * b^5 * c^5 - 128 * a^3 * b^7 * c^3 + 56 * a^4 * b^3 * c^6 + 333 * a^4 * b^5 * c^4 + 452 * a^4 * b^7 * c^2 - 321 * a^5 * b^3 * c^5 - 600 * a^5 * b^5 * c^3 + 328 * a^6 * b^3 * c^4 - 192 * a^6 * b^5 * c^2 + 180 * a^7 * b^3 * c^3)) / c^8 + (2048 * \tan(x/2) * (32 * a * b^{12} - 32 * a^3 * b^{10} + 4 * a^5 * b^8 + 16 * a^5 * c^8 - 48 * a^6 * c^7 + 2 * a^7 * c^6 + 56 * a^8 * c^5 + 12 * a^9 * c^4 + 8 * a * b^8 * c^4 + 32 * a * b^{10} * c^2 - 320 * a^2 * b^{10} * c + 256 * a^4 * b^8 * c - 24 * a^6 * b^6 * c - 64 * a^2 * b^6 * c^5 - 288 * a^2 * b^8 * c^3 + 160 * a^3 * b^4 * c^6 + 888 * a^3 * b^6 * c^4 + 1152 * a^3 * b^8 * c^2 - 128 * a^4 * b^2 * c^7 - 1104 * a^4 * b^4 * c^5 - 1824 * a^4 * b^6 * c^3 + 504 * a^5 * b^2 * c^6 + 1249 * a^5 * b^4 * c^4 - 700 * a^5 * b^6 * c^2 - 292 * a^6 * b^2 * c^5 + 8 * 12 * a^6 * b^4 * c^3 - 392 * a^7 * b^2 * c^4 + 44 * a^7 * b^4 * c^2 - 32 * a^8 * b^2 * c^3)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (- (4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (- (4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7))^{(1/2)} - ((2048 * (16 * a^2 * b^{11} - 12 * a^4 * b^9 - 144 * a^3 * b^9 * c - 28 * a^5 * b * c^7 + 84 * a^5 * b^7 * c + 97 * a^6 * b * c^6 - 52 * a^7 * b * c^5 - 60 * a^8 * b * c^4 + 4 * a^2 * b^7 * c^4 + 16 * a^2 * b^9 * c^2 - 28 * a^3 * b^5 * c^5 - 128 * a^3 * b^7 * c^3 + 56 * a^4 * b^3 * c^6 + 333 * a^4 * b^5 * c^4 + 452 * a^4 * b^7 * c^2 - 321 * a^5 * b^3 * c^5 - 600 * a^5 * b^5 * c^3 + 328 * a^6 * b^3 * c^4 - 192 * a^6 * b^5 * c^2 + 180 * a^7 * b^3 * c^3)) / c^8 - ((2048 * (44 * a^5 * c^9 - 16 * a^4 * c^{10} - 4 * a^6 * c^8 - 64 * a^7 * c^7 + 12 * a^8 * c^6 + 4 * a * b^6 * c^7 + 15 * a * b^8 * c^5 + 14 * a * b^{10} * c^3 - 28 * a^2 * b^4 * c^8 - 119 * a^2 * b^6 * c^6 - 128 * a^2 * b^8 * c^4 - 8 * a^2 * b^{10} * c^2 + 52 * a^3 * b^2 * c^9 + 290 * a^3 * b^4 * c^7 + 397 * a^3 * b^6 * c^5 + 62 * a^3 * b^8 * c^3 - 227 * a^4 * b^2 * c^8 - 491 * a^4 * b^4 * c^6 - 148 * a^4 * b^6 * c^4 + 8 * a^4 * b^8 * c^2 + 221 * a^5 * b^2 * c^7 + 102 * a^5 * b^4 * c^5 - 60 * a^5 * b^6 * c^3 + 68 * a^6 * b^2 * c^6 + 136 * a^6 * b^4 * c^4 - 100 * a^7 * b^2 * c^5)) / c^8 + ((2048 * (4 * a * b^3 * c^{11} + 13 * a * b^5 * c^9 + 4 * a * b^7 * c^7 - 12 * a * b^9 * c^5 - 16 * a^2 * b * c^{12} + 44 * a^3 * b * c^{11} + 4 * a^4 * b * c^{10} + 80 * a^5 * b * c^9 + 12 * a^6 * b * c^8 - 63 * a^2 * b^3 * c^{10} - 16 * a^2 * b^5 * c^8 + 76 * a^2 * b^7 * c^6 - a^3 * b^3 * c^9 - 104 * a^3 * b^5 * c^7 + 12 * a^3 * b^7 * c^5 - 56 * a^4 * b^3 * c^8 - 60 * a^4 * b^5 * c^6 + 48 * a^5 * b^3 * c^7)) / c^8 - (((2048 * (12 * a * b^5 * c^{11} - 16 * a * b^3 * c^{13} + 64 * a^2 * b * c^{14} + 80 * a^3 * b * c^{13} + 48 * a^4 * b * c^{12} - 68 * a^2 * b^3 * c^{12} - 12 * a^3 * b^3 * c^{11})) / c^8 + (2048 * \tan(x/2) * (256 * a^2 * c^{15} + 576 * a^3 * c^{14} + 416 * a^4 * c^{13} + 96 * a^5 * c^{12} - 64 * a * b^2 * c^{14} + 68 * a * b^4 * c^{12} - 8 * a * b^6 * c^{10} - 416 * a^2 * b^2 * c^{13} + 72 * a^2 * b^4 * c^{11} - 264 * a^3 * b^2 * c^{12} + 8 * a^3 * b^4 * c^{10} - 56 * a^4 * b^2 * c^{11})) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (- (4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (- (4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (- (4 * a * c -
\end{aligned}$$

$$\begin{aligned}
& (b^2)^3)^{(1/2)) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7)))^{(1/2)} + (2048 * (32 * a^3 * c^{13} + 64 * a^4 * c^{12} - 16 * a^5 * c^{11} - 48 * a^6 * c^{10} + 2 * a * b^4 * c^{11} - 14 * a * b^6 * c^9 - 16 * a^2 * b^2 * c^{12} + 96 * a^2 * b^4 * c^{10} + 8 * a^2 * b^6 * c^8 - 176 * a^3 * b^2 * c^{11} - 46 * a^3 * b^4 * c^9 + 60 * a^4 * b^2 * c^{10} - 8 * a^4 * b^4 * c^8 + 44 * a^5 * b^2 * c^9)) / c^8 - (2048 * \tan(x/2) * (32 * a * b^5 * c^{10} - 16 * a * b^7 * c^8 + 256 * a^3 * b * c^{12} + 320 * a^4 * b * c^{11} + 128 * a^5 * b * c^{10} - 192 * a^2 * b^3 * c^{11} + 128 * a^2 * b^5 * c^9 - 336 * a^3 * b^3 * c^{10} + 16 * a^3 * b^5 * c^8 - 96 * a^4 * b^3 * c^9)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (-4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (-4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (-4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (-4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (-4 * a * c - b^2)^3)^{(1/2)) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7)))^{(1/2)} + (2048 * \tan(x/2) * (128 * a^3 * c^{12} - 64 * a^2 * c^{13} + 184 * a^4 * c^{11} - 296 * a^5 * c^{10} - 352 * a^6 * c^9 - 72 * a^7 * c^8 + 16 * a * b^2 * c^{12} + 48 * a * b^4 * c^{10} + a * b^6 * c^8 - 92 * a * b^8 * c^6 + 8 * a * b^{10} * c^4 - 224 * a^2 * b^2 * c^{11} + 56 * a^2 * b^4 * c^9 + 732 * a^2 * b^6 * c^7 - 88 * a^2 * b^8 * c^5 - 286 * a^3 * b^2 * c^{10} - 1817 * a^3 * b^4 * c^8 + 440 * a^3 * b^6 * c^6 - 8 * a^3 * b^8 * c^4 + 1502 * a^4 * b^2 * c^9 - 1140 * a^4 * b^4 * c^7 + 72 * a^4 * b^6 * c^5 + 1208 * a^5 * b^2 * c^8 - 220 * a^5 * b^4 * c^6 + 256 * a^6 * b^2 * c^7)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (-4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (-4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (-4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (-4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (-4 * a * c - b^2)^3)^{(1/2)) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7)))^{(1/2)} + (2048 * \tan(x/2) * (8 * a * b^5 * c^8 + 28 * a * b^7 * c^6 + 16 * a * b^9 * c^4 - 16 * a * b^{11} * c^2 + 64 * a^3 * b * c^{10} - 176 * a^4 * b * c^9 - 32 * a^5 * b * c^8 + 128 * a^6 * b * c^7 + 112 * a^7 * b * c^6 - 48 * a^2 * b^3 * c^9 - 192 * a^2 * b^5 * c^7 - 112 * a^2 * b^7 * c^5 + 160 * a^2 * b^9 * c^3 + 364 * a^3 * b^3 * c^8 + 212 * a^3 * b^5 * c^6 - 592 * a^3 * b^7 * c^4 + 16 * a^3 * b^9 * c^2 - 72 * a^4 * b^3 * c^7 + 1008 * a^4 * b^5 * c^5 - 128 * a^4 * b^7 * c^3 - 720 * a^5 * b^3 * c^6 + 336 * a^5 * b^5 * c^4 - 352 * a^6 * b^3 * c^5)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (-4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (-4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (-4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (-4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (-4 * a * c - b^2)^3)^{(1/2)) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7)))^{(1/2)} + (2048 * \tan(x/2) * (32 * a * b^{12} - 32 * a^3 * b^{10} + 4 * a^5 * b^8 + 16 * a^5 * c^8 - 48 * a^6 * c^7 + 2 * a^7 * c^6 + 56 * a^8 * c^5 + 12 * a^9 * c^4 + 8 * a * b^8 * c^4 + 32 * a * b^{10} * c^2 - 320 * a^2 * b^{10} * c + 256 * a^4 * b^8 * c - 24 * a^6 * b^6 * c - 64 * a^2 * b^6 * c^5 - 288 * a^2 * b^8 * c^3 + 160 * a^3 * b^4 * c^6 + 888 * a^3 * b^6 * c^4 + 1152 * a^3 * b^8 * c^2 - 128 * a^4 * b^2 * c^7 - 1104 * a^4 * b^4 * c^5 - 1824 * a^4 * b^6 * c^3 + 504 * a^5 * b^2 * c^6 + 1249 * a^5 * b^4 * c^4 - 700 * a^5 * b^6 * c^2 - 292 * a^6 * b^2 * c^5 + 812 * a^6 * b^4 * c^3 - 392 * a^7 * b^2 * c^4 + 44 * a^7 * b^4 * c^2 - 32 * a^8 * b^2 * c^3)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5 * c^5 - 8 * a^6 * c^4 - b^7 * (-4 * a * c - b^2)^3)^{(1/2)} + 10 * a^3 * b^6 * c + a^2 * b^5 * (-4 * a * c - b^2)^3)^{(1/2)} + 52 * a^2 * b^6 * c^2 - 96 * a^3 * b^4 * c^3 + 66 * a^4 * b^2 * c^4 - 33 * a^4 * b^4 * c^2 + 38 * a^5 * b^2 * c^3 - 12 * a * b^8 * c + 4 * a^3 * b * c^3 * (-4 * a * c - b^2)^3)^{(1/2)} - 4 * a^3 * b^3 * c * (-4 * a * c - b^2)^3)^{(1/2)} + 3 * a^4 * b * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} - 10 * a^2 * b^3 * c^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a * b^5 * c * (-4 * a * c - b^2)^3)^{(1/2)) / (2 * (16 * a^2 * c^{10} + 32 * a^3 * c^9 + 16 * a^4 * c^8 + b^4 * c^8 - b^6 * c^6 - 8 * a * b^2 * c^9 + 10 * a * b^4 * c^7 - 32 * a^2 * b^2 * c^8 + a^2 * b^4 * c^6 - 8 * a^3 * b^2 * c^7)))^{(1/2)} + (4096 * \tan(x/2) * (32 * a^5 * b^7 - 16 * a^7 * b^5 - 16 * a^6 * b * c^5 - 128 * a^6 * b^5 * c + 60 * a^7 * b * c^4 - 48 * a^8 * b * c^3 + 32 * a^8 * b^3 * c - 16 * a^9 * b * c^2 + 8 * a^5 * b^3 * c^4 + 32 * a^5 * b^5 * c^2 - 96 * a^6 * b^3 * c^3 + 144 * a^7 * b^3 * c^2)) / c^8) * ((b^{10} - a^2 * b^8 - 8 * a^5
\end{aligned}$$

$$\begin{aligned}
& *c^5 - 8*a^6*c^4 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a^3*b^6*c + a^2*b^5*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 52*a^2*b^6*c^2 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 - \\
& 33*a^4*b^4*c^2 + 38*a^5*b^2*c^3 - 12*a*b^8*c + 4*a^3*b*c^3*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 4*a^3*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^4*b*c^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c \\
& - b^2)^3)^{(1/2)))/(2*(16*a^2*c^10 + 32*a^3*c^9 + 16*a^4*c^8 + b^4*c^8 - b^6 \\
& *c^6 - 8*a*b^2*c^9 + 10*a*b^4*c^7 - 32*a^2*b^2*c^8 + a^2*b^4*c^6 - 8*a^3*b^ \\
& 2*c^7)))^{(1/2)*2i} - (\operatorname{atan}(\frac{(2048*(16*a^2*b^11 - 12*a^4*b^9 - 144*a^3*b^9*c \\
& - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8*b*c^4 \\
& + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 + 56* \\
& a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 600*a^5 \\
& *b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3))}{c^8} + ((2 \\
& 048*(44*a^5*c^9 - 16*a^4*c^10 - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b \\
& ^6*c^7 + 15*a*b^8*c^5 + 14*a*b^10*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 - \\
& 128*a^2*b^8*c^4 - 8*a^2*b^10*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a \\
& ^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b \\
& ^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6*c^3 \\
& + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5))/c^8 + (2048*\operatorname{tan}(x/2) \\
& )*(8*a*b^5*c^8 + 28*a*b^7*c^6 + 16*a*b^9*c^4 - 16*a*b^11*c^2 + 64*a^3*b*c^1 \\
& 0 - 176*a^4*b*c^9 - 32*a^5*b*c^8 + 128*a^6*b*c^7 + 112*a^7*b*c^6 - 48*a^2*b \\
& ^3*c^9 - 192*a^2*b^5*c^7 - 112*a^2*b^7*c^5 + 160*a^2*b^9*c^3 + 364*a^3*b^3* \\
& c^8 + 212*a^3*b^5*c^6 - 592*a^3*b^7*c^4 + 16*a^3*b^9*c^2 - 72*a^4*b^3*c^7 + \\
& 1008*a^4*b^5*c^5 - 128*a^4*b^7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 3 \\
& 52*a^6*b^3*c^5))/c^8 - (((2048*(4*a*b^3*c^11 + 13*a*b^5*c^9 + 4*a*b^7*c^7 - \\
& 12*a*b^9*c^5 - 16*a^2*b*c^12 + 44*a^3*b*c^11 + 4*a^4*b*c^10 + 80*a^5*b*c^9 \\
& + 12*a^6*b*c^8 - 63*a^2*b^3*c^10 - 16*a^2*b^5*c^8 + 76*a^2*b^7*c^6 - a^3*b \\
& ^3*c^9 - 104*a^3*b^5*c^7 + 12*a^3*b^7*c^5 - 56*a^4*b^3*c^8 - 60*a^4*b^5*c^6 \\
& + 48*a^5*b^3*c^7))/c^8 - (((2048*\operatorname{tan}(x/2)*(32*a*b^5*c^10 - 16*a*b^7*c^8 + \\
& 256*a^3*b*c^12 + 320*a^4*b*c^11 + 128*a^5*b*c^10 - 192*a^2*b^3*c^11 + 128*a \\
& ^2*b^5*c^9 - 336*a^3*b^3*c^10 + 16*a^3*b^5*c^8 - 96*a^4*b^3*c^9))/c^8 - (20 \\
& 48*(32*a^3*c^13 + 64*a^4*c^12 - 16*a^5*c^11 - 48*a^6*c^10 + 2*a*b^4*c^11 - \\
& 14*a*b^6*c^9 - 16*a^2*b^2*c^12 + 96*a^2*b^4*c^10 + 8*a^2*b^6*c^8 - 176*a^3* \\
& b^2*c^11 - 46*a^3*b^4*c^9 + 60*a^4*b^2*c^10 - 8*a^4*b^4*c^8 + 44*a^5*b^2*c^ \\
& 9))/c^8 + (((2048*(12*a*b^5*c^11 - 16*a*b^3*c^13 + 64*a^2*b*c^14 + 80*a^3*b \\
& *c^13 + 48*a^4*b*c^12 - 68*a^2*b^3*c^12 - 12*a^3*b^3*c^11))/c^8 + (2048*\operatorname{tan} \\
& (x/2)*(256*a^2*c^15 + 576*a^3*c^14 + 416*a^4*c^13 + 96*a^5*c^12 - 64*a*b^2* \\
& c^14 + 68*a*b^4*c^12 - 8*a*b^6*c^10 - 416*a^2*b^2*c^13 + 72*a^2*b^4*c^11 - \\
& 264*a^3*b^2*c^12 + 8*a^3*b^4*c^10 - 56*a^4*b^2*c^11))/c^8)*(b^2*2i - a*c*2i \\
& + c^2*1i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*\operatorname{tan}(x/2)* \\
& (128*a^3*c^12 - 64*a^2*c^13 + 184*a^4*c^11 - 296*a^5*c^10 - 352*a^6*c^9 - 72 \\
& *a^7*c^8 + 16*a*b^2*c^12 + 48*a*b^4*c^10 + a*b^6*c^8 - 92*a*b^8*c^6 + 8*a*b \\
& ^10*c^4 - 224*a^2*b^2*c^11 + 56*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8* \\
& c^5 - 286*a^3*b^2*c^10 - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 \\
& + 1502*a^4*b^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 \\
& - 220*a^5*b^4*c^6 + 256*a^6*b^2*c^7))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c \\
& ^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*\operatorname{tan}(x/2)*(32*a*b^12 - 32*a \\
& ^3*b^10 + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12 \\
& *a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^10*c^2 - 320*a^2*b^10*c + 256*a^4*b^8*c - 2 \\
& 4*a^6*b^6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3* \\
& b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4* \\
& b^6*c^3 + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^ \\
& 2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3 \\
& ))/c^8)*(b^2*2i - a*c*2i + c^2*1i)*1i))/(2*c^3) + (((2048*(16*a^2*b^11 - 12* \\
& a^4*b^9 - 144*a^3*b^9*c - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a \\
& ^7*b*c^5 - 60*a^8*b*c^4 + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - \\
& 128*a^3*b^7*c^3 + 56*a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321 \\
& *a^5*b^3*c^5 - 600*a^5*b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^ \\
& 7*b^3*c^3))/c^8 - (((2048*(44*a^5*c^9 - 16*a^4*c^10 - 4*a^6*c^8 - 64*a^7*c^ \\
& 7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^10*c^3 - 28*a^2*b^4*c^
\end{aligned}$$

$$\begin{aligned}
& 8 - 119a^2b^6c^6 - 128a^2b^8c^4 - 8a^2b^{10}c^2 + 52a^3b^2c^9 + 290a^3b^4c^7 + 397a^3b^6c^5 + 62a^3b^8c^3 - 227a^4b^2c^8 - 491a^4b^4c^6 - 148a^4b^6c^4 + 8a^4b^8c^2 + 221a^5b^2c^7 + 102a^5b^4c^5 - 60a^5b^6c^3 + 68a^6b^2c^6 + 136a^6b^4c^4 - 100a^7b^2c^5 \\
& ))/c^8 + (2048\tan(x/2)*(8a^5b^5c^8 + 28a^5b^7c^6 + 16a^5b^9c^4 - 16a^5b^{11}c^2 + 64a^3b^3c^{10} - 176a^4b^5c^9 - 32a^5b^7c^8 + 128a^6b^9c^7 + 112a^7b^{11}c^6 - 48a^2b^3c^9 - 192a^2b^5c^7 - 112a^2b^7c^5 + 160a^2b^9c^3 + 364a^3b^3c^8 + 212a^3b^5c^6 - 592a^3b^7c^4 + 16a^3b^9c^2 - 72a^4b^3c^7 + 1008a^4b^5c^5 - 128a^4b^7c^3 - 720a^5b^3c^6 + 336a^5b^5c^4 - 352a^6b^3c^5))/c^8 + (((2048*(4a^3b^3c^{11} + 13a^3b^5c^9 + 4a^3b^7c^7 - 12a^3b^9c^5 - 16a^2b^3c^{12} + 44a^3b^5c^{11} + 4a^4b^7c^{10} + 80a^5b^9c^9 + 12a^6b^{11}c^8 - 63a^2b^3c^{10} - 16a^2b^5c^8 + 76a^2b^7c^6 - a^3b^3c^9 - 104a^3b^5c^7 + 12a^3b^7c^5 - 56a^4b^3c^8 - 60a^4b^5c^6 + 48a^5b^3c^7))/c^8 - (((2048*(32a^3c^{13} + 64a^4c^{12} - 16a^5c^{11} - 48a^6c^{10} + 2a^7b^4c^{11} - 14a^7b^6c^9 - 16a^2b^2c^{12} + 96a^2b^4c^{10} + 8a^2b^6c^8 - 176a^3b^2c^{11} - 46a^3b^4c^9 + 60a^4b^2c^{10} - 8a^4b^4c^8 + 44a^5b^2c^9))/c^8 - (2048\tan(x/2)*(32a^3b^5c^{10} - 16a^3b^7c^8 + 256a^3b^9c^6 + 320a^4b^3c^{11} + 128a^5b^5c^{10} - 192a^2b^3c^{11} + 128a^2b^5c^9 - 336a^3b^3c^{10} + 16a^3b^5c^8 - 96a^4b^3c^9))/c^8 + (((2048*(12a^3b^5c^{11} - 16a^3b^3c^{13} + 64a^2b^5c^{14} + 80a^3b^7c^{13} + 48a^4b^9c^{12} - 68a^2b^3c^{12} - 12a^3b^3c^{11}))/c^8 + (2048\tan(x/2)*(256a^2c^{15} + 576a^3c^{14} + 416a^4c^{13} + 96a^5c^{12} - 64a^6b^2c^{14} + 68a^6b^4c^{12} - 8a^6b^6c^{10} - 416a^2b^2c^{13} + 72a^2b^4c^{11} - 264a^3b^2c^{12} + 8a^3b^4c^{10} - 56a^4b^2c^{11}))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048\tan(x/2)*(128a^3c^{12} - 64a^2c^{13} + 184a^4c^{11} - 296a^5c^{10} - 352a^6c^9 - 72a^7c^8 + 16a^8b^2c^{12} + 48a^8b^4c^{10} + a^8b^6c^8 - 92a^8b^8c^6 + 8a^8b^{10}c^4 - 224a^2b^2c^{11} + 56a^2b^4c^9 + 732a^2b^6c^7 - 88a^2b^8c^5 - 286a^3b^2c^{10} - 1817a^3b^4c^8 + 440a^3b^6c^6 - 8a^3b^8c^4 + 1502a^4b^2c^9 - 1140a^4b^4c^7 + 72a^4b^6c^5 + 1208a^5b^2c^8 - 220a^5b^4c^6 + 256a^6b^2c^7))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048\tan(x/2)*(32a^3b^{12} - 32a^3b^{10} + 4a^5b^8 + 16a^5c^8 - 48a^6c^7 + 2a^7c^6 + 56a^8c^5 + 12a^9c^4 + 8a^8b^8c^4 + 32a^8b^{10}c^2 - 320a^2b^{10}c + 256a^4b^8c - 24a^6b^6c - 64a^2b^6c^5 - 288a^2b^8c^3 + 160a^3b^4c^6 + 888a^3b^6c^4 + 1152a^3b^8c^2 - 128a^4b^2c^7 - 1104a^4b^4c^5 - 1824a^4b^6c^3 + 504a^5b^2c^6 + 1249a^5b^4c^4 - 700a^5b^6c^2 - 292a^6b^2c^5 + 812a^6b^4c^3 - 392a^7b^2c^4 + 44a^7b^4c^2 - 32a^8b^2c^3))/c^8)*(b^2*2i - a*c*2i + c^2*1i)*1i))/(2*c^3))/((4096*(16a^6b^6 - 4a^8b^4 - 4a^7c^5 + 15a^8c^4 - 14a^9c^3 - 48a^7b^4c + 4a^9b^2c + 4a^6b^2c^4 + 16a^6b^4c^2 - 32a^7b^2c^3 + 44a^8b^2c^2))/c^8 + (4096\tan(x/2)*(32a^5b^7 - 16a^7b^5 - 16a^6b^3c^5 - 128a^6b^5c + 60a^7b^3c^4 - 48a^8b^3c^3 + 32a^8b^3c - 16a^9b^3c^2 + 8a^5b^3c^4 + 32a^5b^5c^2 - 96a^6b^3c^3 + 144a^7b^3c^2))/c^8 + (((2048*(16a^2b^{11} - 12a^4b^9 - 144a^3b^9c - 28a^5b^7c + 84a^5b^7c + 97a^6b^5c^6 - 52a^7b^5c^5 - 60a^8b^5c^4 + 4a^2b^7c^4 + 16a^2b^9c^2 - 28a^3b^5c^5 - 128a^3b^7c^3 + 56a^4b^3c^6 + 333a^4b^5c^4 + 452a^4b^7c^2 - 321a^5b^3c^5 - 600a^5b^5c^3 + 328a^6b^3c^4 - 192a^6b^5c^2 + 180a^7b^3c^3))/c^8 + (((2048*(44a^5c^9 - 16a^4c^{10} - 4a^6c^8 - 64a^7c^7 + 12a^8c^6 + 4a^8b^6c^7 + 15a^8b^8c^5 + 14a^8b^{10}c^3 - 28a^2b^4c^8 - 119a^2b^6c^6 - 128a^2b^8c^4 - 8a^2b^{10}c^2 + 52a^3b^2c^9 + 290a^3b^4c^7 + 397a^3b^6c^5 + 62a^3b^8c^3 - 227a^4b^2c^8 - 491a^4b^4c^6 - 148a^4b^6c^4 + 8a^4b^8c^2 + 221a^5b^2c^7 + 102a^5b^4c^5 - 60a^5b^6c^3 + 68a^6b^2c^6 + 136a^6b^4c^4 - 100a^7b^2c^5))/c^8 + (2048\tan(x/2)*(8a^5b^5c^8 + 28a^5b^7c^6 + 16a^5b^9c^4 - 16a^5b^{11}c^2 + 64a^3b^3c^{10} - 176a^4b^5c^9 - 32a^5b^7c^8 + 128a^6b^9c^7 + 112a^7b^11c^6 - 48a^2b^3c^9 - 192a^2b^5c^7 - 112a^2b^7c^5 + 160a^2b^9c^3 + 364a^3b^3c^8 + 212a^3b^5c^6 - 592a^3b^7c^4 + 16a^3b^9c^2 - 72a^4b^3c^7 + 1008a^4b^5c^5 - 128a^4
\end{aligned}$$



$$\begin{aligned}
& *b^7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 352*a^6*b^3*c^5)/c^8 - (((2048*(4*a*b^3*c^11 + 13*a*b^5*c^9 + 4*a*b^7*c^7 - 12*a*b^9*c^5 - 16*a^2*b*c^12 + 44*a^3*b*c^11 + 4*a^4*b*c^10 + 80*a^5*b*c^9 + 12*a^6*b*c^8 - 63*a^2*b^3*c^10 - 16*a^2*b^5*c^8 + 76*a^2*b^7*c^6 - a^3*b^3*c^9 - 104*a^3*b^5*c^7 + 12*a^3*b^7*c^5 - 56*a^4*b^3*c^8 - 60*a^4*b^5*c^6 + 48*a^5*b^3*c^7))/c^8 - ((2048*\tan(x/2)*(32*a*b^5*c^10 - 16*a*b^7*c^8 + 256*a^3*b*c^12 + 320*a^4*b*c^11 + 128*a^5*b*c^10 - 192*a^2*b^3*c^11 + 128*a^2*b^5*c^9 - 336*a^3*b^3*c^10 + 16*a^3*b^5*c^8 - 96*a^4*b^3*c^9))/c^8 - (2048*(32*a^3*c^13 + 64*a^4*c^12 - 16*a^5*c^11 - 48*a^6*c^10 + 2*a*b^4*c^11 - 14*a*b^6*c^9 - 16*a^2*b^2*c^12 + 96*a^2*b^4*c^10 + 8*a^2*b^6*c^8 - 176*a^3*b^2*c^11 - 46*a^3*b^4*c^9 + 60*a^4*b^2*c^10 - 8*a^4*b^4*c^8 + 44*a^5*b^2*c^9))/c^8 + (((2048*(12*a*b^5*c^11 - 16*a*b^3*c^13 + 64*a^2*b*c^14 + 80*a^3*b*c^13 + 48*a^4*b*c^12 - 68*a^2*b^3*c^12 - 12*a^3*b^3*c^11))/c^8 + (2048*\tan(x/2)*(256*a^2*c^15 + 576*a^3*c^14 + 416*a^4*c^13 + 96*a^5*c^12 - 64*a*b^2*c^14 + 68*a*b^4*c^12 - 8*a*b^6*c^10 - 416*a^2*b^2*c^13 + 72*a^2*b^4*c^11 - 264*a^3*b^2*c^12 + 8*a^3*b^4*c^10 - 56*a^4*b^2*c^11))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*\tan(x/2)*(128*a^3*c^12 - 64*a^2*c^13 + 184*a^4*c^11 - 296*a^5*c^10 - 352*a^6*c^9 - 72*a^7*c^8 + 16*a*b^2*c^12 + 48*a*b^4*c^10 + a*b^6*c^8 - 92*a*b^8*c^6 + 8*a*b^10*c^4 - 224*a^2*b^2*c^11 + 56*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8*c^5 - 286*a^3*b^2*c^10 - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 + 1502*a^4*b^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 - 220*a^5*b^4*c^6 + 256*a^6*b^2*c^7))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*\tan(x/2)*(32*a*b^12 - 32*a^3*b^10 + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 + 8*a*b^8*c^4 + 32*a*b^10*c^2 - 320*a^2*b^10*c + 256*a^4*b^8*c - 24*a^6*b^6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) - (((2048*(16*a^2*b^11 - 12*a^4*b^9 - 144*a^3*b^9*c - 28*a^5*b*c^7 + 84*a^5*b^7*c + 97*a^6*b*c^6 - 52*a^7*b*c^5 - 60*a^8*b*c^4 + 4*a^2*b^7*c^4 + 16*a^2*b^9*c^2 - 28*a^3*b^5*c^5 - 128*a^3*b^7*c^3 + 56*a^4*b^3*c^6 + 333*a^4*b^5*c^4 + 452*a^4*b^7*c^2 - 321*a^5*b^3*c^5 - 600*a^5*b^5*c^3 + 328*a^6*b^3*c^4 - 192*a^6*b^5*c^2 + 180*a^7*b^3*c^3))/c^8 - (((2048*(44*a^5*c^9 - 16*a^4*c^10 - 4*a^6*c^8 - 64*a^7*c^7 + 12*a^8*c^6 + 4*a*b^6*c^7 + 15*a*b^8*c^5 + 14*a*b^10*c^3 - 28*a^2*b^4*c^8 - 119*a^2*b^6*c^6 - 128*a^2*b^8*c^4 - 8*a^2*b^10*c^2 + 52*a^3*b^2*c^9 + 290*a^3*b^4*c^7 + 397*a^3*b^6*c^5 + 62*a^3*b^8*c^3 - 227*a^4*b^2*c^8 - 491*a^4*b^4*c^6 - 148*a^4*b^6*c^4 + 8*a^4*b^8*c^2 + 221*a^5*b^2*c^7 + 102*a^5*b^4*c^5 - 60*a^5*b^6*c^3 + 68*a^6*b^2*c^6 + 136*a^6*b^4*c^4 - 100*a^7*b^2*c^5))/c^8 + (2048*\tan(x/2)*(8*a*b^5*c^8 + 28*a*b^7*c^6 + 16*a*b^9*c^4 - 16*a*b^11*c^2 + 64*a^3*b*c^10 - 176*a^4*b*c^9 - 32*a^5*b*c^8 + 128*a^6*b*c^7 + 112*a^7*b*c^6 - 48*a^2*b^3*c^9 - 192*a^2*b^5*c^7 - 112*a^2*b^7*c^5 + 160*a^2*b^9*c^3 + 364*a^3*b^3*c^8 + 212*a^3*b^5*c^6 - 592*a^3*b^7*c^4 + 16*a^3*b^9*c^2 - 72*a^4*b^3*c^7 + 1008*a^4*b^5*c^5 - 128*a^4*b^7*c^3 - 720*a^5*b^3*c^6 + 336*a^5*b^5*c^4 - 352*a^6*b^3*c^5))/c^8 + (((2048*(4*a*b^3*c^11 + 13*a*b^5*c^9 + 4*a*b^7*c^7 - 12*a*b^9*c^5 - 16*a^2*b*c^12 + 44*a^3*b*c^11 + 4*a^4*b*c^10 + 80*a^5*b*c^9 + 12*a^6*b*c^8 - 63*a^2*b^3*c^10 - 16*a^2*b^5*c^8 + 76*a^2*b^7*c^6 - a^3*b^3*c^9 - 104*a^3*b^5*c^7 + 12*a^3*b^7*c^5 - 56*a^4*b^3*c^8 - 60*a^4*b^5*c^6 + 48*a^5*b^3*c^7))/c^8 - (((2048*(32*a^3*c^13 + 64*a^4*c^12 - 16*a^5*c^11 - 48*a^6*c^10 + 2*a*b^4*c^11 - 14*a*b^6*c^9 - 16*a^2*b^2*c^12 + 96*a^2*b^4*c^10 + 8*a^2*b^6*c^8 - 176*a^3*b^2*c^11 - 46*a^3*b^4*c^9 + 60*a^4*b^2*c^10 - 8*a^4*b^4*c^8 + 44*a^5*b^2*c^9))/c^8 - (2048*\tan(x/2)*(32*a*b^5*c^10 - 16*a*b^7*c^8 + 256*a^3*b*c^12 + 320*a^4*b*c^11 + 128*a^5*b*c^10 - 192*a^2*b^3*c^11 + 128*a^2*b^5*c^9 - 336*a^3*b^3*c^10 + 16*a^3*b^5*c^8 - 96*a^4*b^3*c^9))/c^8 + (((2048*(12*a*b^5*c^11 - 16*a*b^3*c^13 + 64*a^2*b*c^14 + 80*a^3*b*c^13 + 48*a^4*b*c^12 - 68*a^2*b^3*c^12 - 12*a^3*b^3*c^11))/c^8 + (2048*\tan(x/2)*(256*a^2*c^15 + 576*a^3*c^14 + 416*a^4*c^13 + 96*a^5*c^12 - 64*a*b^2*c^14 + 68
\end{aligned}$$

$$\begin{aligned}
& 8*a*b^4*c^{12} - 8*a*b^6*c^{10} - 416*a^2*b^2*c^{13} + 72*a^2*b^4*c^{11} - 264*a^3* \\
& b^2*c^{12} + 8*a^3*b^4*c^{10} - 56*a^4*b^2*c^{11}))/c^8)*(b^2*2i - a*c*2i + c^2*1 \\
& i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*tan(x/2)*(128*a^3* \\
& c^{12} - 64*a^2*c^{13} + 184*a^4*c^{11} - 296*a^5*c^{10} - 352*a^6*c^9 - 72*a^7*c^8 \\
& + 16*a*b^2*c^{12} + 48*a*b^4*c^{10} + a*b^6*c^8 - 92*a*b^8*c^6 + 8*a*b^{10}*c^4 \\
& - 224*a^2*b^2*c^{11} + 56*a^2*b^4*c^9 + 732*a^2*b^6*c^7 - 88*a^2*b^8*c^5 - 28 \\
& 6*a^3*b^2*c^{10} - 1817*a^3*b^4*c^8 + 440*a^3*b^6*c^6 - 8*a^3*b^8*c^4 + 1502*a \\
& a^4*b^2*c^9 - 1140*a^4*b^4*c^7 + 72*a^4*b^6*c^5 + 1208*a^5*b^2*c^8 - 220*a^ \\
& 5*b^4*c^6 + 256*a^6*b^2*c^7))/c^8)*(b^2*2i - a*c*2i + c^2*1i))/(2*c^3))*(b^ \\
& 2*2i - a*c*2i + c^2*1i))/(2*c^3) + (2048*tan(x/2)*(32*a*b^{12} - 32*a^3*b^{10} \\
& + 4*a^5*b^8 + 16*a^5*c^8 - 48*a^6*c^7 + 2*a^7*c^6 + 56*a^8*c^5 + 12*a^9*c^4 \\
& + 8*a*b^8*c^4 + 32*a*b^{10}*c^2 - 320*a^2*b^{10}*c + 256*a^4*b^8*c - 24*a^6*b^ \\
& 6*c - 64*a^2*b^6*c^5 - 288*a^2*b^8*c^3 + 160*a^3*b^4*c^6 + 888*a^3*b^6*c^4 \\
& + 1152*a^3*b^8*c^2 - 128*a^4*b^2*c^7 - 1104*a^4*b^4*c^5 - 1824*a^4*b^6*c^3 \\
& + 504*a^5*b^2*c^6 + 1249*a^5*b^4*c^4 - 700*a^5*b^6*c^2 - 292*a^6*b^2*c^5 + \\
& 812*a^6*b^4*c^3 - 392*a^7*b^2*c^4 + 44*a^7*b^4*c^2 - 32*a^8*b^2*c^3))/c^8)* \\
& (b^2*2i - a*c*2i + c^2*1i))/(2*c^3))*(b^2*2i - a*c*2i + c^2*1i)*1i)/c^3
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)\*\*4/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

$$3.2 \quad \int \frac{\sin^3(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

**Optimal.** Leaf size=298

$$\frac{\sqrt{2} b \left( -\frac{b^2}{\sqrt{b^2-4ac}} + \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \tan^{-1} \left( \frac{\tan(\frac{x}{2})(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^2\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} b \left( \frac{b^2}{\sqrt{b^2-4ac}} - \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \tan^{-1} \left( \frac{\tan(\frac{x}{2})(b+\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^2\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out]  $-\frac{b^2 x}{c^2} - \frac{\cos(x)}{c} + b \arctan\left(\frac{1}{2} \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{b^2 - 2c(a+c) - b^2 \tan^2(x/2)}}\right) + \frac{2c \sqrt{b^2 - 2c(a+c) - b^2 \tan^2(x/2)}}{c^2} + \frac{b \arctan\left(\frac{1}{2} \frac{2c + (b + \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{b^2 - 2c(a+c) + b^2 \tan^2(x/2)}}\right) + \frac{2c \sqrt{b^2 - 2c(a+c) + b^2 \tan^2(x/2)}}{c^2} - \frac{3ac}{c^2} + \frac{b^2}{c^2} + \frac{b}{c}$

**Rubi [A]** time = 3.74, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {3256, 2638, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} b \left( -\frac{b^2}{\sqrt{b^2-4ac}} + \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \tan^{-1} \left( \frac{\tan(\frac{x}{2})(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^2\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} b \left( \frac{b^2}{\sqrt{b^2-4ac}} - \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \tan^{-1} \left( \frac{\tan(\frac{x}{2})(b+\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{c^2\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3/(a + bSin[x] + cSin[x]^2), x]

[Out]  $-\frac{(b^2 x)}{c^2} + \frac{\sqrt{2} b \left( \frac{b^2}{\sqrt{b^2-4ac}} - \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \arctan\left(\frac{2c + (b - \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{b^2 - 2c(a+c) - b^2 \tan^2(x/2)}}\right) + \frac{2c \sqrt{b^2 - 2c(a+c) - b^2 \tan^2(x/2)}}{c^2} + \frac{\sqrt{2} b \left( -\frac{b^2}{\sqrt{b^2-4ac}} + \frac{3ac}{\sqrt{b^2-4ac}} - \frac{ac}{b} + b \right) \arctan\left(\frac{2c + (b + \sqrt{b^2 - 4ac}) \tan(x/2)}{\sqrt{b^2 - 2c(a+c) + b^2 \tan^2(x/2)}}\right) + \frac{2c \sqrt{b^2 - 2c(a+c) + b^2 \tan^2(x/2)}}{c^2} - \frac{3ac}{c^2} + \frac{b^2}{c^2} + \frac{b}{c}}{c^2}$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 2638**

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

**Rule 2660**

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a

$e^{2*x^2}$ ), x], x, Tan[(c + d\*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3256

Int[sin[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n\_.) + (c\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n2\_.))^(p\_), x\_Symbol] :> Int[ExpandTrig[sin[d + e\*x]^m\*(a + b\*sin[d + e\*x]^n + c\*sin[d + e\*x]^(2\*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && IntegersQ[m, n, p]

Rule 3292

Int[((A\_) + (B\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])/((a\_.) + (b\_.)\*sin[(d\_.) + (e\_.)\*(x\_)] + (c\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^2), x\_Symbol] :> Module[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[B + (b\*B - 2\*A\*c)/q, Int[1/(b + q + 2\*c\*Sin[d + e\*x]), x], x] + Dist[B - (b\*B - 2\*A\*c)/q, Int[1/(b - q + 2\*c\*Sin[d + e\*x]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\int \frac{\sin^3(x)}{a + b \sin(x) + c \sin^2(x)} dx = \int \left( -\frac{b}{c^2} + \frac{\sin(x)}{c} + \frac{ab + b^2 \left(1 - \frac{ac}{b^2}\right) \sin(x)}{c^2 (a + b \sin(x) + c \sin^2(x))} \right) dx$$

$$= -\frac{bx}{c^2} + \frac{\int \frac{ab + b^2 \left(1 - \frac{ac}{b^2}\right) \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{c^2} + \frac{\int \sin(x) dx}{c}$$

$$= -\frac{bx}{c^2} - \frac{\cos(x)}{c} + \frac{\left(b^2 - ac + \frac{b^3}{\sqrt{b^2 - 4ac}} - \frac{3abc}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{c^2} + \frac{(b^2 - ac)}{c^2}$$

$$= -\frac{bx}{c^2} - \frac{\cos(x)}{c} + \frac{\left(2 \left(b^2 - ac + \frac{b^3}{\sqrt{b^2 - 4ac}} - \frac{3abc}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst} \left( \int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})x^2} dx \right)}{c^2}$$

$$= -\frac{bx}{c^2} - \frac{\cos(x)}{c} - \frac{\left(4 \left(b^2 - ac + \frac{b^3}{\sqrt{b^2 - 4ac}} - \frac{3abc}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst} \left( \int \frac{1}{4 \left(4c^2 - (b + \sqrt{b^2 - 4ac})^2\right) - x^2} dx \right)}{c^2}$$

$$= -\frac{bx}{c^2} + \frac{\sqrt{2} \left(b^2 - ac - \frac{b^3}{\sqrt{b^2 - 4ac}} + \frac{3abc}{\sqrt{b^2 - 4ac}}\right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{c^2 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}}{c^2}$$

**Mathematica** [C] time = 0.96, size = 358, normalized size = 1.20

$$\frac{\left(b^2 \sqrt{4ac - b^2} - ac \sqrt{4ac - b^2} - 3iabc + ib^3\right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) (b - i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} + \frac{\left(b^2 \sqrt{4ac - b^2} - ac \sqrt{4ac - b^2} + 3iabc - ib^3\right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) (b + i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3/(a + b\*SIN[x] + c\*SIN[x]^2),x]

[Out]  $(-b*x) + ((I*b^3 - (3*I)*a*b*c + b^2*sqrt[-b^2 + 4*a*c] - a*c*sqrt[-b^2 + 4*a*c])*ArcTan[(2*c + (b - I*sqrt[-b^2 + 4*a*c])*Tan[x/2])/(sqrt[2]*sqrt[b^2$

$$\frac{2 - 2*c*(a + c) - I*b*\sqrt{-b^2 + 4*a*c}}{\sqrt{-1/2*b^2 + 2*a*c}*\sqrt{b^2 - 2*c*(a + c) - I*b*\sqrt{-b^2 + 4*a*c}}} + \frac{((-I)*b^3 + (3*I)*a*b*c + b^2*\sqrt{-b^2 + 4*a*c} - a*c*\sqrt{-b^2 + 4*a*c})*\text{ArcTan}[(2*c + (b + I*\sqrt{-b^2 + 4*a*c}))*\text{Tan}[x/2]]}{\sqrt{2}*\sqrt{b^2 - 2*c*(a + c) + I*b*\sqrt{-b^2 + 4*a*c}}}}{\sqrt{-1/2*b^2 + 2*a*c}*\sqrt{b^2 - 2*c*(a + c) + I*b*\sqrt{-b^2 + 4*a*c}}} - c*\text{Cos}[x])/c^2$$

**fricas [B]** time = 2.27, size = 6531, normalized size = 21.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*(\sqrt{2})*c^2*\sqrt{(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2} \\ & - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3* \\ & a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*\sqrt{-(a^4*b^6 - 2*a^2*b^8 + b^{10} + 9*a^4 \\ & *b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^ \\ & 2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c}/(4*a*c^{13} + (16*a^2 - b^2 \\ & )*c^{12} + 12*(2*a^3 - a*b^2)*c^{11} + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^{10} + 4*(a \\ & ^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 \\ & + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*\log( \\ & 12*a^5*b*c^3 + 8*(a^6*b - 2*a^4*b^3)*c^2 + 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2) \\ & )*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*\sqrt{-(a^4*b^6 \\ & - 2*a^2*b^8 + b^{10} + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^ \\ & 6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c} \\ & )/(4*a*c^{13} + (16*a^2 - b^2)*c^{12} + 12*(2*a^3 - a*b^2)*c^{11} + 2*(8*a^4 - 11* \\ & a^2*b^2 + b^4)*c^{10} + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2* \\ & b^4 + b^6)*c^8))*\sin(x) - 4*(a^5*b^3 - a^3*b^5)*c - \sqrt{2}*((12*a^2*b*c^9 \\ & + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - \\ & 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*\sqrt{-(a^4*b^6 \\ & - 2*a^2*b^8 + b^{10} + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^ \\ & 6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c} \\ & )/(4*a*c^{13} + (16*a^2 - b^2)*c^{12} + 12*(2*a^3 - a*b^2)*c^{11} + 2*(8*a^4 - 11* \\ & a^2*b^2 + b^4)*c^{10} + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2* \\ & b^4 + b^6)*c^8))*\cos(x) + (12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8 \\ & *a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7) \\ & )*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c)*\cos(x))*\sqrt{(a^2*b^4 - b^6 + 2*a^3*c^3 \\ & + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 \\ & - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*\sqrt{-(a^4*b^6 \\ & - 2*a^2*b^8 + b^{10} + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^ \\ & 6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c} \\ & )/(4*a*c^{13} + (16*a^2 - b^2)*c^{12} + 12*(2*a^3 - a*b^2)*c^{11} + 2*(8*a^4 - 11* \\ & a^2*b^2 + b^4)*c^{10} + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2* \\ & b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 \\ & - (a^2*b^2 - b^4)*c^4)) - 2*(a^5*b^4 - a^3*b^6 - 3*a^5*b^2*c^2 - 2*(a^6*b^2 \\ & - 2*a^4*b^4)*c)*\sin(x)) - \sqrt{2}*c^2*\sqrt{(a^2*b^4 - b^6 + 2*a^3*c^3 + (2 \\ & *a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2) \\ & )*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*\sqrt{-(a^4*b^6 - 2*a^ \\ & 2*b^8 + b^{10} + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 \\ & - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c}/(4*a \\ & *c^{13} + (16*a^2 - b^2)*c^{12} + 12*(2*a^3 - a*b^2)*c^{11} + 2*(8*a^4 - 11*a^2*b^ \\ & ^2 + b^4)*c^{10} + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + \\ & b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^ \\ & 2 - b^4)*c^4))*\log(12*a^5*b*c^3 + 8*(a^6*b - 2*a^4*b^3)*c^2 - 2*(4*a^4*c^7 \\ & + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4) \\ & )*c^4)*\sqrt{-(a^4*b^6 - 2*a^2*b^8 + b^{10} + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^ \\ & 3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3* \\ & a^3*b^6 + 2*a*b^8)*c}/(4*a*c^{13} + (16*a^2 - b^2)*c^{12} + 12*(2*a^3 - a*b^2)* \\ & c^{11} + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^{10} + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 \end{aligned}$$



$$\begin{aligned}
& ^3 - 8*(a^6*b - 2*a^4*b^3)*c^2 - 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*( \\
& 2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*\sqrt{-(a^4*b^6 - 2*a^2*b^ \\
& 8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11 \\
& *a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 \\
& + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + \\
& b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6) \\
& *c^8))*\sin(x) + 4*(a^5*b^3 - a^3*b^5)*c - \sqrt{2}*((12*a^2*b*c^9 + 7*(4*a^3 \\
& *b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 \\
& + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*\sqrt{-(a^4*b^6 - 2*a^2*b^ \\
& ^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 1 \\
& 1*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^1 \\
& 3 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + \\
& b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6 \\
& )*c^8))*\cos(x) + (12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 3 \\
& 3*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^ \\
& 4*b^5 - 2*a^2*b^7 + b^9)*c)*\cos(x))*\sqrt{((a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^ \\
& 4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c \\
& ^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*\sqrt{-(a^4*b^6 - 2*a^2* \\
& b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - \\
& 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^ \\
& 13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 \\
& + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^ \\
& 6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 \\
& - b^4)*c^4)) + 2*(a^5*b^4 - a^3*b^6 - 3*a^5*b^2*c^2 - 2*(a^6*b^2 - 2*a^4*b^ \\
& ^4)*c)*\sin(x)) + 4*b*x + 4*c*\cos(x))/c^2
\end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.29, size = 890, normalized size = 2.99

$$\frac{8a^2 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2}}{c(8ca - 2b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} + \frac{4a \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2} b^2}{c^2(8ca - 2b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} - c \left(
\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 
$$\begin{aligned}
& -8/c*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)}*\arcc \\
& \tan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1 \\
& /2)+4*a^2}^{(1/2)})*(-4*a*c+b^2)^{(1/2)}+4/c^2*a/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b \\
& *(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)} \\
& -b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*b^ \\
& 2-16/c*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)}*a \\
& rctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{( \\
& 1/2)+4*a^2}^{(1/2)})*b+4/c^2*a/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{( \\
& 1/2)+4*a^2}^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^ \\
& 2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)})*b^3-8/c*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^ \\
& ^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2 \\
& )^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)})*(-4*a*c+b^2)^{(1/ \\
& 2)+4/c^2*a/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2}^{(1/2)}*a \\
& rctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2} + 4a^2)^{1/2}) * (-4ac + b^2)^{1/2} * b^2 + 16/c * a^2 / (8ac - 2b^2) / (4ca - 2b^2 - 2b * (-4ac + b^2)^{1/2} + 4a^2)^{1/2} * \arctan((2a * \tan(1/2 * x) + b + (-4ac + b^2)^{1/2}) / (4ca - 2b^2 - 2b * (-4ac + b^2)^{1/2} + 4a^2)^{1/2}) * b - 4/c^2 * a / (8ac - 2b^2) / (4ca - 2b^2 - 2b * (-4ac + b^2)^{1/2} + 4a^2)^{1/2} * \arctan((2a * \tan(1/2 * x) + b + (-4ac + b^2)^{1/2}) / (4ca - 2b^2 - 2b * (-4ac + b^2)^{1/2} + 4a^2)^{1/2}) * b^3 - 2/c / (\tan(1/2 * x)^2 + 1) - 2/c^2 * b * \arctan(\tan(1/2 * x)) \end{aligned}$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Timed out

**mupad** [B] time = 25.03, size = 21407, normalized size = 71.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(a + c\*sin(x)^2 + b\*sin(x)),x)

[Out] 
$$\begin{aligned} & -2/(c * (\tan(x/2)^2 + 1)) - \operatorname{atan}\left(\frac{(8192 * (4a^2b^7 - 3a^4b^5 - 20a^3b^5 * c + 9a^5b^3c + 20a^4b^3c^2)) / c^4 + ((8192 * (4ab^7c^2 - 2a^2b^7c + 2a^4b^5c + 12a^5b^3c^4 + 8a^6b^3c^3 - 24a^2b^5c^3 + 32a^3b^3c^4 + 10a^3b^5c^2 - 10a^4b^3c^3 - 10a^5b^3c^2)) / c^4 + ((b^8 - a^2b^6 + 8a^4c^4 + 8a^5c^3 + b^5 * (-4ac - b^2)^3)^{1/2} + 8a^3b^4c - a^2b^3 * (-4ac - b^2)^3)^{1/2} + 33a^2b^4c^2 - 38a^3b^2c^3 - 18a^4b^2c^2 - 10ab^6c + 3a^2b^3c^2 * (-4ac - b^2)^3)^{1/2} - 4ab^3c * (-4ac - b^2)^3)^{1/2} + 2a^3b^3c * (-4ac - b^2)^3)^{1/2}}{2 * (16a^2c^8 + 32a^3c^7 + 16a^4c^6 + b^4c^6 - b^6c^4 - 8ab^2c^7 + 10ab^4c^5 - 32a^2b^2c^6 + a^2b^4c^4 - 8a^3b^2c^5))^{1/2}} * ((8192 * (3ab^7c^3 - 4ab^5c^5 + 20a^4b^3c^6 + 9a^5b^3c^5 + 16a^2b^3c^6 - 13a^2b^5c^4 - 3a^3b^5c^3 + 9a^4b^3c^4)) / c^4 + ((b^8 - a^2b^6 + 8a^4c^4 + 8a^5c^3 + b^5 * (-4ac - b^2)^3)^{1/2} + 8a^3b^4c - a^2b^3 * (-4ac - b^2)^3)^{1/2} + 33a^2b^4c^2 - 38a^3b^2c^3 - 18a^4b^2c^2 - 10ab^6c + 3a^2b^3c^2 * (-4ac - b^2)^3)^{1/2} - 4ab^3c * (-4ac - b^2)^3)^{1/2} + 2a^3b^3c * (-4ac - b^2)^3)^{1/2}}{2 * (16a^2c^8 + 32a^3c^7 + 16a^4c^6 + b^4c^6 - b^6c^4 - 8ab^2c^7 + 10ab^4c^5 - 32a^2b^2c^6 + a^2b^4c^4 - 8a^3b^2c^5))^{1/2}} * ((8192 * (3ab^5c^6 + 16a^3b^3c^8 - 4a^4b^3c^7 - 8a^5b^3c^6 - 16a^2b^3c^7 - 2a^2b^5c^5 + 9a^3b^3c^6 + 2a^4b^3c^5)) / c^4 + ((8192 * (3ab^5c^7 - 4ab^3c^9 + 16a^2b^3c^10 + 20a^3b^3c^9 + 12a^4b^3c^8 - 17a^2b^3c^8 - 3a^3b^3c^7)) / c^4 + (8192 * \tan(x/2) * (64a^2c^11 + 144a^3c^10 + 104a^4c^9 + 24a^5c^8 - 16ab^2c^10 + 17ab^4c^8 - 2ab^6c^6 - 104a^2b^2c^9 + 18a^2b^4c^7 - 66a^3b^2c^8 + 2a^3b^4c^6 - 14a^4b^2c^7)) / c^4 * ((b^8 - a^2b^6 + 8a^4c^4 + 8a^5c^3 + b^5 * (-4ac - b^2)^3)^{1/2} + 8a^3b^4c - a^2b^3 * (-4ac - b^2)^3)^{1/2} + 33a^2b^4c^2 - 38a^3b^2c^3 - 18a^4b^2c^2 - 10ab^6c + 3a^2b^3c^2 * (-4ac - b^2)^3)^{1/2} - 4ab^3c * (-4ac - b^2)^3)^{1/2} + 2a^3b^3c * (-4ac - b^2)^3)^{1/2}}{2 * (16a^2c^8 + 32a^3c^7 + 16a^4c^6 + b^4c^6 - b^6c^4 - 8ab^2c^7 + 10ab^4c^5 - 32a^2b^2c^6 + a^2b^4c^4 - 8a^3b^2c^5))^{1/2}} + (8192 * \tan(x/2) * (32a^3c^9 + 48a^4c^8 + 16a^5c^7 + 8ab^4c^7 - 4ab^6c^5 - 40a^2b^2c^8 + 28a^2b^4c^6 - 60a^3b^2c^7 + 4a^3b^4c^5 - 20a^4b^2c^6)) / c^4) - (8192 * \tan(x/2) * (16a^4c^7 + 24a^5c^6 + 10a^6c^5 + 16ab^4c^6 - 24ab^6c^4 + 2ab^8c^2 - 64a^2b^2c^7 + 144a^2b^4c^5 - 18a^2b^6c^3 - 200a^3b^2c^6 + 75a^3b^4c^4 - 2a^3b^6c^2 - 142a^4b^2c^5 + 14a^4b^4c^3 - 27a^5b^2c^4)) / c^4) - (8192 * \tan(x/2) * (8a^5c^5 + 4a^6c^4 - 8ab^6c^3 - 4a^3b^6c + 40a^2b^4c^4 - 28a^2b^6c^2 - 32a^3b^2c^5 + \end{aligned}$$



$$\begin{aligned}
& 60*a^3*b^4*c^3 - 56*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a*b^8*c) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} + (8192*\tan(x/2)*(8*a*b^8 - 8*a^3*b^6 + a^5*b^4 + a^7*c^2 - 48*a^2*b^6*c + 32*a^4*b^4*c - 2*a^6*b^2*c + 72*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 16*a^5*b^2*c^2)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} * i + ((8192*(4*a^2*b^7 - 3*a^4*b^5 - 20*a^3*b^5*c + 9*a^5*b^3*c + 20*a^4*b^3*c^2)) / c^4 - ((8192*(4*a*b^7*c^2 - 2*a^2*b^7*c + 2*a^4*b^5*c + 12*a^5*b*c^4 + 8*a^6*b*c^3 - 24*a^2*b^5*c^3 + 32*a^3*b^3*c^4 + 10*a^3*b^5*c^2 - 10*a^4*b^3*c^3 - 10*a^5*b^3*c^2)) / c^4 + ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} * (((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} * ((8192*(3*a*b^5*c^6 + 16*a^3*b*c^8 - 4*a^4*b*c^7 - 8*a^5*b*c^6 - 16*a^2*b^3*c^7 - 2*a^2*b^5*c^5 + 9*a^3*b^3*c^6 + 2*a^4*b^3*c^5)) / c^4 - ((8192*(3*a*b^5*c^7 - 4*a*b^3*c^9 + 16*a^2*b*c^10 + 20*a^3*b*c^9 + 12*a^4*b*c^8 - 17*a^2*b^3*c^8 - 3*a^3*b^3*c^7)) / c^4 + (8192*\tan(x/2)*(64*a^2*c^11 + 144*a^3*c^10 + 104*a^4*c^9 + 24*a^5*c^8 - 16*a*b^2*c^10 + 17*a*b^4*c^8 - 2*a*b^6*c^6 - 104*a^2*b^2*c^9 + 18*a^2*b^4*c^7 - 66*a^3*b^2*c^8 + 2*a^3*b^4*c^6 - 14*a^4*b^2*c^7)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} + (8192*\tan(x/2)*(32*a^3*c^9 + 48*a^4*c^8 + 16*a^5*c^7 + 8*a*b^4*c^7 - 4*a*b^6*c^5 - 40*a^2*b^2*c^8 + 28*a^2*b^4*c^6 - 60*a^3*b^2*c^7 + 4*a^3*b^4*c^5 - 20*a^4*b^2*c^6)) / c^4 - (8192*(3*a*b^7*c^3 - 4*a*b^5*c^5 + 20*a^4*b*c^6 + 9*a^5*b*c^5 + 16*a^2*b^3*c^6 - 13*a^2*b^5*c^4 - 3*a^3*b^5*c^3 + 9*a^4*b^3*c^4)) / c^4 + (8192*\tan(x/2)*(16*a^4*c^7 + 24*a^5*c^6 + 10*a^6*c^5 + 16*a*b^4*c^6 - 24*a*b^6*c^4 + 2*a*b^8*c^2 - 64*a^2*b^2*c^7 + 144*a^2*b^4*c^5 - 18*a^2*b^6*c^3 - 200*a^3*b^2*c^6 + 75*a^3*b^4*c^4 - 2*a^3*b^6*c^2 - 142*a^4*b^2*c^5 + 14*a^4*b^4*c^3 - 27*a^5*b^2*c^4)) / c^4 - (8192*\tan(x/2)*(8*a^5*c^5 + 4*a^6*c^4 - 8*a*b^6*c^3 - 4*a^3*b^6*c + 40*a^2*b^4*c^4 - 28*a^2*b^6*c^2 - 32*a^3*b^2*c^5 + 60*a^3*b^4*c^3 - 56*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a*b^8*c)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 + b^5*(-(4*a*c - b^2)^3)^{1/2} + 8*a^3*b^4*c - a^2*b^3*(-(4*a*c - b^2)^3)^{1/2} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2} + 2*a^3*b*c*(-(4*a*c - b^2)^3)^{1/2}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{1/2} + (8192*\tan(x/
\end{aligned}$$

$$\begin{aligned}
& 2) * (8 * a * b^8 - 8 * a^3 * b^6 + a^5 * b^4 + a^7 * c^2 - 48 * a^2 * b^6 * c + 32 * a^4 * b^4 * c - \\
& 2 * a^6 * b^2 * c + 72 * a^3 * b^4 * c^2 - 16 * a^4 * b^2 * c^3 - 16 * a^5 * b^2 * c^2)) / c^4 * ((b^8 - \\
& a^2 * b^6 + 8 * a^4 * c^4 + 8 * a^5 * c^3 + b^5 * (- (4 * a * c - b^2)^3)^{1/2} + 8 * a^3 * \\
& b^4 * c - a^2 * b^3 * (- (4 * a * c - b^2)^3)^{1/2} + 33 * a^2 * b^4 * c^2 - 38 * a^3 * b^2 * c^3 \\
& - 18 * a^4 * b^2 * c^2 - 10 * a * b^6 * c + 3 * a^2 * b * c^2 * (- (4 * a * c - b^2)^3)^{1/2} - 4 * a * \\
& b^3 * c * (- (4 * a * c - b^2)^3)^{1/2} + 2 * a^3 * b * c * (- (4 * a * c - b^2)^3)^{1/2})) / (2 * (16 \\
& * a^2 * c^8 + 32 * a^3 * c^7 + 16 * a^4 * c^6 + b^4 * c^6 - b^6 * c^4 - 8 * a * b^2 * c^7 + 10 * a \\
& * b^4 * c^5 - 32 * a^2 * b^2 * c^6 + a^2 * b^4 * c^4 - 8 * a^3 * b^2 * c^5)))^{1/2} * i) / (((819 \\
& 2 * (4 * a^2 * b^7 - 3 * a^4 * b^5 - 20 * a^3 * b^5 * c + 9 * a^5 * b^3 * c + 20 * a^4 * b^3 * c^2)) / c^4 - \\
& ((8192 * (4 * a * b^7 * c^2 - 2 * a^2 * b^7 * c + 2 * a^4 * b^5 * c + 12 * a^5 * b * c^4 + 8 * a^6 * \\
& b * c^3 - 24 * a^2 * b^5 * c^3 + 32 * a^3 * b^3 * c^4 + 10 * a^3 * b^5 * c^2 - 10 * a^4 * b^3 * c^3 - \\
& 10 * a^5 * b^3 * c^2)) / c^4 + ((b^8 - a^2 * b^6 + 8 * a^4 * c^4 + 8 * a^5 * c^3 + b^5 * (- (4 * \\
& a * c - b^2)^3)^{1/2} + 8 * a^3 * b^4 * c - a^2 * b^3 * (- (4 * a * c - b^2)^3)^{1/2} + 33 * a \\
& ^2 * b^4 * c^2 - 38 * a^3 * b^2 * c^3 - 18 * a^4 * b^2 * c^2 - 10 * a * b^6 * c + 3 * a^2 * b * c^2 * (- (4 \\
& * a * c - b^2)^3)^{1/2} - 4 * a * b^3 * c * (- (4 * a * c - b^2)^3)^{1/2} + 2 * a^3 * b * c * (- (4 \\
& * a * c - b^2)^3)^{1/2})) / (2 * (16 * a^2 * c^8 + 32 * a^3 * c^7 + 16 * a^4 * c^6 + b^4 * c^6 - b^6 * c^4 - \\
& 8 * a * b^2 * c^7 + 10 * a * b^4 * c^5 - 32 * a^2 * b^2 * c^6 + a^2 * b^4 * c^4 - 8 * a^3 * b^2 * \\
& * b^2 * c^5)))^{1/2} * (((b^8 - a^2 * b^6 + 8 * a^4 * c^4 + 8 * a^5 * c^3 + b^5 * (- (4 * a * c - \\
& b^2)^3)^{1/2} + 8 * a^3 * b^4 * c - a^2 * b^3 * (- (4 * a * c - b^2)^3)^{1/2} + 33 * a^2 * b^4 \\
& * c^2 - 38 * a^3 * b^2 * c^3 - 18 * a^4 * b^2 * c^2 - 10 * a * b^6 * c + 3 * a^2 * b * c^2 * (- (4 * a * c \\
& - b^2)^3)^{1/2} - 4 * a * b^3 * c * (- (4 * a * c - b^2)^3)^{1/2} + 2 * a^3 * b * c * (- (4 * a * c \\
& - b^2)^3)^{1/2})) / (2 * (16 * a^2 * c^8 + 32 * a^3 * c^7 + 16 * a^4 * c^6 + b^4 * c^6 - b^6 * c^4 - \\
& 8 * a * b^2 * c^7 + 10 * a * b^4 * c^5 - 32 * a^2 * b^2 * c^6 + a^2 * b^4 * c^4 - 8 * a^3 * b^2 * \\
& c^5)))^{1/2} * ((8192 * (3 * a * b^5 * c^6 + 16 * a^3 * b * c^8 - 4 * a^4 * b * c^7 - 8 * a^5 * b * c^6 \\
& - 16 * a^2 * b^3 * c^7 - 2 * a^2 * b^5 * c^5 + 9 * a^3 * b^3 * c^6 + 2 * a^4 * b^3 * c^5)) / c^4 - ( \\
& (8192 * (3 * a * b^5 * c^7 - 4 * a * b^3 * c^9 + 16 * a^2 * b * c^10 + 20 * a^3 * b * c^9 + 12 * a^4 * b * \\
& c^8 - 17 * a^2 * b^3 * c^8 - 3 * a^3 * b^3 * c^7)) / c^4 + (8192 * \tan(x/2) * (64 * a^2 * c^11 + \\
& 144 * a^3 * c^10 + 104 * a^4 * c^9 + 24 * a^5 * c^8 - 16 * a * b^2 * c^10 + 17 * a * b^4 * c^8 - 2 * \\
& a * b^6 * c^6 - 104 * a^2 * b^2 * c^9 + 18 * a^2 * b^4 * c^7 - 66 * a^3 * b^2 * c^8 + 2 * a^3 * b^4 * c^6 \\
& - 14 * a^4 * b^2 * c^7)) / c^4 * ((b^8 - a^2 * b^6 + 8 * a^4 * c^4 + 8 * a^5 * c^3 + b^5 * (- \\
& (4 * a * c - b^2)^3)^{1/2} + 8 * a^3 * b^4 * c - a^2 * b^3 * (- (4 * a * c - b^2)^3)^{1/2} + 3 \\
& 3 * a^2 * b^4 * c^2 - 38 * a^3 * b^2 * c^3 - 18 * a^4 * b^2 * c^2 - 10 * a * b^6 * c + 3 * a^2 * b * c^2 * \\
& (- (4 * a * c - b^2)^3)^{1/2} - 4 * a * b^3 * c * (- (4 * a * c - b^2)^3)^{1/2} + 2 * a^3 * b * c * \\
& (- (4 * a * c - b^2)^3)^{1/2})) / (2 * (16 * a^2 * c^8 + 32 * a^3 * c^7 + 16 * a^4 * c^6 + b^4 * c^6 - \\
& b^6 * c^4 - 8 * a * b^2 * c^7 + 10 * a * b^4 * c^5 - 32 * a^2 * b^2 * c^6 + a^2 * b^4 * c^4 - 8 * \\
& a^3 * b^2 * c^5)))^{1/2} + (8192 * \tan(x/2) * (32 * a^3 * c^9 + 48 * a^4 * c^8 + 16 * a^5 * c^7 \\
& + 8 * a * b^4 * c^7 - 4 * a * b^6 * c^5 - 40 * a^2 * b^2 * c^8 + 28 * a^2 * b^4 * c^6 - 60 * a^3 * b^2 \\
& * c^7 + 4 * a^3 * b^4 * c^5 - 20 * a^4 * b^2 * c^6)) / c^4) - (8192 * (3 * a * b^7 * c^3 - 4 * a * b^5 \\
& * c^5 + 20 * a^4 * b * c^6 + 9 * a^5 * b * c^5 + 16 * a^2 * b^3 * c^6 - 13 * a^2 * b^5 * c^4 - 3 * a^3 \\
& * b^5 * c^3 + 9 * a^4 * b^3 * c^4)) / c^4 + (8192 * \tan(x/2) * (16 * a^4 * c^7 + 24 * a^5 * c^6 + \\
& 10 * a^6 * c^5 + 16 * a * b^4 * c^6 - 24 * a * b^6 * c^4 + 2 * a * b^8 * c^2 - 64 * a^2 * b^2 * c^7 + 1 \\
& 44 * a^2 * b^4 * c^5 - 18 * a^2 * b^6 * c^3 - 200 * a^3 * b^2 * c^6 + 75 * a^3 * b^4 * c^4 - 2 * a^3 * \\
& b^6 * c^2 - 142 * a^4 * b^2 * c^5 + 14 * a^4 * b^4 * c^3 - 27 * a^5 * b^2 * c^4)) / c^4) - (8192 * \\
& \tan(x/2) * (8 * a^5 * c^5 + 4 * a^6 * c^4 - 8 * a * b^6 * c^3 - 4 * a^3 * b^6 * c + 40 * a^2 * b^4 * c^4 \\
& - 28 * a^2 * b^6 * c^2 - 32 * a^3 * b^2 * c^5 + 60 * a^3 * b^4 * c^3 - 56 * a^4 * b^2 * c^4 + 20 * \\
& a^4 * b^4 * c^2 - 16 * a^5 * b^2 * c^3 + 4 * a * b^8 * c)) / c^4 * ((b^8 - a^2 * b^6 + 8 * a^4 * c^4 \\
& + 8 * a^5 * c^3 + b^5 * (- (4 * a * c - b^2)^3)^{1/2} + 8 * a^3 * b^4 * c - a^2 * b^3 * (- (4 * a * \\
& c - b^2)^3)^{1/2} + 33 * a^2 * b^4 * c^2 - 38 * a^3 * b^2 * c^3 - 18 * a^4 * b^2 * c^2 - 10 * a \\
& * b^6 * c + 3 * a^2 * b * c^2 * (- (4 * a * c - b^2)^3)^{1/2} - 4 * a * b^3 * c * (- (4 * a * c - b^2)^3 \\
& )^{1/2} + 2 * a^3 * b * c * (- (4 * a * c - b^2)^3)^{1/2})) / (2 * (16 * a^2 * c^8 + 32 * a^3 * c^7 + \\
& 16 * a^4 * c^6 + b^4 * c^6 - b^6 * c^4 - 8 * a * b^2 * c^7 + 10 * a * b^4 * c^5 - 32 * a^2 * b^2 * c^6 \\
& + a^2 * b^4 * c^4 - 8 * a^3 * b^2 * c^5)))^{1/2} + (8192 * \tan(x/2) * (8 * a * b^8 - 8 * a^3 \\
& * b^6 + a^5 * b^4 + a^7 * c^2 - 48 * a^2 * b^6 * c + 32 * a^4 * b^4 * c - 2 * a^6 * b^2 * c + 72 * a \\
& ^3 * b^4 * c^2 - 16 * a^4 * b^2 * c^3 - 16 * a^5 * b^2 * c^2)) / c^4 * ((b^8 - a^2 * b^6 + 8 * a^4 \\
& * c^4 + 8 * a^5 * c^3 + b^5 * (- (4 * a * c - b^2)^3)^{1/2} + 8 * a^3 * b^4 * c - a^2 * b^3 * (- (4 \\
& * a * c - b^2)^3)^{1/2} + 33 * a^2 * b^4 * c^2 - 38 * a^3 * b^2 * c^3 - 18 * a^4 * b^2 * c^2 - \\
& 10 * a * b^6 * c + 3 * a^2 * b * c^2 * (- (4 * a * c - b^2)^3)^{1/2} - 4 * a * b^3 * c * (- (4 * a * c - b^2 \\
& )^3)^{1/2} + 2 * a^3 * b * c * (- (4 * a * c - b^2)^3)^{1/2})) / (2 * (16 * a^2 * c^8 + 32 * a^3 * c^7 \\
& + 16 * a^4 * c^6 + b^4 * c^6 - b^6 * c^4 - 8 * a * b^2 * c^7 + 10 * a * b^4 * c^5 - 32 * a^2 * b^2 *
\end{aligned}$$

$$\begin{aligned}
& \left( a^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5 \right)^{1/2} - \left( (8192 (4 a^2 b^7 - 3 a^4 b^5 - 20 a^3 b^5 c + 9 a^5 b^3 c + 20 a^4 b^3 c^2)) / c^4 + \left( (8192 (4 a^2 b^7 c^2 - 2 a^2 b^7 c + 2 a^4 b^5 c + 12 a^5 b^3 c^4 + 8 a^6 b^3 c^3 - 24 a^2 b^5 c^3 + 32 a^3 b^3 c^4 + 10 a^3 b^5 c^2 - 10 a^4 b^3 c^3 - 10 a^5 b^3 c^2)) / c^4 + \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big* \left( (8192 (3 a^2 b^7 c^3 - 4 a^2 b^5 c^5 + 20 a^4 b^3 c^6 + 9 a^5 b^3 c^5 + 16 a^2 b^3 c^6 - 13 a^2 b^5 c^4 - 3 a^3 b^5 c^3 + 9 a^4 b^3 c^4)) / c^4 + \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big* \left( (8192 (3 a^2 b^5 c^6 + 16 a^3 b^3 c^8 - 4 a^4 b^3 c^7 - 8 a^5 b^3 c^6 - 16 a^2 b^3 c^7 - 2 a^2 b^5 c^5 + 9 a^3 b^3 c^6 + 2 a^4 b^3 c^5)) / c^4 + \left( (8192 (3 a^2 b^5 c^7 - 4 a^2 b^3 c^9 + 16 a^2 b^3 c^{10} + 20 a^3 b^3 c^9 + 12 a^4 b^3 c^8 - 17 a^2 b^3 c^8 - 3 a^3 b^3 c^7)) / c^4 + (8192 \tan(x/2) (64 a^2 c^{11} + 144 a^3 c^{10} + 104 a^4 c^9 + 24 a^5 c^8 - 16 a b^2 c^{10} + 17 a b^4 c^8 - 2 a b^6 c^6 - 104 a^2 b^2 c^9 + 18 a^2 b^4 c^7 - 66 a^3 b^2 c^8 + 2 a^3 b^4 c^6 - 14 a^4 b^2 c^7)) / c^4 \right) \Big* \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big+ (8192 \tan(x/2) (32 a^3 c^9 + 48 a^4 c^8 + 16 a^5 c^7 + 8 a b^4 c^7 - 4 a b^6 c^5 - 40 a^2 b^2 c^8 + 28 a^2 b^4 c^6 - 60 a^3 b^2 c^7 + 4 a^3 b^4 c^5 - 20 a^4 b^2 c^6)) / c^4 - (8192 \tan(x/2) (16 a^4 c^7 + 24 a^5 c^6 + 10 a^6 c^5 + 16 a b^4 c^6 - 24 a b^6 c^4 + 2 a b^8 c^2 - 64 a^2 b^2 c^7 + 144 a^2 b^4 c^5 - 18 a^2 b^6 c^3 - 200 a^3 b^2 c^6 + 75 a^3 b^4 c^4 - 2 a^3 b^6 c^2 - 142 a^4 b^2 c^5 + 14 a^4 b^4 c^3 - 27 a^5 b^2 c^4)) / c^4 - (8192 \tan(x/2) (8 a^5 c^5 + 4 a^6 c^4 - 8 a b^6 c^3 - 4 a^3 b^6 c + 40 a^2 b^4 c^4 - 28 a^2 b^6 c^2 - 32 a^3 b^2 c^5 + 60 a^3 b^4 c^3 - 56 a^4 b^2 c^4 + 20 a^4 b^4 c^2 - 16 a^5 b^2 c^3 + 4 a b^8 c)) / c^4 \Big* \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big+ (8192 \tan(x/2) (8 a^2 b^8 - 8 a^3 b^6 + a^5 b^4 + a^7 c^2 - 48 a^2 b^6 c + 32 a^4 b^4 c - 2 a^6 b^2 c + 72 a^3 b^4 c^2 - 16 a^4 b^2 c^3 - 16 a^5 b^2 c^2)) / c^4 \Big* \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big+ (16384 (a^7 b - 4 a^5 b^3)) / c^4 + (16384 \tan(x/2) (4 a^6 b^2 - 8 a^4 b^4 + 8 a^5 b^2 c)) / c^4 \Big* \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big+ (16384 \tan(x/2) (4 a^6 b^2 - 8 a^4 b^4 + 8 a^5 b^2 c)) / c^4 \Big* \left( (b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 + b^5 (-4 a c - b^2)^3 \right)^{1/2} + 8 a^3 b^4 c - a^2 b^3 (-4 a c - b^2)^3 \right)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a b^6 c + 3 a^2 b^2 c^2 (-4 a c - b^2)^3 \right)^{1/2} - 4 a b^3 c (-4 a c - b^2)^3 \right)^{1/2} + 2 a^3 b^2 c (-4 a c - b^2)^3 \right)^{1/2} \Big/ \left( 2 (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a b^2 c^7 + 10 a b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5) \right)^{1/2} \Big
\end{aligned}$$



$$\begin{aligned}
& *a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^8 + \\
& 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - \\
& 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)*(((b^8 - a^2*b^6 + 8 \\
& *a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^ \\
& 2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^8 + 32*a \\
& ^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a \\
& ^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)*((8192*(3*a*b^5*c^6 + 16* \\
& a^3*b*c^8 - 4*a^4*b*c^7 - 8*a^5*b*c^6 - 16*a^2*b^3*c^7 - 2*a^2*b^5*c^5 + 9* \\
& a^3*b^3*c^6 + 2*a^4*b^3*c^5))/c^4 - ((8192*(3*a*b^5*c^7 - 4*a*b^3*c^9 + 16* \\
& a^2*b*c^10 + 20*a^3*b*c^9 + 12*a^4*b*c^8 - 17*a^2*b^3*c^8 - 3*a^3*b^3*c^7)) \\
& /c^4 + (8192*\tan(x/2)*(64*a^2*c^11 + 144*a^3*c^10 + 104*a^4*c^9 + 24*a^5*c^ \\
& 8 - 16*a*b^2*c^10 + 17*a*b^4*c^8 - 2*a*b^6*c^6 - 104*a^2*b^2*c^9 + 18*a^2*b \\
& ^4*c^7 - 66*a^3*b^2*c^8 + 2*a^3*b^4*c^6 - 14*a^4*b^2*c^7))/c^4)*((b^8 - a^2 \\
& *b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + \\
& a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^ \\
& 4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(- \\
& -(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^ \\
& 8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^ \\
& 5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)} + (8192*\tan(x/2)* \\
& (32*a^3*c^9 + 48*a^4*c^8 + 16*a^5*c^7 + 8*a*b^4*c^7 - 4*a*b^6*c^5 - 40*a^2* \\
& b^2*c^8 + 28*a^2*b^4*c^6 - 60*a^3*b^2*c^7 + 4*a^3*b^4*c^5 - 20*a^4*b^2*c^6) \\
& )/c^4) - (8192*(3*a*b^7*c^3 - 4*a*b^5*c^5 + 20*a^4*b*c^6 + 9*a^5*b*c^5 + 16 \\
& *a^2*b^3*c^6 - 13*a^2*b^5*c^4 - 3*a^3*b^5*c^3 + 9*a^4*b^3*c^4))/c^4 + (8192 \\
& *\tan(x/2)*(16*a^4*c^7 + 24*a^5*c^6 + 10*a^6*c^5 + 16*a*b^4*c^6 - 24*a*b^6*c \\
& ^4 + 2*a*b^8*c^2 - 64*a^2*b^2*c^7 + 144*a^2*b^4*c^5 - 18*a^2*b^6*c^3 - 200* \\
& a^3*b^2*c^6 + 75*a^3*b^4*c^4 - 2*a^3*b^6*c^2 - 142*a^4*b^2*c^5 + 14*a^4*b^4 \\
& *c^3 - 27*a^5*b^2*c^4))/c^4) - (8192*\tan(x/2)*(8*a^5*c^5 + 4*a^6*c^4 - 8*a* \\
& b^6*c^3 - 4*a^3*b^6*c + 40*a^2*b^4*c^4 - 28*a^2*b^6*c^2 - 32*a^3*b^2*c^5 + \\
& 60*a^3*b^4*c^3 - 56*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a*b^8 \\
& *c))/c^4)*((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^ \\
& ^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 3 \\
& 8*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3 \\
& )^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3 \\
& )^{(1/2)})/(2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a* \\
& b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1 \\
& /2)} + (8192*\tan(x/2)*(8*a*b^8 - 8*a^3*b^6 + a^5*b^4 + a^7*c^2 - 48*a^2*b^6* \\
& c + 32*a^4*b^4*c - 2*a^6*b^2*c + 72*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 16*a^5*b \\
& ^2*c^2))/c^4)*((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 \\
& - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2 \\
& )^3)^{(1/2)})/(2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - \\
& 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)) \\
& )^{(1/2)}*i)/(((8192*(4*a^2*b^7 - 3*a^4*b^5 - 20*a^3*b^5*c + 9*a^5*b^3*c + 2 \\
& 0*a^4*b^3*c^2))/c^4 - ((8192*(4*a*b^7*c^2 - 2*a^2*b^7*c + 2*a^4*b^5*c + 12* \\
& a^5*b*c^4 + 8*a^6*b*c^3 - 24*a^2*b^5*c^3 + 32*a^3*b^3*c^4 + 10*a^3*b^5*c^2 \\
& - 10*a^4*b^3*c^3 - 10*a^5*b^3*c^2))/c^4 + ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a \\
& ^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c \\
& - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
& ) - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^ \\
& 4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a \\
& ^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)*(((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^ \\
& 3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3 \\
& )^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3* \\
& a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2 \\
& *a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6
\end{aligned}$$

$$\begin{aligned}
& + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} * ((8192 * (3 a^2 b^5 c^6 + 16 a^3 b^2 c^8 - 4 a^4 b^2 c^7 - 8 a^5 b^2 c^6 - 16 a^2 b^3 c^7 - 2 a^2 b^5 c^5 + 9 a^3 b^3 c^6 + 2 a^4 b^3 c^5)) / c^4 - ((8192 * (3 a^2 b^5 c^7 - 4 a^2 b^3 c^9 + 16 a^2 b^2 c^{10} + 20 a^3 b^2 c^9 + 12 a^4 b^2 c^8 - 17 a^2 b^3 c^8 - 3 a^3 b^3 c^7)) / c^4 + (8192 * \tan(x/2) * (64 a^2 c^{11} + 144 a^3 c^{10} + 104 a^4 c^9 + 24 a^5 c^8 - 16 a^2 b^2 c^{10} + 17 a^2 b^4 c^8 - 2 a^2 b^6 c^6 - 104 a^2 b^2 c^9 + 18 a^2 b^4 c^7 - 66 a^3 b^2 c^8 + 2 a^3 b^4 c^6 - 14 a^4 b^2 c^7)) / c^4) * ((b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 - b^5 * (-4 a^2 c - b^2)^3)^{1/2} + 8 a^3 b^4 c + a^2 b^3 * (-4 a^2 c - b^2)^3)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a^2 b^6 c - 3 a^2 b^2 c^2 * (-4 a^2 c - b^2)^3)^{1/2} + 4 a^2 b^3 c * (-4 a^2 c - b^2)^3)^{1/2} - 2 a^3 b^2 c * (-4 a^2 c - b^2)^3)^{1/2} / (2 * (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} + (8192 * \tan(x/2) * (32 a^3 c^9 + 48 a^4 c^8 + 16 a^5 c^7 + 8 a^2 b^4 c^7 - 4 a^2 b^6 c^5 - 40 a^2 b^2 c^8 + 28 a^2 b^4 c^6 - 60 a^3 b^2 c^7 + 4 a^3 b^4 c^5 - 20 a^4 b^2 c^6)) / c^4 - (8192 * (3 a^2 b^7 c^3 - 4 a^2 b^5 c^5 + 20 a^4 b^2 c^6 + 9 a^5 b^2 c^5 + 16 a^2 b^3 c^6 - 13 a^2 b^5 c^4 - 3 a^3 b^5 c^3 + 9 a^4 b^3 c^4)) / c^4 + (8192 * \tan(x/2) * (16 a^4 c^7 + 24 a^5 c^6 + 10 a^6 c^5 + 16 a^2 b^4 c^6 - 24 a^2 b^6 c^4 + 2 a^2 b^8 c^2 - 64 a^2 b^2 c^7 + 144 a^2 b^4 c^5 - 18 a^2 b^6 c^3 - 200 a^3 b^2 c^6 + 75 a^3 b^4 c^4 - 2 a^3 b^6 c^2 - 142 a^4 b^2 c^5 + 14 a^4 b^4 c^3 - 27 a^5 b^2 c^4)) / c^4 - (8192 * \tan(x/2) * (8 a^5 c^5 + 4 a^6 c^4 - 8 a^2 b^6 c^3 - 4 a^3 b^6 c + 40 a^2 b^4 c^4 - 28 a^2 b^6 c^2 - 32 a^3 b^2 c^5 + 60 a^3 b^4 c^3 - 56 a^4 b^2 c^4 + 20 a^4 b^4 c^2 - 16 a^5 b^2 c^3 + 4 a^2 b^8 c)) / c^4) * ((b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 - b^5 * (-4 a^2 c - b^2)^3)^{1/2} + 8 a^3 b^4 c + a^2 b^3 * (-4 a^2 c - b^2)^3)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a^2 b^6 c - 3 a^2 b^2 c^2 * (-4 a^2 c - b^2)^3)^{1/2} + 4 a^2 b^3 c * (-4 a^2 c - b^2)^3)^{1/2} - 2 a^3 b^2 c * (-4 a^2 c - b^2)^3)^{1/2} / (2 * (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} + (8192 * \tan(x/2) * (8 a^2 b^8 - 8 a^3 b^6 + a^5 b^4 + a^7 c^2 - 48 a^2 b^6 c + 32 a^4 b^4 c - 2 a^6 b^2 c + 72 a^3 b^4 c^2 - 16 a^4 b^2 c^3 - 16 a^5 b^2 c^2)) / c^4) * ((b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 - b^5 * (-4 a^2 c - b^2)^3)^{1/2} + 8 a^3 b^4 c + a^2 b^3 * (-4 a^2 c - b^2)^3)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a^2 b^6 c - 3 a^2 b^2 c^2 * (-4 a^2 c - b^2)^3)^{1/2} + 4 a^2 b^3 c * (-4 a^2 c - b^2)^3)^{1/2} - 2 a^3 b^2 c * (-4 a^2 c - b^2)^3)^{1/2} / (2 * (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} - ((8192 * (4 a^2 b^7 - 3 a^4 b^5 - 20 a^3 b^5 c + 9 a^5 b^3 c + 20 a^4 b^3 c^2)) / c^4 + ((8192 * (4 a^2 b^7 c^2 - 2 a^2 b^7 c + 2 a^4 b^5 c + 12 a^5 b^2 c^4 + 8 a^6 b^2 c^3 - 24 a^2 b^5 c^3 + 32 a^3 b^3 c^4 + 10 a^3 b^5 c^2 - 10 a^4 b^3 c^3 - 10 a^5 b^3 c^2)) / c^4 + ((b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 - b^5 * (-4 a^2 c - b^2)^3)^{1/2} + 8 a^3 b^4 c + a^2 b^3 * (-4 a^2 c - b^2)^3)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a^2 b^6 c - 3 a^2 b^2 c^2 * (-4 a^2 c - b^2)^3)^{1/2} + 4 a^2 b^3 c * (-4 a^2 c - b^2)^3)^{1/2} - 2 a^3 b^2 c * (-4 a^2 c - b^2)^3)^{1/2} / (2 * (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} * ((8192 * (3 a^2 b^7 c^3 - 4 a^2 b^5 c^5 + 20 a^4 b^2 c^6 + 9 a^5 b^2 c^5 + 16 a^2 b^3 c^6 - 13 a^2 b^5 c^4 - 3 a^3 b^5 c^3 + 9 a^4 b^3 c^4)) / c^4 + ((b^8 - a^2 b^6 + 8 a^4 c^4 + 8 a^5 c^3 - b^5 * (-4 a^2 c - b^2)^3)^{1/2} + 8 a^3 b^4 c + a^2 b^3 * (-4 a^2 c - b^2)^3)^{1/2} + 33 a^2 b^4 c^2 - 38 a^3 b^2 c^3 - 18 a^4 b^2 c^2 - 10 a^2 b^6 c - 3 a^2 b^2 c^2 * (-4 a^2 c - b^2)^3)^{1/2} + 4 a^2 b^3 c * (-4 a^2 c - b^2)^3)^{1/2} - 2 a^3 b^2 c * (-4 a^2 c - b^2)^3)^{1/2} / (2 * (16 a^2 c^8 + 32 a^3 c^7 + 16 a^4 c^6 + b^4 c^6 - b^6 c^4 - 8 a^2 b^2 c^7 + 10 a^2 b^4 c^5 - 32 a^2 b^2 c^6 + a^2 b^4 c^4 - 8 a^3 b^2 c^5))^{1/2} * ((8192 * (3 a^2 b^5 c^6 + 16 a^3 b^2 c^8 - 4 a^4 b^2 c^7 - 8 a^5 b^2 c^6 - 16 a^2 b^3 c^7 - 2 a^2 b^5 c^5 + 9 a^3 b^3 c^6 + 2 a^4 b^3 c^5)) / c^4 + ((8192 * (3 a^2 b^5 c^7 - 4 a^2 b^3 c^9 + 16 a^2 b^2 c^{10} + 20 a^3 b^2 c^9 + 12 a^4 b^2 c^8 - 17 a^2 b^3 c^8 - 3 a^3 b^3 c^7)) / c^4 + (8192 * \tan(x/2) * (64 a^2 c^{11} + 144 a^3 c^{10} + 104
\end{aligned}$$

$$\begin{aligned}
& *a^4*c^9 + 24*a^5*c^8 - 16*a*b^2*c^10 + 17*a*b^4*c^8 - 2*a*b^6*c^6 - 104*a^2*b^2*c^9 + 18*a^2*b^4*c^7 - 66*a^3*b^2*c^8 + 2*a^3*b^4*c^6 - 14*a^4*b^2*c^7) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)} + (8192*\tan(x/2)*(32*a^3*c^9 + 48*a^4*c^8 + 16*a^5*c^7 + 8*a*b^4*c^7 - 4*a*b^6*c^5 - 40*a^2*b^2*c^8 + 28*a^2*b^4*c^6 - 60*a^3*b^2*c^7 + 4*a^3*b^4*c^5 - 20*a^4*b^2*c^6)) / c^4 - (8192*\tan(x/2)*(16*a^4*c^7 + 24*a^5*c^6 + 10*a^6*c^5 + 16*a*b^4*c^6 - 24*a*b^6*c^4 + 2*a*b^8*c^2 - 64*a^2*b^2*c^7 + 144*a^2*b^4*c^5 - 18*a^2*b^6*c^3 - 200*a^3*b^2*c^6 + 75*a^3*b^4*c^4 - 2*a^3*b^6*c^2 - 142*a^4*b^2*c^5 + 14*a^4*b^4*c^3 - 27*a^5*b^2*c^4)) / c^4 - (8192*\tan(x/2)*(8*a^5*c^5 + 4*a^6*c^4 - 8*a*b^6*c^3 - 4*a^3*b^6*c + 40*a^2*b^4*c^4 - 28*a^2*b^6*c^2 - 32*a^3*b^2*c^5 + 60*a^3*b^4*c^3 - 56*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a*b^8*c)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)} + (8192*\tan(x/2)*(8*a*b^8 - 8*a^3*b^6 + a^5*b^4 + a^7*c^2 - 48*a^2*b^6*c + 32*a^4*b^4*c - 2*a^6*b^2*c + 72*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 16*a^5*b^2*c^2)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)} + (16384*(a^7*b - 4*a^5*b^3)) / c^4 + (16384*\tan(x/2)*(4*a^6*b^2 - 8*a^4*b^4 + 8*a^5*b^2*c)) / c^4 * ((b^8 - a^2*b^6 + 8*a^4*c^4 + 8*a^5*c^3 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^3*b^4*c + a^2*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 18*a^4*b^2*c^2 - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^3*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(16*a^2*c^8 + 32*a^3*c^7 + 16*a^4*c^6 + b^4*c^6 - b^6*c^4 - 8*a*b^2*c^7 + 10*a*b^4*c^5 - 32*a^2*b^2*c^6 + a^2*b^4*c^4 - 8*a^3*b^2*c^5)))^{(1/2)} * 2i - (2*b*atan((16384*a*b^9*\tan(x/2)) / (16384*a*b^9 + 16384*a^3*b^7 - 32768*a^5*b^5 - 131072*a^2*b^7*c - 98304*a^4*b^5*c + 131072*a^6*b^3*c + 16384*a^7*b*c^2 + 262144*a^3*b^5*c^2 + 131072*a^5*b^3*c^2) + (16384*a^7*b*\tan(x/2)) / (16384*a^7*b + 262144*a^3*b^5 + 131072*a^5*b^3 + (16384*a*b^9) / c^2 - (131072*a^2*b^7) / c - (98304*a^4*b^5) / c + (131072*a^6*b^3) / c + (16384*a^3*b^7) / c^2 - (32768*a^5*b^5) / c^2) - (131072*a^2*b^7*\tan(x/2)) / (131072*a^6*b^3 - 98304*a^4*b^5 - 131072*a^2*b^7 + 262144*a^3*b^5*c + 131072*a^5*b^3*c + (16384*a*b^9) / c + (16384*a^3*b^7) / c - (32768*a^5*b^5) / c + 16384*a^7*b*c) - (98304*a^4*b^5*\tan(x/2)) / (131072*a^6*b^3 - 98304*a^4*b^5 - 131072*a^2*b^7 + 262144*a^3*b^5*c + 131072*a^5*b^3*c + (16384*a*b^9) / c + (16384*a^3*b^7) / c - (32768*a^5*b^5) / c + 16384*a^7*b*c) + (131072*a^6*b^3*\tan(x/2)) / (131072*a^6*b^3 - 98304*a^4*b^5 - 131072*a^2*b^7 + 262144*a^3*b^5*c + 131072*a^5*b^3*c + (16384*a*b^9) / c + (16384*a^3*b^7) / c - (32768*a^5*b^5) / c + 16384*a^7*b*c) + (16384*a^3*b^7*\tan(x/2)) / (16384*a*b^9 + 16384*a^3*b^7 - 32768*a^5*b^5 - 131072*a^2*b^7*c - 98304*a^4*b^5*c + 131072*a^6*b^3*c + 16384*a^7*b*c^2 + 262144*a^3*b^5*c^2 + 131072*a^5*b^3*c^2) - (32768*a^5*b^5*\tan(x/2)) / (16384*a*b^9 + 16384*a^3*b^7 - 32768*a^5*b^5 - 131072*a^2*b^7*c - 98304*a^4*b^5*c + 131072*a^6*b^3*c + 16384*a^7*b*c^2 + 262144*a^3*b^5*c^2 + 131072*a^5*b^3*c^2) + (262144*a^3*b^5*\tan(x/2)) / (16384*a^7*b + 262144*a^3*b^5 + 131072*a^5*b^3 + (16384*a*b^9) / c^2 - (131072*a^2*b^7) / c - (98304*a^4*b^5) / c + (131072*a^6*b^3) / c + (16384*a^3*b^7) / c^2 - (32768*a^5*b^5) / c^2) + (131072*a^5*b^3*\tan(x/2)) / (16384*a^7*b + 2
\end{aligned}$$

```
62144*a^3*b^5 + 131072*a^5*b^3 + (16384*a*b^9)/c^2 - (131072*a^2*b^7)/c - (
98304*a^4*b^5)/c + (131072*a^6*b^3)/c + (16384*a^3*b^7)/c^2 - (32768*a^5*b^
5)/c^2)))/c^2
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Timed out
```



### 3.3 $\int \frac{\sin^2(x)}{a+b \sin(x)+c \sin^2(x)} dx$

**Optimal.** Leaf size=253

$$\frac{\sqrt{2} \left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) \left( b - \sqrt{b^2-4ac} \right) + 2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} \right)}{c\sqrt{-b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} - \frac{\sqrt{2} \left( \frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) \left( \sqrt{b^2-4ac} + b \right) + 2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} \right)}{c\sqrt{b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} + \frac{x}{c}$$

[Out]  $x/c - \arctan\left(\frac{1}{2} \cdot (2c + (b - (-4ac + b^2)^{1/2})) \cdot \tan(1/2x)\right) \cdot 2^{1/2} / (b^2 - 2c(a+c) - b(-4ac + b^2)^{1/2})^{1/2} \cdot 2^{1/2} \cdot (b + (2ac - b^2) / (-4ac + b^2)^{1/2}) / (b^2 - 2c(a+c) - b(-4ac + b^2)^{1/2})^{1/2} - \arctan\left(\frac{1}{2} \cdot (2c + (b + (-4ac + b^2)^{1/2})) \cdot \tan(1/2x)\right) \cdot 2^{1/2} / (b^2 - 2c(a+c) + b(-4ac + b^2)^{1/2})^{1/2} \cdot 2^{1/2} \cdot (b + (-2ac + b^2) / (-4ac + b^2)^{1/2}) / (b^2 - 2c(a+c) + b(-4ac + b^2)^{1/2})^{1/2}$

**Rubi [A]** time = 1.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {3256, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} \left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) \left( b - \sqrt{b^2-4ac} \right) + 2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} \right)}{c\sqrt{-b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} - \frac{\sqrt{2} \left( \frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) \left( \sqrt{b^2-4ac} + b \right) + 2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} \right)}{c\sqrt{b\sqrt{b^2-4ac} - 2c(a+c) + b^2}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]^2/(a + b*Sin[x] + c*Sin[x]^2),x]`

[Out]  $x/c - (\text{Sqrt}[2] \cdot (b - (b^2 - 2ac)/\text{Sqrt}[b^2 - 4ac]) \cdot \text{ArcTan}[(2c + (b - \text{Sqrt}[b^2 - 4ac]) \cdot \text{Tan}[x/2]) / (\text{Sqrt}[2] \cdot \text{Sqrt}[b^2 - 2c(a+c) - b\text{Sqrt}[b^2 - 4ac]])]) / (c \cdot \text{Sqrt}[b^2 - 2c(a+c) - b\text{Sqrt}[b^2 - 4ac]]) - (\text{Sqrt}[2] \cdot (b + (b^2 - 2ac)/\text{Sqrt}[b^2 - 4ac]) \cdot \text{ArcTan}[(2c + (b + \text{Sqrt}[b^2 - 4ac]) \cdot \text{Tan}[x/2]) / (\text{Sqrt}[2] \cdot \text{Sqrt}[b^2 - 2c(a+c) + b\text{Sqrt}[b^2 - 4ac]])]) / (c \cdot \text{Sqrt}[b^2 - 2c(a+c) + b\text{Sqrt}[b^2 - 4ac]])$

#### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

#### Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]`

#### Rule 2660

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + dx)/2], x]}, Dist[(2e)/d, Subst[Int[1/(a + 2be*x + ae^2*x^2), x], x, Tan[(c + dx)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

#### Rule 3256

`Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(-p_), x_Symbol] := Int[ExpandTr`

```
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

### Rule 3292

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])/((a_) + (b_)*sin[(d_) + (e_)
*(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2), x_Symbol] := Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
;/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( \frac{1}{c} + \frac{-a - b \sin(x)}{c(a + b \sin(x) + c \sin^2(x))} \right) dx \\ &= \frac{x}{c} + \frac{\int \frac{-a - b \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{c} - \frac{\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{c} \\ &= \frac{x}{c} - \frac{\left(2\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b - \sqrt{b^2 - 4ac} + 4cx + (b - \sqrt{b^2 - 4ac})x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{c} - \frac{\left(2\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{c} \\ &= \frac{x}{c} + \frac{\left(4\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{-8(b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}) - x^2} dx, x, 4c + 2\left(b - \sqrt{b^2 - 4ac}\right)\right)}{c} \\ &= \frac{x}{c} - \frac{\sqrt{2}\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{2c + (b - \sqrt{b^2 - 4ac})\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}}\right)}{c\sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} - \frac{\sqrt{2}\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{2c + (b + \sqrt{b^2 - 4ac})\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}}\right)}{c\sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} \end{aligned}$$

**Mathematica** [C] time = 0.64, size = 310, normalized size = 1.23

$$\frac{\left(b\sqrt{4ac - b^2} - 2iac + ib^2\right) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b - i\sqrt{4ac - b^2})}{\sqrt{2}\sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}}\sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} - \frac{\left(b\sqrt{4ac - b^2} + 2iac - ib^2\right) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b + i\sqrt{4ac - b^2})}{\sqrt{2}\sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}}\sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x] + c*Sin[x]^2), x]
```

```
[Out] (x - ((I*b^2 - (2*I)*a*c + b*Sqrt[-b^2 + 4*a*c])*ArcTan[(2*c + (b - I*Sqrt[-b^2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]]) - (((-I)*b^2 + (2*I)*a*c + b*Sqrt[-b^2 + 4*a*c])*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]]))/c
```

**fricas** [B] time = 1.20, size = 4985, normalized size = 19.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*c*sqrt((a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4*a
*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*s
qrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4
*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b
^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^
2 - b^4)*c^2))*log(8*a^3*b*c^2 + 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(
2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^
4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*
c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))*sin(x) + 4*(a^
4*b - a^2*b^3)*c - sqrt(2)*((8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (24*a^4 -
22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 -
3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*
(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7
+ 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (
a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))*cos(x) - (8*a^2*b^2*c^3 + 2*(2*a^3*b^2 - 3
*a*b^4)*c^2 - (a^2*b^4 - b^6)*c)*cos(x))*sqrt((a^2*b^2 - b^4 - 2*a^2*c^2 -
2*(a^3 - 2*a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^
3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 +
4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*
c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5
- (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^
3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 2*(a^4*b^2 - a^2*b^4 + 2*a^3*b^2
*c)*sin(x) - sqrt(2)*c*sqrt((a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)
*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^
4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b
^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 -
11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^
2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3
- (a^2*b^2 - b^4)*c^2))*log(8*a^3*b*c^2 - 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*
c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 -
2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^
2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 +
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))*sin(x
) + 4*(a^4*b - a^2*b^3)*c - sqrt(2)*((8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (
24*a^4 - 22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a
^4*b^2 - 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2
*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a
*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))*cos(x) + (8*a^2*b^2*c^3 + 2*(2*a^
3*b^2 - 3*a*b^4)*c^2 - (a^2*b^4 - b^6)*c)*cos(x))*sqrt((a^2*b^2 - b^4 - 2*a
^2*c^2 - 2*(a^3 - 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*
a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*
b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3
- a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4
+ 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 2*(a^4*b^2 - a^2*b^4 +
2*a^3*b^2*c)*sin(x) + sqrt(2)*c*sqrt((a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 -
2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2
*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*
b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*
b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a
*b^2)*c^3 - (a^2*b^2 - b^4)*c^2))*log(-8*a^3*b*c^2 + 2*(4*a^3*c^5 + (8*a^4 -
a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-
```

$$\begin{aligned} & (a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c) / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \sin(x) - 4(a^4 b - a^2 b^3) c - \sqrt{2} \left( (8a^2 c^7 + 6(4a^3 - a b^2) c^6 + (24a^4 - 22a^2 b^2 + b^4) c^5 + 2(4a^5 - 9a^3 b^2 + 4a b^4) c^4 - (2a^4 b^2 - 3a^2 b^4 + b^6) c^3) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \right) \cos(x) + (8a^2 b^2 c^3 + 2(2a^3 b^2 - 3a b^4) c^2 - (a^2 b^4 - b^6) c) \cos(x) \sqrt{(a^2 b^2 - b^4 - 2a^2 c^2 - 2(a^3 - 2a b^2) c - (4a^2 c^5 + (8a^2 - b^2) c^4 + 2(2a^3 - 3a b^2) c^3 - (a^2 b^2 - b^4) c^2)) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \log(-8a^3 b c^2 - 2(4a^3 c^5 + (8a^4 - a^2 b^2) c^4 + 2(2a^5 - 3a^3 b^2) c^3 - (a^4 b^2 - a^2 b^4) c^2) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \sin(x) - 4(a^4 b - a^2 b^3) c - \sqrt{2} \left( (8a^2 c^7 + 6(4a^3 - a b^2) c^6 + (24a^4 - 22a^2 b^2 + b^4) c^5 + 2(4a^5 - 9a^3 b^2 + 4a b^4) c^4 - (2a^4 b^2 - 3a^2 b^4 + b^6) c^3) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \right) \cos(x) - (8a^2 b^2 c^3 + 2(2a^3 b^2 - 3a b^4) c^2 - (a^2 b^4 - b^6) c) \cos(x) \sqrt{(a^2 b^2 - b^4 - 2a^2 c^2 - 2(a^3 - 2a b^2) c + (4a^2 c^5 + (8a^2 - b^2) c^4 + 2(2a^3 - 3a b^2) c^3 - (a^2 b^2 - b^4) c^2)) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \sin(x) - 4(a^4 b - a^2 b^3) c - \sqrt{2} \left( (8a^2 c^7 + 6(4a^3 - a b^2) c^6 + (24a^4 - 22a^2 b^2 + b^4) c^5 + 2(4a^5 - 9a^3 b^2 + 4a b^4) c^4 - (2a^4 b^2 - 3a^2 b^4 + b^6) c^3) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \right) \cos(x) - (8a^2 b^2 c^3 + 2(2a^3 b^2 - 3a b^4) c^2 - (a^2 b^4 - b^6) c) \cos(x) \sqrt{(a^2 b^2 - b^4 - 2a^2 c^2 - 2(a^3 - 2a b^2) c + (4a^2 c^5 + (8a^2 - b^2) c^4 + 2(2a^3 - 3a b^2) c^3 - (a^2 b^2 - b^4) c^2)) \sqrt{-(a^4 b^2 - 2a^2 b^4 + b^6 + 4a^2 b^2 c^2 + 4(a^3 b^2 - a b^4) c)} / (4a^2 c^9 + (16a^2 - b^2) c^8 + 12(2a^3 - a b^2) c^7 + 2(8a^4 - 11a^2 b^2 + b^4) c^6 + 4(a^5 - 3a^3 b^2 + 2a b^4) c^5 - (a^4 b^2 - 2a^2 b^4 + b^6) c^4) \sin(x) + 4x) / c \end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.28, size = 638, normalized size = 2.52

$$\frac{4a \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) b \sqrt{-4ca + b^2}}{c(8ca - 2b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} + \frac{16a^2 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right)}{c(8ca - 2b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} - \frac{4a}{c(8ca - 2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\sin(x)^2/(a+b*\sin(x)+c*\sin(x)^2),x)$

[Out] 
$$\begin{aligned} & -4*a/c/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan \\ & n((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)} \\ & +4*a^2)^{(1/2)})*b*(-4*a*c+b^2)^{(1/2)}+16*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2+2*b* \\ & (-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}- \\ & b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})-4*a/c/(8*a*c-2*b^2)/(4 \\ & *c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4* \\ & a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*b^2-4*a \\ & /c/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2 \\ & *a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a \\ & ^2)^{(1/2)})*b*(-4*a*c+b^2)^{(1/2)}-16*a^2/(8*a*c-2*b^2)/(4*c*a-2*b^2-2*b*(-4*a \\ & *c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4* \\ & c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})+4*a/c/(8*a*c-2*b^2)/(4*c*a-2 \\ & *b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b \\ & ^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*b^2+2/c*\arctan \\ & (\tan(1/2*x)) \end{aligned}$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sin(x)^2/(a+b*\sin(x)+c*\sin(x)^2),x, \text{algorithm}=\text{"maxima"})$

[Out] Timed out

**mupad** [B] time = 27.75, size = 15461, normalized size = 61.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\sin(x)^2/(a + c*\sin(x)^2 + b*\sin(x)),x)$

[Out] 
$$\begin{aligned} & (2*\text{atan}((147456*a^5*\tan(x/2))/(16384*a*b^4 + 393216*a^4*c + 147456*a^5 - 22 \\ & 9376*a^3*b^2 + 262144*a^3*c^2 - 131072*a^2*b^2*c + (32768*a^2*b^4)/c - (327 \\ & 68*a^4*b^2)/c) + (393216*a^4*\tan(x/2))/(262144*a^3*c + 393216*a^4 - 131072* \\ & a^2*b^2 + (147456*a^5)/c + (16384*a*b^4)/c - (229376*a^3*b^2)/c + (32768*a^ \\ & 2*b^4)/c^2 - (32768*a^4*b^2)/c^2) + (16384*a*b^4*\tan(x/2))/(16384*a*b^4 + 3 \\ & 93216*a^4*c + 147456*a^5 - 229376*a^3*b^2 + 262144*a^3*c^2 - 131072*a^2*b^2 \\ & *c + (32768*a^2*b^4)/c - (32768*a^4*b^2)/c) + (262144*a^3*c*\tan(x/2))/(2621 \\ & 44*a^3*c + 393216*a^4 - 131072*a^2*b^2 + (147456*a^5)/c + (16384*a*b^4)/c - \\ & (229376*a^3*b^2)/c + (32768*a^2*b^4)/c^2 - (32768*a^4*b^2)/c^2) - (229376* \\ & a^3*b^2*\tan(x/2))/(16384*a*b^4 + 393216*a^4*c + 147456*a^5 - 229376*a^3*b^2 \\ & + 262144*a^3*c^2 - 131072*a^2*b^2*c + (32768*a^2*b^4)/c - (32768*a^4*b^2)/ \\ & c) - (131072*a^2*b^2*\tan(x/2))/(262144*a^3*c + 393216*a^4 - 131072*a^2*b^2 \\ & + (147456*a^5)/c + (16384*a*b^4)/c - (229376*a^3*b^2)/c + (32768*a^2*b^4)/c \\ & ^2 - (32768*a^4*b^2)/c^2) + (32768*a^2*b^4*\tan(x/2))/(147456*a^5*c + 32768* \\ & a^2*b^4 - 32768*a^4*b^2 + 262144*a^3*c^3 + 393216*a^4*c^2 - 229376*a^3*b^2* \\ & c - 131072*a^2*b^2*c^2 + 16384*a*b^4*c) - (32768*a^4*b^2*\tan(x/2))/(147456* \\ & a^5*c + 32768*a^2*b^4 - 32768*a^4*b^2 + 262144*a^3*c^3 + 393216*a^4*c^2 - 2 \\ & 29376*a^3*b^2*c - 131072*a^2*b^2*c^2 + 16384*a*b^4*c))/c - \text{atan}(((b^6 - a \\ & ^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4* \\ & a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(- \\ & (4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 \\ & - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a \\ & ^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(65536*a*b^4 + 131072*a^4*c + 24576*a^5 - 655 \\ & 36*a^3*b^2 + 131072*a^3*c^2 - 262144*a^2*b^2*c) + ((b^6 - a^2*b^4 - 8*a^3*c \\ & ^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1 \\ & /2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3) \end{aligned}$$

$$\begin{aligned}
& \left. \frac{1}{2} \right) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (32768 * a * b^5 - 32768 * a^3 * b^3 - 65536 * a * b^3 * c^2 + 262144 * a^2 * b * c^3 - 196608 * a^2 * b^3 * c + 196608 * a^3 * b * c^2 + 131072 * a^4 * b * c) + 24576 * a^5 * c + 8192 * a^2 * b^4 - 8192 * a^4 * b^2 - 131072 * a^3 * c^3 - 131072 * a^4 * c^2 + ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (16384 * a^3 * b^4 - 16384 * a * b^6 + 524288 * a^2 * c^5 + 1179648 * a^3 * c^4 + 786432 * a^4 * c^3 + 147456 * a^5 * c^2 - 131072 * a * b^2 * c^4 + 196608 * a * b^4 * c^2 + 131072 * a^2 * b^4 * c - 98304 * a^4 * b^2 * c - 1048576 * a^2 * b^2 * c^3 - 491520 * a^3 * b^2 * c^2) + ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (524288 * a^2 * c^7 + 1179648 * a^3 * c^6 + 851968 * a^4 * c^5 + 196608 * a^5 * c^4 - 131072 * a * b^2 * c^6 + 139264 * a * b^4 * c^4 - 16384 * a * b^6 * c^2 - 851968 * a^2 * b^2 * c^5 + 147456 * a^2 * b^4 * c^3 - 540672 * a^3 * b^2 * c^4 + 16384 * a^3 * b^4 * c^2 - 114688 * a^4 * b^2 * c^3) - 32768 * a * b^3 * c^5 + 24576 * a * b^5 * c^3 + 131072 * a^2 * b * c^6 + 163840 * a^3 * b * c^5 + 98304 * a^4 * b * c^4 - 139264 * a^2 * b^3 * c^4 - 24576 * a^3 * b^3 * c^3) + \tan(x/2) * (32768 * a * b^5 * c^2 - 65536 * a * b^3 * c^4 + 262144 * a^2 * b * c^5 + 262144 * a^3 * b * c^4 + 131072 * a^4 * b * c^3 - 196608 * a^2 * b^3 * c^3 - 32768 * a^3 * b^3 * c^2) + 98304 * a^4 * c^4 + 98304 * a^5 * c^3 - 24576 * a * b^4 * c^3 + 98304 * a^2 * b^2 * c^4 + 24576 * a^2 * b^4 * c^2 - 122880 * a^3 * b^2 * c^3 - 24576 * a^4 * b^2 * c^2) - 32768 * a * b^3 * c^3 + 131072 * a^2 * b * c^4 + 65536 * a^3 * b * c^3 - 24576 * a^3 * b^3 * c + 73728 * a^4 * b * c^2 - 106496 * a^2 * b^3 * c^2 + 24576 * a * b^5 * c) - 8192 * a^3 * b^2 * c + 163840 * a^2 * b^2 * c^2 - 32768 * a * b^4 * c) - 24576 * a^4 * b + 32768 * a^2 * b^3 - 98304 * a^3 * b * c) * i + ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (65536 * a * b^4 + 131072 * a^4 * c + 24576 * a^5 - 65536 * a^3 * b^2 + 131072 * a^3 * c^2 - 262144 * a^2 * b^2 * c) + ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (8192 * a^4 * b^2 - 24576 * a^5 * c - 8192 * a^2 * b^4 - \tan(x/2) * (32768 * a * b^5 - 32768 * a^3 * b^3 - 65536 * a * b^3 * c^2 + 262144 * a^2 * b * c^3 - 196608 * a^2 * b^3 * c + 196608 * a^3 * b * c^2 + 131072 * a^4 * b * c) + 131072 * a^3 * c^3 + 131072 * a^4 * c^2 + ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (16384 * a^3 * b^4 - 16384 * a * b^6 + 524288 * a^2 * c^5 + 1179648 * a^3 * c^4 + 786432 * a^4 * c^3 + 147456 * a^5 * c^2 - 131072 * a * b^2 * c^4 + 196608 * a * b^4 * c^2 + 131072 * a^2 * b^4 * c - 98304 * a^4 * b^2 * c - 1048576 * a^2 * b^2 * c^3 - 491520 * a^3 * b^2 * c^2) - ((b^6 - a^2 * b^4 - 8 * a^3 * c^3 - 8 * a^4 * c^2 - b^3 * (-4 * a * c - b^2)^3)^{(1/2)} + a^2 * b * (-4 * a * c - b^2)^3)^{(1/2)} + 6 * a^3 * b^2 * c + 18 * a^2 * b^2 * c^2 - 8 * a * b^4 * c + 2 * a * b * c * (-4 * a * c - b^2)^3)^{(1/2)}) / (2 * (16 * a^2 * c^6 + 32 * a^3 * c^5 + 16 * a^4 * c^4 + b^4 * c^4 - b^6 * c^2 - 8 * a * b^2 * c^5 + 10 * a * b^4 * c^3 - 32 * a^2 * b^2 * c^4 + a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3))^{(1/2)} * (\tan(x/2) * (32768 * a * b^5 * c^2 - 65536 * a * b^3 * c^4 + 262144 * a^2 * b * c^5 + 262144 * a^3 * b * c^4 + 131072 * a^
\end{aligned}$$

$$\begin{aligned}
& 4*b*c^3 - 196608*a^2*b^3*c^3 - 32768*a^3*b^3*c^2) - ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(524288*a^2*c^7 + 1179648*a^3*c^6 + 851968*a^4*c^5 + 196608*a^5*c^4 - 131072*a*b^2*c^6 + 139264*a*b^4*c^4 - 16384*a*b^6*c^2 - 851968*a^2*b^2*c^5 + 147456*a^2*b^4*c^3 - 540672*a^3*b^2*c^4 + 16384*a^3*b^4*c^2 - 114688*a^4*b^2*c^3) - 32768*a*b^3*c^5 + 24576*a*b^5*c^3 + 131072*a^2*b*c^6 + 163840*a^3*b*c^5 + 98304*a^4*b*c^4 - 139264*a^2*b^3*c^4 - 24576*a^3*b^3*c^3) + 98304*a^4*c^4 + 98304*a^5*c^3 - 24576*a*b^4*c^3 + 98304*a^2*b^2*c^4 + 24576*a^2*b^4*c^2 - 122880*a^3*b^2*c^3 - 24576*a^4*b^2*c^2) - 32768*a*b^3*c^3 + 131072*a^2*b*c^4 + 65536*a^3*b*c^3 - 24576*a^3*b^3*c + 73728*a^4*b*c^2 - 106496*a^2*b^3*c^2 + 24576*a*b^5*c) + 8192*a^3*b^2*c - 163840*a^2*b^2*c^2 + 32768*a*b^4*c) - 24576*a^4*b + 32768*a^2*b^3 - 98304*a^3*b*c)*1i)/(65536*a^4 - ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(65536*a*b^4 + 131072*a^4*c + 24576*a^5 - 65536*a^3*b^2 + 131072*a^3*c^2 - 262144*a^2*b^2*c) + ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(32768*a*b^5 - 32768*a^3*b^3 - 65536*a*b^3*c^2 + 262144*a^2*b*c^3 - 196608*a^2*b^3*c + 196608*a^3*b*c^2 + 131072*a^4*b*c) + 24576*a^5*c + 8192*a^2*b^4 - 8192*a^4*b^2 - 131072*a^3*c^3 - 131072*a^4*c^2 + ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(16384*a^3*b^4 - 16384*a*b^6 + 524288*a^2*c^5 + 1179648*a^3*c^4 + 786432*a^4*c^3 + 147456*a^5*c^2 - 131072*a*b^2*c^4 + 196608*a*b^4*c^2 + 131072*a^2*b^4*c - 98304*a^4*b^2*c - 1048576*a^2*b^2*c^3 - 491520*a^3*b^2*c^2) + ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(524288*a^2*c^7 + 1179648*a^3*c^6 + 851968*a^4*c^5 + 196608*a^5*c^4 - 131072*a*b^2*c^6 + 139264*a*b^4*c^4 - 16384*a*b^6*c^2 - 851968*a^2*b^2*c^5 + 147456*a^2*b^4*c^3 - 540672*a^3*b^2*c^4 + 16384*a^3*b^4*c^2 - 114688*a^4*b^2*c^3) - 32768*a*b^3*c^5 + 24576*a*b^5*c^3 + 131072*a^2*b*c^6 + 163840*a^3*b*c^5 + 98304*a^4*b*c^4 - 139264*a^2*b^3*c^4 - 24576*a^3*b^3*c^3) + \tan(x/2)*(32768*a*b^5*c^2 - 65536*a*b^3*c^4 + 262144*a^2*b*c^5 + 262144*a^3*b*c^4 + 131072*a^4*b*c^3 - 196608*a^2*b^3*c^3 - 32768*a^3*b^3*c^2) + 98304*a^4*c^4 + 98304*a^5*c^3 - 24576*a*b^4*c^3 + 98304*a^2*b^2*c^4 + 24576*a^2*b^4*c^2 - 122880*a^3*b^2*c^3 - 24576*a^4*b^2*c^2) - 32768*a*b^3*c^3 + 131072*a^2*b*c^4 + 65536*a^3*b*c^3 - 24576*a^3*b^3*c + 73728*a^4*b*c^2 - 106496*a^2*b^3*c^2 + 24576*a*b^5*c) - 8192*a^3*b^2*c + 163840*a^2*b^2*c^2 - 32768*a*b^4*c) - 24576*a^4*b + 32768*a^2*b^3 - 98304*a^3*b*c) + ((b^6 - a^2*b^4 - 8*a^3*c^3 - 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b^2*c + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*
\end{aligned}$$

$$\begin{aligned}
& a^4c^4 + b^4c^4 - b^6c^2 - 8a^3b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + \\
& a^2b^4c^2 - 8a^3b^2c^3))^{(1/2)} * (\tan(x/2) * (65536a^4b^4 + 131072a^4c^4 + \\
& 24576a^5 - 65536a^3b^2 + 131072a^3c^2 - 262144a^2b^2c) + ((b^6 - \\
& a^2b^4 - 8a^3c^3 - 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2b * (- \\
& 4ac - b^2)^3)^{(1/2)} + 6a^3b^2c + 18a^2b^2c^2 - 8a^2b^4c + 2a^2b^2c * \\
& (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8 \\
& a^3b^2c^3))^{(1/2)} * (8192a^4b^2 - 24576a^5c - 8192a^2b^4 - \tan(x/2) \\
& * (32768a^2b^5 - 32768a^3b^3 - 65536a^2b^3c^2 + 262144a^2b^2c^3 - 196608 \\
& a^2b^3c + 196608a^3b^2c^2 + 131072a^4b^2c) + 131072a^3c^3 + 131072a^4 \\
& c^2 + ((b^6 - a^2b^4 - 8a^3c^3 - 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + \\
& a^2b * (-4ac - b^2)^3)^{(1/2)} + 6a^3b^2c + 18a^2b^2c^2 - 8a^2b^4c + 2a^2b^2c * \\
& (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8 \\
& a^3b^2c^3))^{(1/2)} * (\tan(x/2) * (16384a^3b^4 - 16384a^2b^6 + \\
& 524288a^2c^5 + 1179648a^3c^4 + 786432a^4c^3 + 147456a^5c^2 - 13 \\
& 1072a^2b^2c^4 + 196608a^2b^4c^2 + 131072a^2b^4c - 98304a^4b^2c - 10 \\
& 48576a^2b^2c^3 - 491520a^3b^2c^2) - ((b^6 - a^2b^4 - 8a^3c^3 - 8a^4 \\
& c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2b * (-4ac - b^2)^3)^{(1/2)} + 6a^3 \\
& b^2c + 18a^2b^2c^2 - 8a^2b^4c + 2a^2b^2c * (-4ac - b^2)^3)^{(1/2)}) / \\
& (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8a^2b^2c^5 + \\
& 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{(1/2)} * (\tan \\
& (x/2) * (32768a^2b^5c^2 - 65536a^2b^3c^4 + 262144a^2b^2c^5 + 262144a^3b^2 \\
& c^4 + 131072a^4b^2c^3 - 196608a^2b^3c^3 - 32768a^3b^3c^2) - ((b^6 - \\
& a^2b^4 - 8a^3c^3 - 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2b * (-4 \\
& ac - b^2)^3)^{(1/2)} + 6a^3b^2c + 18a^2b^2c^2 - 8a^2b^4c + 2a^2b^2c * ( \\
& -4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3 \\
& b^2c^3))^{(1/2)} * (\tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4 \\
& c^5 + 196608a^5c^4 - 131072a^2b^2c^6 + 139264a^2b^4c^4 - 16384a^2b^6 \\
& c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384 \\
& a^3b^4c^2 - 114688a^4b^2c^3) - 32768a^2b^3c^5 + 24576a^2b^5c^3 + 13 \\
& 1072a^2b^2c^6 + 163840a^3b^2c^5 + 98304a^4b^2c^4 - 139264a^2b^3c^4 - \\
& 24576a^3b^3c^3) + 98304a^4c^4 + 98304a^5c^3 - 24576a^2b^4c^3 + 9830 \\
& 4a^2b^2c^4 + 24576a^2b^4c^2 - 122880a^3b^2c^3 - 24576a^4b^2c^2) - \\
& 32768a^2b^3c^3 + 131072a^2b^2c^4 + 65536a^3b^2c^3 - 24576a^3b^3c + \\
& 73728a^4b^2c^2 - 106496a^2b^3c^2 + 24576a^2b^5c) + 8192a^3b^2c - 1 \\
& 63840a^2b^2c^2 + 32768a^2b^4c) - 24576a^4b + 32768a^2b^3 - 98304a^3 \\
& b^2c) + 131072a^3b^2 * \tan(x/2)) * ((b^6 - a^2b^4 - 8a^3c^3 - 8a^4c^2 - \\
& b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2b * (-4ac - b^2)^3)^{(1/2)} + 6a^3b^2c \\
& + 18a^2b^2c^2 - 8a^2b^4c + 2a^2b^2c * (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2 \\
& c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8a^2b^2c^5 + 10a^2b^4 \\
& c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{(1/2)} * 2i - \operatorname{atan}((( - \\
& a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2 \\
& b * (-4ac - b^2)^3)^{(1/2)} - 6a^3b^2c - 18a^2b^2c^2 + 8a^2b^4c + 2a \\
& a^2b^2c * (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + \\
& b^4c^4 - b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - \\
& 8a^3b^2c^3))^{(1/2)} * (\tan(x/2) * (65536a^4b^4 + 131072a^4c^4 + 24576a^5 \\
& - 65536a^3b^2 + 131072a^3c^2 - 262144a^2b^2c) - 24576a^4b + (- \\
& a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + a^2b * \\
& (-4ac - b^2)^3)^{(1/2)} - 6a^3b^2c - 18a^2b^2c^2 + 8a^2b^4c + 2a \\
& a^2b^2c * (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4 \\
& c^4 - b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3 \\
& b^2c^3))^{(1/2)} * ((-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3 * (-4ac - b^2)^3)^{(1/2)} + \\
& a^2b * (-4ac - b^2)^3)^{(1/2)} - 6a^3b^2c - 18a^2b^2c^2 + 8a^2b^4c + 2a \\
& a^2b^2c^2 + 8a^2b^4c + 2a^2b^2c * (-4ac - b^2)^3)^{(1/2)}) / (2 * (16a^2c^6 \\
& + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8a^2b^2c^5 + 10a^2b^4c^3 - \\
& 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{(1/2)} * (\tan(x/2) * (16384a^3 \\
& b^4 - 16384a^2b^6 + 524288a^2c^5 + 1179648a^3c^4 + 786432a^4c^3 +
\end{aligned}$$



$$\begin{aligned}
& 147456a^5c^2 - 131072ab^2c^4 + 196608ab^4c^2 + 131072a^2b^4c - \\
& 98304a^4b^2c - 1048576a^2b^2c^3 - 491520a^3b^2c^2) + (-a^2b^4 - \\
& b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + a^2b(-4ac - \\
& b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + 8ab^4c + 2abc(-4ac - \\
& b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - \\
& 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (32768ab^5c^2 - 65536ab^3c^4 + 262144a^2b^2c^5 + \\
& 262144a^3b^2c^4 + 131072a^4b^2c^3 - 196608a^2b^3c^3 - 32768a^3b^3c^2) + \\
& (-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + 8ab^4c + \\
& 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - \\
& 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4c^5 + 196608a^5c^4 - 131072ab^2c^6 + 139264ab^4c^4 - \\
& 16384ab^6c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 - 114688a^4b^2c^3) - 32768ab^3c^5 + 24576ab^5c^3 + \\
& 131072a^2b^2c^6 + 163840a^3b^2c^5 + 98304a^4b^2c^4 - 139264a^2b^3c^4 - 24576a^3b^3c^3) + 98304a^4c^4 + 98304a^5c^3 - \\
& 24576ab^4c^3 + 98304a^2b^2c^4 + 24576a^2b^4c^2 - 122880a^3b^2c^3 - 24576a^4b^2c^2) - 32768ab^3c^3 + 131072a^2b^2c^4 + 65536a^3b^2c^3 - \\
& 24576a^3b^3c + 73728a^4b^2c^2 - 106496a^2b^3c^2 + 24576ab^5c) + \tan(x/2) * (32768ab^5 - 32768a^3b^3 - 65536ab^3c^2 + 262144a^2b^2c^3 - \\
& 196608a^2b^3c + 196608a^3b^2c^2 + 131072a^4b^2c) + 24576a^5c + 8192a^2b^4 - 8192a^4b^2 - 131072a^3c^3 - 131072a^4c^2 - 8192a^3b^2c + \\
& 163840a^2b^2c^2 - 32768ab^4c) + 32768a^2b^3 - 98304a^3b^2c) * i + (-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + \\
& a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + 8ab^4c + 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (65536ab^4 + 131072a^4c + 24576a^5 - \\
& 65536a^3b^2 + 131072a^3c^2 - 262144a^2b^2c) - 24576a^4b + (-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + \\
& a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + 8ab^4c + 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * ((-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + \\
& a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + 8ab^4c + 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - \\
& b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (16384a^3b^4 - 16384ab^6 + 524288a^2c^5 + \\
& 1179648a^3c^4 + 786432a^4c^3 + 147456a^5c^2 - 131072ab^2c^4 + 196608ab^4c^2 + 131072a^2b^4c - 98304a^4b^2c - 1048576a^2b^2c^3 - \\
& 491520a^3b^2c^2) - (-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - 18a^2b^2c^2 + \\
& 8ab^4c + 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - 32a^2b^2c^4 + \\
& a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (32768ab^5c^2 - 65536ab^3c^4 + 262144a^2b^2c^5 + 262144a^3b^2c^4 + 131072a^4b^2c^3 - \\
& 196608a^2b^3c^3 - 32768a^3b^3c^2) - (-a^2b^4 - b^6 + 8a^3c^3 + 8a^4c^2 - b^3(-4ac - b^2)^3)^{1/2} + a^2b(-4ac - b^2)^3)^{1/2} - 6a^3b^2c - \\
& 18a^2b^2c^2 + 8ab^4c + 2abc(-4ac - b^2)^3)^{1/2} / (2(16a^2c^6 + 32a^3c^5 + 16a^4c^4 + b^4c^4 - b^6c^2 - 8ab^2c^5 + 10ab^4c^3 - \\
& 32a^2b^2c^4 + a^2b^4c^2 - 8a^3b^2c^3))^{1/2} * (\tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4c^5 + 196608a^5c^4 - 131072ab^2c^6 + \\
& 139264ab^4c^4 - 16384ab^6c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 - 114688a^4b^2c^3) - 32768ab^3c^5 + \\
& 24576ab^5c^3 + 131072a^2b^2c^6 + 163840a^3b^2c^5 + 98304a^4b^2c^4 - 139264a^2b^3c^4 - 24576a^3b^3c^3) + 98304a^4c^4 + 98304a^5c^3
\end{aligned}$$

$$\begin{aligned}
& 5*c^3 - 24576*a*b^4*c^3 + 98304*a^2*b^2*c^4 + 24576*a^2*b^4*c^2 - 122880*a^3*b^2*c^3 - 24576*a^4*b^2*c^2) - 32768*a*b^3*c^3 + 131072*a^2*b*c^4 + 65536 \\
& *a^3*b*c^3 - 24576*a^3*b^3*c + 73728*a^4*b*c^2 - 106496*a^2*b^3*c^2 + 24576 \\
& *a*b^5*c) - \tan(x/2)*(32768*a*b^5 - 32768*a^3*b^3 - 65536*a*b^3*c^2 + 26214 \\
& 4*a^2*b*c^3 - 196608*a^2*b^3*c + 196608*a^3*b*c^2 + 131072*a^4*b*c) - 24576 \\
& *a^5*c - 8192*a^2*b^4 + 8192*a^4*b^2 + 131072*a^3*c^3 + 131072*a^4*c^2 + 81 \\
& 92*a^3*b^2*c - 163840*a^2*b^2*c^2 + 32768*a*b^4*c) + 32768*a^2*b^3 - 98304* \\
& a^3*b*c)*i)/(65536*a^4 - (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(- \\
& (4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c - 18* \\
& a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*(16*a^2*c^6 \\
& + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 \\
& - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*(tan(x/2)*(65536*a* \\
& b^4 + 131072*a^4*c + 24576*a^5 - 65536*a^3*b^2 + 131072*a^3*c^2 - 262144*a^ \\
& 2*b^2*c) - 24576*a^4*b + (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-( \\
& 4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c - 18*a \\
& ^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*(16*a^2*c^6 + \\
& 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - \\
& 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*((-(a^2*b^4 - b^6 + \\
& 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2 \\
& )^3)^(1/2) - 6*a^3*b^2*c - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - \\
& b^2)^3)^(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 \\
& - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^ \\
& 3)))^(1/2)*(tan(x/2)*(16384*a^3*b^4 - 16384*a*b^6 + 524288*a^2*c^5 + 117964 \\
& 8*a^3*c^4 + 786432*a^4*c^3 + 147456*a^5*c^2 - 131072*a*b^2*c^4 + 196608*a*b \\
& ^4*c^2 + 131072*a^2*b^4*c - 98304*a^4*b^2*c - 1048576*a^2*b^2*c^3 - 491520* \\
& a^3*b^2*c^2) + (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-(4*a*c - b^ \\
& 2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c - 18*a^2*b^2*c^2 \\
& + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*(16*a^2*c^6 + 32*a^3*c^ \\
& 5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^ \\
& 2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*(tan(x/2)*(32768*a*b^5*c^2 - 6 \\
& 5536*a*b^3*c^4 + 262144*a^2*b*c^5 + 262144*a^3*b*c^4 + 131072*a^4*b*c^3 - 1 \\
& 96608*a^2*b^3*c^3 - 32768*a^3*b^3*c^2) + (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a \\
& ^4*c^2 - b^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6* \\
& a^3*b^2*c - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/ \\
& (2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 \\
& + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*(tan \\
& (x/2)*(524288*a^2*c^7 + 1179648*a^3*c^6 + 851968*a^4*c^5 + 196608*a^5*c^4 - \\
& 131072*a*b^2*c^6 + 139264*a*b^4*c^4 - 16384*a*b^6*c^2 - 851968*a^2*b^2*c^5 \\
& + 147456*a^2*b^4*c^3 - 540672*a^3*b^2*c^4 + 16384*a^3*b^4*c^2 - 114688*a^4 \\
& *b^2*c^3) - 32768*a*b^3*c^5 + 24576*a*b^5*c^3 + 131072*a^2*b*c^6 + 163840*a \\
& ^3*b*c^5 + 98304*a^4*b*c^4 - 139264*a^2*b^3*c^4 - 24576*a^3*b^3*c^3) + 9830 \\
& 4*a^4*c^4 + 98304*a^5*c^3 - 24576*a*b^4*c^3 + 98304*a^2*b^2*c^4 + 24576*a^2 \\
& *b^4*c^2 - 122880*a^3*b^2*c^3 - 24576*a^4*b^2*c^2) - 32768*a*b^3*c^3 + 1310 \\
& 72*a^2*b*c^4 + 65536*a^3*b*c^3 - 24576*a^3*b^3*c + 73728*a^4*b*c^2 - 106496 \\
& *a^2*b^3*c^2 + 24576*a*b^5*c) + \tan(x/2)*(32768*a*b^5 - 32768*a^3*b^3 - 655 \\
& 36*a*b^3*c^2 + 262144*a^2*b*c^3 - 196608*a^2*b^3*c + 196608*a^3*b*c^2 + 131 \\
& 072*a^4*b*c) + 24576*a^5*c + 8192*a^2*b^4 - 8192*a^4*b^2 - 131072*a^3*c^3 - \\
& 131072*a^4*c^2 - 8192*a^3*b^2*c + 163840*a^2*b^2*c^2 - 32768*a*b^4*c) + 32 \\
& 768*a^2*b^3 - 98304*a^3*b*c) + (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b \\
& ^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c \\
& - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*(16*a^2 \\
& *c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4 \\
& *c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*(tan(x/2)*(655 \\
& 36*a*b^4 + 131072*a^4*c + 24576*a^5 - 65536*a^3*b^2 + 131072*a^3*c^2 - 2621 \\
& 44*a^2*b^2*c) - 24576*a^4*b + (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^ \\
& 3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c - \\
& 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*(16*a^2* \\
& c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4* \\
& c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^(1/2)*((-(a^2*b^4 - b
\end{aligned}$$

$$\begin{aligned}
&^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c \\
&- b^2)^3)^{(1/2)} - 6*a^3*b^2*c - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a \\
&*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6 \\
&*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b \\
&^2*c^3)))^{(1/2)}*(\tan(x/2)*(16384*a^3*b^4 - 16384*a*b^6 + 524288*a^2*c^5 + 1 \\
&179648*a^3*c^4 + 786432*a^4*c^3 + 147456*a^5*c^2 - 131072*a*b^2*c^4 + 19660 \\
&8*a*b^4*c^2 + 131072*a^2*b^4*c - 98304*a^4*b^2*c - 1048576*a^2*b^2*c^3 - 49 \\
&1520*a^3*b^2*c^2) - (-(a^2*b^4 - b^6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-(4*a*c \\
&- b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b^2*c - 18*a^2*b^ \\
&2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^6 + 32*a \\
&^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a \\
&^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)}*(\tan(x/2)*(32768*a*b^5*c^ \\
&2 - 65536*a*b^3*c^4 + 262144*a^2*b*c^5 + 262144*a^3*b*c^4 + 131072*a^4*b*c^ \\
&3 - 196608*a^2*b^3*c^3 - 32768*a^3*b^3*c^2) - (-(a^2*b^4 - b^6 + 8*a^3*c^3 \\
&+ 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - b^2)^3)^{(1/2)} \\
&- 6*a^3*b^2*c - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1 \\
&/2)})/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6*c^2 - 8*a*b^2 \\
&*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^2*c^3)))^{(1/2)} \\
&*(\tan(x/2)*(524288*a^2*c^7 + 1179648*a^3*c^6 + 851968*a^4*c^5 + 196608*a^5* \\
&c^4 - 131072*a*b^2*c^6 + 139264*a*b^4*c^4 - 16384*a*b^6*c^2 - 851968*a^2*b^ \\
&2*c^5 + 147456*a^2*b^4*c^3 - 540672*a^3*b^2*c^4 + 16384*a^3*b^4*c^2 - 11468 \\
&8*a^4*b^2*c^3) - 32768*a*b^3*c^5 + 24576*a*b^5*c^3 + 131072*a^2*b*c^6 + 163 \\
&840*a^3*b*c^5 + 98304*a^4*b*c^4 - 139264*a^2*b^3*c^4 - 24576*a^3*b^3*c^3) + \\
&98304*a^4*c^4 + 98304*a^5*c^3 - 24576*a*b^4*c^3 + 98304*a^2*b^2*c^4 + 2457 \\
&6*a^2*b^4*c^2 - 122880*a^3*b^2*c^3 - 24576*a^4*b^2*c^2) - 32768*a*b^3*c^3 + \\
&131072*a^2*b*c^4 + 65536*a^3*b*c^3 - 24576*a^3*b^3*c + 73728*a^4*b*c^2 - 1 \\
&06496*a^2*b^3*c^2 + 24576*a*b^5*c) - \tan(x/2)*(32768*a*b^5 - 32768*a^3*b^3 \\
&- 65536*a*b^3*c^2 + 262144*a^2*b*c^3 - 196608*a^2*b^3*c + 196608*a^3*b*c^2 \\
&+ 131072*a^4*b*c) - 24576*a^5*c - 8192*a^2*b^4 + 8192*a^4*b^2 + 131072*a^3* \\
&c^3 + 131072*a^4*c^2 + 8192*a^3*b^2*c - 163840*a^2*b^2*c^2 + 32768*a*b^4*c) \\
&+ 32768*a^2*b^3 - 98304*a^3*b*c) + 131072*a^3*b*\tan(x/2)))*(-(a^2*b^4 - b^ \\
&6 + 8*a^3*c^3 + 8*a^4*c^2 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b*(-(4*a*c - \\
&b^2)^3)^{(1/2)} - 6*a^3*b^2*c - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a* \\
&c - b^2)^3)^{(1/2)})/(2*(16*a^2*c^6 + 32*a^3*c^5 + 16*a^4*c^4 + b^4*c^4 - b^6 \\
&*c^2 - 8*a*b^2*c^5 + 10*a*b^4*c^3 - 32*a^2*b^2*c^4 + a^2*b^4*c^2 - 8*a^3*b^ \\
&2*c^3)))^{(1/2)}*2i
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)\*\*2/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

### 3.4 $\int \frac{\sin(x)}{a+b \sin(x)+c \sin^2(x)} dx$

**Optimal.** Leaf size=226

$$\frac{\sqrt{2} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out] arctan(1/2\*(2\*c+(b-(-4\*a\*c+b^2)^(1/2))\*tan(1/2\*x))\*2^(1/2)/(b^2-2\*c\*(a+c)-b\*(-4\*a\*c+b^2)^(1/2))^(1/2))\*2^(1/2)\*(1-b/(-4\*a\*c+b^2)^(1/2))/(b^2-2\*c\*(a+c)-b\*(-4\*a\*c+b^2)^(1/2))^(1/2)+arctan(1/2\*(2\*c+(b+(-4\*a\*c+b^2)^(1/2))\*tan(1/2\*x))\*2^(1/2)/(b^2-2\*c\*(a+c)+b\*(-4\*a\*c+b^2)^(1/2))^(1/2))\*2^(1/2)\*(1+b/(-4\*a\*c+b^2)^(1/2))/(b^2-2\*c\*(a+c)+b\*(-4\*a\*c+b^2)^(1/2))^(1/2)

**Rubi [A]** time = 0.55, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {3256, 2660, 618, 204}

$$\frac{\sqrt{2} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] (Sqrt[2]\*(1 - b/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - b\*Sqrt[b^2 - 4\*a\*c]])]/Sqrt[b^2 - 2\*c\*(a + c) - b\*Sqrt[b^2 - 4\*a\*c]] + (Sqrt[2]\*(1 + b/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + b\*Sqrt[b^2 - 4\*a\*c]])]/Sqrt[b^2 - 2\*c\*(a + c) + b\*Sqrt[b^2 - 4\*a\*c]])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3256

Int[sin[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n\_.) + (c\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n2\_.))^(-p\_), x\_Symbol] := Int[ExpandTrig[sin[d + e\*x]^m\*(a + b\*sin[d + e\*x]^n + c\*sin[d + e\*x]^(2\*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && Integ

ersQ[m, n, p]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sin(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( \frac{1 - \frac{b}{\sqrt{b^2 - 4ac}}}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} + \frac{1 + \frac{b}{\sqrt{b^2 - 4ac}}}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} \right) dx \\
 &= \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} dx + \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx \\
 &= \left( 2 \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{b - \sqrt{b^2 - 4ac} + 4cx + (b - \sqrt{b^2 - 4ac}) x^2} dx, \right. \\
 &= - \left( 4 \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{-8(b^2 - 2c(a + c) - b\sqrt{b^2 - 4ac}) - x^2} dx, \right. \\
 &= \frac{\sqrt{2} \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a + c) - b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 2c(a + c) - b\sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left( \frac{2c + (b + \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a + c) + b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 2c(a + c) + b\sqrt{b^2 - 4ac}}}
 \end{aligned}$$

**Mathematica [C]** time = 0.72, size = 268, normalized size = 1.19

$$\frac{\left( \sqrt{4ac - b^2} + ib \right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) (b - i\sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a + c) + b^2}} \right)}{\sqrt{-ib\sqrt{4ac - b^2} - 2c(a + c) + b^2}} + \frac{\left( \sqrt{4ac - b^2} - ib \right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) (b + i\sqrt{4ac - b^2})}{\sqrt{2} \sqrt{ib\sqrt{4ac - b^2} - 2c(a + c) + b^2}} \right)}{\sqrt{ib\sqrt{4ac - b^2} - 2c(a + c) + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] (((I\*b + Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]] + (((-I)\*b + Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])/Sqrt[-1/2\*b^2 + 2\*a\*c]

**fricas [B]** time = 0.79, size = 3519, normalized size = 15.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*sqrt(-(2\*a^2 - b^2 + 2\*a\*c - (a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c)\*sqrt(b^2/(a^4\*b^2 - 2\*a^2\*b^4 + b^6 - 4\*a\*c^5 - (16\*a^2 - b^2)\*c^4 - 12\*(2\*a^3 - a\*b^2)\*c^3 - 2\*(8\*a^4 - 11\*a^2\*b^2 + b^4)\*c^2 - 4\*(a^5 - 3\*a^3\*b^2 + 2\*a\*b^4)\*c)))/(a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c))\*log(2\*a\*b^2\*sin(x) + 4\*a\*b\*c + 2\*(a^3\*b^2 - a\*b^4 - 4\*a^2\*c^3 - (8\*a^3 - a\*b^2)\*c^2 - 2\*(2\*a^4 - 3\*a^2\*c



$c) \cdot \cos(x) \cdot \sqrt{-(2a^2 - b^2 + 2ac - (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2) \cdot c^2 - 2(2a^3 - 3ab^2) \cdot c) \cdot \sqrt{b^2/(a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2) \cdot c^4 - 12(2a^3 - ab^2) \cdot c^3 - 2(8a^4 - 11a^2b^2 + b^4) \cdot c^2 - 4(a^5 - 3a^3b^2 + 2ab^4) \cdot c)))/(a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2) \cdot c^2 - 2(2a^3 - 3ab^2) \cdot c)}$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 0.28, size = 216, normalized size = 0.96

$$\frac{8a\sqrt{-4ca+b^2} \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca+b^2} - b}{\sqrt{4ca-2b^2+2b\sqrt{-4ca+b^2}+4a^2}}\right)}{(8ca-2b^2)\sqrt{4ca-2b^2+2b\sqrt{-4ca+b^2}+4a^2}} + \frac{8a\sqrt{-4ca+b^2} \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + b + \sqrt{-4ca+b^2}}{\sqrt{4ca-2b^2-2b\sqrt{-4ca+b^2}+4a^2}}\right)}{(8ca-2b^2)\sqrt{4ca-2b^2-2b\sqrt{-4ca+b^2}+4a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out]  $8a \cdot (-4ac + b^2)^{(1/2)} / (8ac - 2b^2) / (4ca - 2b^2 + 2b \cdot (-4ac + b^2)^{(1/2)} + 4a^2)^{(1/2)} \cdot \arctan\left(\frac{-2a \cdot \tan(1/2 \cdot x) + (-4ac + b^2)^{(1/2)} - b}{(4ca - 2b^2 + 2b \cdot (-4ac + b^2)^{(1/2)} + 4a^2)^{(1/2)}}\right) + 8a \cdot (-4ac + b^2)^{(1/2)} / (8ac - 2b^2) / (4ca - 2b^2 - 2b \cdot (-4ac + b^2)^{(1/2)} + 4a^2)^{(1/2)} \cdot \arctan\left(\frac{2a \cdot \tan(1/2 \cdot x) + b + (-4ac + b^2)^{(1/2)}}{(4ca - 2b^2 - 2b \cdot (-4ac + b^2)^{(1/2)} + 4a^2)^{(1/2)}}\right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{c \sin(x)^2 + b \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] integrate(sin(x)/(c\*sin(x)^2 + b\*sin(x) + a), x)

**mupad** [B] time = 24.99, size = 5048, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a + c\*sin(x)^2 + b\*sin(x)),x)

[Out]  $\operatorname{atan}\left(\frac{\left(\left(\left(8a^3c + b \cdot (-4ac - b^2)^3\right)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c\right) / \left(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c)\right)\right)^{1/2} \cdot \tan(x/2) \cdot (256a^3c - 64a^2b^2 + 256a^2c^2 - 64ab^2c) - 32ab^3 + (8a^3c + b \cdot (-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c}{\left(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c)\right)^{1/2} \cdot \tan(x/2) \cdot (96ab^4 + 256a^4c - 64a^3b^2 + 512a^2c^3 + 768a^3c^2 - 128ab^2c^2 - 576a^2b^2c) + 32a^2b^3 + 128a^2bc^2 - 32ab^3c - 128a^3bc + 128a^2bc} - \tan(x/2) \cdot (128a^2c - 64ab^2 + 64a^3) + 32a^2b\right) \cdot \left(\left(8a^3c + b \cdot (-4ac - b^2)^3\right)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2\right)^{1/2}$

$$\begin{aligned}
& 2*c)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8 \\
& *a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*i - (((8*a \\
& ^3*c + b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c \\
& )/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a* \\
& b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(256 \\
& *a^3*c - 64*a^2*b^2 + 256*a^2*c^2 - 64*a*b^2*c) - 32*a*b^3 - ((8*a^3*c + b* \\
& (-4*a*c - b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2 \\
& *b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - \\
& 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 2 \\
& 56*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2 \\
& *b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + 128*a^2* \\
& b*c) + \tan(x/2)*(128*a^2*c - 64*a*b^2 + 64*a^3) - 32*a^2*b)*((8*a^3*c + b*( \\
& -(4*a*c - b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2* \\
& b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - \\
& 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*i)/((((8*a^3*c + b*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 \\
& - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a \\
& ^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(256*a^3*c - 64*a \\
& ^2*b^2 + 256*a^2*c^2 - 64*a*b^2*c) - 32*a*b^3 + ((8*a^3*c + b*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + \\
& 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c \\
& - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 64 \\
& *a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32* \\
& a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + 128*a^2*b*c) - \tan(x/ \\
& 2)*(128*a^2*c - 64*a*b^2 + 64*a^3) + 32*a^2*b)*((8*a^3*c + b*(-(4*a*c - b^2 \\
& )^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 1 \\
& 6*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - \\
& 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)} - 128*a^2*\tan(x/2) + (((8*a^3*c + b*( \\
& -(4*a*c - b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2* \\
& b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - \\
& 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(256*a^3*c - 6 \\
& 4*a^2*b^2 + 256*a^2*c^2 - 64*a*b^2*c) - 32*a*b^3 - ((8*a^3*c + b*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 \\
& + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2 \\
& *c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - \\
& 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + \\
& 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + 128*a^2*b*c) + \tan \\
& (x/2)*(128*a^2*c - 64*a*b^2 + 64*a^3) - 32*a^2*b)*((8*a^3*c + b*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 \\
& + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2* \\
& c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)})*((8*a^3*c + b*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 16*a^ \\
& 2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32* \\
& a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*2i + \operatorname{atan}(-((((8*a^3*c - b*(-(4*a*c - b^2 \\
& )^3)^{(1/2)} + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 1 \\
& 6*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - \\
& 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(256*a^3*c - 64*a^2*b^2 + 2 \\
& 56*a^2*c^2 - 64*a*b^2*c) - 32*a*b^3 + ((8*a^3*c - b*(-(4*a*c - b^2)^3)^{(1/2) \\
& ) + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 \\
& + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b \\
& ^2*c^2 + 10*a*b^4*c)))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + \\
& 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + \\
& 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + 128*a^2*b*c) - \tan(x/2)*(128*a^ \\
& 2*c - 64*a*b^2 + 64*a^3) + 32*a^2*b)*((8*a^3*c - b*(-(4*a*c - b^2)^3)^{(1/2) \\
& + b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 \\
& + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b \\
& ^2*c^2 + 10*a*b^4*c)))^{(1/2)}*i - (((8*a^3*c - b*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& b^4 - 2*a^2*b^2 + 8*a^2*c^2 - 6*a*b^2*c)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 3 \\
& 2*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c
\end{aligned}$$



$$\begin{aligned} & \left( (256a^3c - 64a^2b^2 + 256a^2c^2 - 64ab^2c) - 32ab^3 - \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \right)^{1/2} \\ & \left( (96ab^4 + 256a^4c - 64a^3b^2 + 512a^2c^3 + 768a^3c^2 - 128ab^2c^2 - 576a^2b^2c) + 32a^2b^3 + 128a^2b^2c^2 - 32ab^3c - 128a^3b^2c \right) + \tan(x/2) \left( 128a^2c - 64ab^2 + 64a^3 \right) \\ & - 32a^2b \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \\ & \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \\ & \left( (256a^3c - 64a^2b^2 + 256a^2c^2 - 64ab^2c) - 32ab^3 + \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \right)^{1/2} \\ & \left( (96ab^4 + 256a^4c - 64a^3b^2 + 512a^2c^3 + 768a^3c^2 - 128ab^2c^2 - 576a^2b^2c) + 32a^2b^3 + 128a^2b^2c^2 - 32ab^3c - 128a^3b^2c \right) + \tan(x/2) \left( 128a^2c - 64ab^2 + 64a^3 \right) \\ & - 32a^2b \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \\ & - 128a^2 \tan(x/2) + \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \\ & \left( (256a^3c - 64a^2b^2 + 256a^2c^2 - 64ab^2c) - 32ab^3 - \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \right)^{1/2} \\ & \left( (96ab^4 + 256a^4c - 64a^3b^2 + 512a^2c^3 + 768a^3c^2 - 128ab^2c^2 - 576a^2b^2c) + 32a^2b^3 + 128a^2b^2c^2 - 32ab^3c - 128a^3b^2c \right) + \tan(x/2) \left( 128a^2c - 64ab^2 + 64a^3 \right) \\ & - 32a^2b \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \\ & \left( (8a^3c - b(-4ac - b^2)^3)^{1/2} + b^4 - 2a^2b^2 + 8a^2c^2 - 6ab^2c \right) / \left( 2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c) \right) \end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

$$3.5 \quad \int \frac{1}{a+b \sin(x)+c \sin^2(x)} dx$$

**Optimal.** Leaf size=221

$$\frac{2\sqrt{2}c \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{2\sqrt{2}c \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out]  $2*c*\arctan(1/2*(2*c+(b-(-4*a*c+b^2)^(1/2))*\tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2)-2*c*\arctan(1/2*(2*c+(b+(-4*a*c+b^2)^(1/2))*\tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2)$

**Rubi [A]** time = 0.40, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3248, 2660, 618, 204}

$$\frac{2\sqrt{2}c \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{2\sqrt{2}c \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*SIN[x] + c\*SIN[x]^2)^(-1), x]

[Out]  $(2*\text{Sqrt}[2]*c*\text{ArcTan}[(2*c + (b - \text{Sqrt}[b^2 - 4*a*c])*\text{Tan}[x/2])]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) - b*\text{Sqrt}[b^2 - 4*a*c]]))/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b^2 - 2*c*(a + c) - b*\text{Sqrt}[b^2 - 4*a*c]]) - (2*\text{Sqrt}[2]*c*\text{ArcTan}[(2*c + (b + \text{Sqrt}[b^2 - 4*a*c])*\text{Tan}[x/2])]/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) + b*\text{Sqrt}[b^2 - 4*a*c]]))/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b^2 - 2*c*(a + c) + b*\text{Sqrt}[b^2 - 4*a*c]])$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3248

Int[((a\_.) + (b\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n\_.) + (c\_.)\*sin[(d\_.) + (e\_.)\*(x\_)^(n2\_.)]^(-1), x\_Symbol] := Module[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[1/(b - q + 2\*c\*SIN[d + e\*x]^n), x], x] - Dist[(2\*c)/q, Int[1/(b + q + 2\*c\*SIN[d + e\*x]^n), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,

2\*n] && NeQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + b \sin(x) + c \sin^2(x)} dx &= \frac{(2c) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{\sqrt{b^2 - 4ac}} \\ &= \frac{(4c) \text{Subst} \left( \int \frac{1}{b - \sqrt{b^2 - 4ac} + 4cx + (b - \sqrt{b^2 - 4ac})x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{\sqrt{b^2 - 4ac}} - \frac{(4c) \text{Subst} \left( \int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{\sqrt{b^2 - 4ac}} \\ &= -\frac{(8c) \text{Subst} \left( \int \frac{1}{-8(b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}) - x^2} dx, x, 4c + 2(b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right) \right)}{\sqrt{b^2 - 4ac}} \\ &= \frac{2\sqrt{2} c \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} - \frac{2\sqrt{2} c \tan^{-1} \left( \frac{2c + (b + \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [C] time = 0.60, size = 233, normalized size = 1.05

$$\frac{2ic \left( \frac{\tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right)(b - i\sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} - \frac{\tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right)(b + i\sqrt{4ac - b^2})}{\sqrt{2} \sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sin[x] + c\*Sin[x]^2)^(-1), x]

[Out] ((-2\*I)\*c\*(ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])]/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])/Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]] - ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])]/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])/Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])/Sqrt[-1/2\*b^2 + 2\*a\*c]

fricas [B] time = 0.81, size = 3495, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 + (a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c)\*sqrt(b^2/(a^4\*b^2 - 2\*a^2\*b^4 + b^6 - 4\*a\*c^5 - (16\*a^2 - b^2)\*c^4 - 12\*(2\*a^3 - a\*b^2)\*c^3 - 2\*(8\*a^4 - 11\*a^2\*b^2 + b^4)\*c^2 - 4\*(a^5 - 3\*a^3\*b^2 + 2\*a\*b^4)\*c)))/(a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c)\*log(2\*b^2\*c\*sin(x) + 4\*b\*c^2 + 2\*(4\*a\*c^4 + (8\*a^2 - b^2)\*c^3 + 2\*(2\*a^3 - 3\*a\*b^2)\*c^2 - (a^2\*b^2 - b^4)\*c)\*sqrt(b^2/(a^4\*b^2 - 2\*a^2\*b^4 + b^6 - 4\*a\*c^5 - (16\*a^2 - b^2)\*c^4 - 12\*(2\*a^3 - a\*b^2)\*c^3 - 2\*(8\*a^4 - 11\*a^2\*b^2 + b^4)\*c^2 - 4\*(a^5 - 3\*a^3\*b^2 + 2\*a\*b^4)\*c))\*sin(x) - sqrt(2)\*((a^2\*b^4 - b^6 + 8\*a\*c^5 + 2\*(12\*a^2 -



$$\frac{a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c}{(a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c)}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.28, size = 610, normalized size = 2.76

$$\frac{2 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) b \sqrt{-4ca + b^2} + 8a \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) c}{(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2} + (4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2} + (4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 
$$\frac{-2/(4ac - b^2)/(4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2)^{1/2} + 4a^2)^{1/2} \arctan\left(\frac{-2a \tan(1/2x) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2)^{1/2}}\right) b \sqrt{-4ca + b^2} + 8a \arctan\left(\frac{-2a \tan(1/2x) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2)^{1/2}}\right) c}{(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2} + (4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2} + (4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c \sin(x)^2 + b \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] integrate(1/(c\*sin(x)^2 + b\*sin(x) + a), x)

**mupad** [B] time = 25.73, size = 5064, normalized size = 22.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c\*sin(x)^2 + b\*sin(x)),x)

[Out] 
$$\frac{\operatorname{atan}\left(\frac{(-8ac^3 + b(-4ac - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6ab^2c}{2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c)}\right)^{1/2} \left(\frac{-8ac^3 + b(-4ac - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6ab^2c}{2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c)}\right)^{1/2}}{\left(\frac{-8ac^3 + b(-4ac - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6ab^2c}{2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c)}\right)^{1/2}}$$



$$\begin{aligned}
& (1/2) * (\tan(x/2) * (96 * a * b^4 + 256 * a^4 * c - 64 * a^3 * b^2 + 512 * a^2 * c^3 + 768 * a^3 * c^2 - 128 * a * b^2 * c^2 - 576 * a^2 * b^2 * c) + 32 * a^2 * b^3 + 128 * a^2 * b * c^2 - 32 * a * b^3 * c - 128 * a^3 * b * c) + 32 * a^2 * b^2 - 128 * a^2 * c^2 + 32 * a * b^2 * c) + \tan(x/2) * (128 * a * c^2 - 32 * a * b^2 + 64 * a^2 * c) + 32 * a * b * c) * i + (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (128 * a * c^2 - 32 * a * b^2 + 64 * a^2 * c) - (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (64 * a * b^3 - 256 * a^2 * b * c) - 128 * a^3 * c - (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (96 * a * b^4 + 256 * a^4 * c - 64 * a^3 * b^2 + 512 * a^2 * c^3 + 768 * a^3 * c^2 - 128 * a * b^2 * c^2 - 576 * a^2 * b^2 * c) + 32 * a^2 * b^3 + 128 * a^2 * b * c^2 - 32 * a * b^3 * c - 128 * a^3 * b * c) + 32 * a^2 * b^2 - 128 * a^2 * c^2 + 32 * a * b^2 * c) + 32 * a * b * c) * i) / (64 * a * c - (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * ((- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (64 * a * b^3 - 256 * a^2 * b * c) - 128 * a^3 * c + (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (96 * a * b^4 + 256 * a^4 * c - 64 * a^3 * b^2 + 512 * a^2 * c^3 + 768 * a^3 * c^2 - 128 * a * b^2 * c^2 - 576 * a^2 * b^2 * c) + 32 * a^2 * b^3 + 128 * a^2 * b * c^2 - 32 * a * b^3 * c - 128 * a^3 * b * c) + 32 * a^2 * b^2 - 128 * a^2 * c^2 + 32 * a * b^2 * c) + \tan(x/2) * (128 * a * c^2 - 32 * a * b^2 + 64 * a^2 * c) + 32 * a * b * c) + (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (128 * a * c^2 - 32 * a * b^2 + 64 * a^2 * c) - (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (64 * a * b^3 - 256 * a^2 * b * c) - 128 * a^3 * c - (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * (\tan(x/2) * (96 * a * b^4 + 256 * a^4 * c - 64 * a^3 * b^2 + 512 * a^2 * c^3 + 768 * a^3 * c^2 - 128 * a * b^2 * c^2 - 576 * a^2 * b^2 * c) + 32 * a^2 * b^3 + 128 * a^2 * b * c^2 - 32 * a * b^3 * c - 128 * a^3 * b * c) + 32 * a^2 * b^2 - 128 * a^2 * c^2 + 32 * a * b^2 * c) + 32 * a * b * c) * (- (8 * a * c^3 - b * (- (4 * a * c - b^2)^3)^{1/2} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (a^2 * b^4 - b^6 + 16 * a^2 * c^4 + 32 * a^3 * c^3 + 16 * a^4 * c^2 + b^4 * c^2 - 8 * a * b^2 * c^3 - 8 * a^3 * b^2 * c - 32 * a^2 * b^2 * c^2 + 10 * a * b^4 * c)))^{1/2} * 2i
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

### 3.6 $\int \frac{\csc(x)}{a+b \sin(x)+c \sin^2(x)} dx$

**Optimal.** Leaf size=244

$$\frac{\sqrt{2} c \left( \frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{\sqrt{2} c \left( 1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \tanh^{-1}$$

[Out]  $-\operatorname{arctanh}(\cos(x))/a-c*\operatorname{arctan}(1/2*(2*c+(b-(-4*a*c+b^2)^{(1/2)}))*\tan(1/2*x))*2^{(1/2)}/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}*(1+b/(-4*a*c+b^2)^{(1/2)})/a/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}-c*\operatorname{arctan}(1/2*(2*c+(b+(-4*a*c+b^2)^{(1/2)}))*\tan(1/2*x))*2^{(1/2)}/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}*(1-b/(-4*a*c+b^2)^{(1/2)})/a/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.76, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {3256, 3770, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} c \left( \frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \frac{\sqrt{2} c \left( 1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} - \tanh^{-1}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]/(a + b*Sin[x] + c*Sin[x]^2),x]`

[Out]  $-\left(\left(\operatorname{Sqrt}[2]*c*(1 + b/\operatorname{Sqrt}[b^2 - 4*a*c])*\operatorname{ArcTan}[(2*c + (b - \operatorname{Sqrt}[b^2 - 4*a*c]))*\operatorname{Tan}[x/2]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2 - 2*c*(a + c) - b*\operatorname{Sqrt}[b^2 - 4*a*c]])\right)\right)/(a*\operatorname{Sqrt}[b^2 - 2*c*(a + c) - b*\operatorname{Sqrt}[b^2 - 4*a*c]]) - \left(\operatorname{Sqrt}[2]*c*(1 - b/\operatorname{Sqrt}[b^2 - 4*a*c])*\operatorname{ArcTan}[(2*c + (b + \operatorname{Sqrt}[b^2 - 4*a*c]))*\operatorname{Tan}[x/2]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b^2 - 2*c*(a + c) + b*\operatorname{Sqrt}[b^2 - 4*a*c]])\right)\right)/(a*\operatorname{Sqrt}[b^2 - 2*c*(a + c) + b*\operatorname{Sqrt}[b^2 - 4*a*c]]) - \operatorname{ArcTanh}[\operatorname{Cos}[x]]/a$

#### Rule 204

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

#### Rule 618

`Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

#### Rule 2660

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

#### Rule 3256

`Int[sin[(d_) + (e_)*(x_)]^(m_)*((a_) + (b_)*sin[(d_) + (e_)*(x_)]^(n_) + (c_)*sin[(d_) + (e_)*(x_)]^(n2_))^(p_), x_Symbol] := Int[ExpandTr`



ig[sin[d + e\*x]^m\*(a + b\*sin[d + e\*x]^n + c\*sin[d + e\*x]^(2\*n))^p, x], x] /  
 ; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && Integ  
 ersQ[m, n, p]

### Rule 3292

Int[((A\_) + (B\_)\*sin[(d\_) + (e\_)\*(x\_)])/((a\_) + (b\_)\*sin[(d\_) + (e\_)\*  
 \*(x\_)] + (c\_)\*sin[(d\_) + (e\_)\*(x\_)]^2), x\_Symbol] :> Module[{q = Rt[b^2  
 - 4\*a\*c, 2]}, Dist[B + (b\*B - 2\*A\*c)/q, Int[1/(b + q + 2\*c\*Sin[d + e\*x]), x  
 ], x] + Dist[B - (b\*B - 2\*A\*c)/q, Int[1/(b - q + 2\*c\*Sin[d + e\*x]), x], x]]  
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3770

Int[csc[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x]  
 /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\csc(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( \frac{\csc(x)}{a} + \frac{-b - c \sin(x)}{a(a + b \sin(x) + c \sin^2(x))} \right) dx \\
 &= \frac{\int \csc(x) dx}{a} + \frac{\int \frac{-b - c \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{a} \\
 &= \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\left( c \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{a} - \frac{\left( c \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{b + \sqrt{b^2 - 4ac} - 2c \sin(x)} dx}{a} \\
 &= \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\left( 2c \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})x^2} dx, x, 4c \right)}{a} - \frac{\left( 2c \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{b + \sqrt{b^2 - 4ac} - 4cx + (b - \sqrt{b^2 - 4ac})x^2} dx, x, -4c \right)}{a} \\
 &= \frac{\tanh^{-1}(\cos(x))}{a} + \frac{\left( 4c \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{4 \left( 4c^2 - (b + \sqrt{b^2 - 4ac})^2 \right) - x^2} dx, x, 4c \right)}{a} - \frac{\left( 4c \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left( \int \frac{1}{4 \left( 4c^2 - (b - \sqrt{b^2 - 4ac})^2 \right) - x^2} dx, x, -4c \right)}{a} \\
 &= \frac{\sqrt{2} c \left( 1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan \left( \frac{x}{2} \right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}}} \right)}{a \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} c \left( 1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left( \frac{2c + (b + \sqrt{b^2 - 4ac}) \tan \left( \frac{x}{2} \right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}}} \right)}{a \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}}}
 \end{aligned}$$

**Mathematica [C]** time = 1.30, size = 306, normalized size = 1.25

$$\frac{c \left( \sqrt{4ac - b^2} - ib \right) \tan^{-1} \left( \frac{2c + \tan \left( \frac{x}{2} \right) (b - i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} + \frac{c \left( \sqrt{4ac - b^2} + ib \right) \tan^{-1} \left( \frac{2c + \tan \left( \frac{x}{2} \right) (b + i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} - \log \left( \sin \left( \frac{x}{2} \right) \right) + \log \left( \cos \left( \frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] -(((c\*((-I)\*b + Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])  
 )\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])))/(S  
 qrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]]) + (  
 c\*(I\*b + Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x

$$\frac{1}{2}) / (\sqrt{2} \sqrt{b^2 - 2c(a+c) + I b \sqrt{-b^2 + 4ac}}) / (\sqrt{-1/2b^2 + 2ac} \sqrt{b^2 - 2c(a+c) + I b \sqrt{-b^2 + 4ac}}) + \text{Log}[\text{Cos}[x/2]] - \text{Log}[\text{Sin}[x/2]])/a$$

**fricas [B]** time = 131.81, size = 5296, normalized size = 21.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{2} a \sqrt{-(b^4 - 4ab^2c + 2ac^3 + (2a^2 - b^2)c^2 + (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} / (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \log(4b^3c^3 - 8ab^3c^4 - 4b^3c^5 + 2(4a^3c^5 + (8a^4 - a^2b^2)c^4 + 2(2a^5 - 3a^3b^2)c^3 - (a^4b^2 - a^2b^4)c^2) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} \sin(x) - \sqrt{2} ((a^4b^5 - a^2b^7 + 4a^3b^3c^5 + (20a^4b - a^2b^3)c^4 + (28a^5b - 13a^3b^3)c^3 + (12a^6b - 27a^4b^3 + 2a^2b^5)c^2 - (7a^5b^3 - 9a^3b^5)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} \cos(x) - (b^7 - 7a^5b^3c - 4ab^3c^5 - (12a^2b - b^3)c^4 - (8a^3b - 11a^2b^3)c^3 + 2(7a^2b^3 - b^5)c^2) \cos(x) \sqrt{-(b^4 - 4ab^2c + 2ac^3 + (2a^2 - b^2)c^2 + (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}}) / (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) + 2(b^4c^2 - 2ab^2c^3 - b^2c^4) \sin(x) - \sqrt{2} a \sqrt{-(b^4 - 4ab^2c + 2ac^3 + (2a^2 - b^2)c^2 + (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}}) / (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \log(4b^3c^3 - 8ab^3c^4 - 4b^3c^5 - 2(4a^3c^5 + (8a^4 - a^2b^2)c^4 + 2(2a^5 - 3a^3b^2)c^3 - (a^4b^2 - a^2b^4)c^2) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} \sin(x) - \sqrt{2} ((a^4b^5 - a^2b^7 + 4a^3b^3c^5 + (20a^4b - a^2b^3)c^4 + (28a^5b - 13a^3b^3)c^3 + (12a^6b - 27a^4b^3 + 2a^2b^5)c^2 - (7a^5b^3 - 9a^3b^5)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} \cos(x) + (b^7 - 7a^5b^3c - 4ab^3c^5 - (12a^2b - b^3)c^4 - (8a^3b - 11a^2b^3)c^3 + 2(7a^2b^3 - b^5)c^2) \cos(x) \sqrt{-(b^4 - 4ab^2c + 2ac^3 + (2a^2 - b^2)c^2 + (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}}) \sqrt{(b^6 - 4ab^4c + 4ab^2c^3 + b^2c^4 + 2(2a^2b^2 - b^4)c^2) / (a^8b^2 - 2a^6b^4 + a^4b^6 - 4a^5c^5 - (16a^6 - a^4b^2)c^4 - 12(2a^7 - a^5b^2)c^3 - 2(8a^8 - 11a^6b^2 + a^4b^4)c^2 - 4(a^9 - 3a^7b^2 + 2a^5b^4)c)}} / (a^4b^2 - a^2b^4 - 4a^3c^3 - (8a^4 - a^2b^2)c^2 - 2(2a^5 - 3a^3b^2)c)$

$$\begin{aligned}
& *b^2 + 2*a^5*b^4)*c)))/(a^4*b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c \\
& ^2 - 2*(2*a^5 - 3*a^3*b^2)*c)) + 2*(b^4*c^2 - 2*a*b^2*c^3 - b^2*c^4)*\sin(x) \\
& ) + \sqrt{2}*a*\sqrt{-(b^4 - 4*a*b^2*c + 2*a*c^3 + (2*a^2 - b^2)*c^2 - (a^4*b^2 \\
& ^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)*c) \\
& )*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2)/( \\
& a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^5*c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*(2* \\
& a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^7* \\
& b^2 + 2*a^5*b^4)*c)))/(a^4*b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c^ \\
& 2 - 2*(2*a^5 - 3*a^3*b^2)*c))*\log(-4*b^3*c^3 + 8*a*b*c^4 + 4*b*c^5 + 2*(4*a \\
& ^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2 \\
& *b^4)*c^2)*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 + b^2*c^4 + 2*(2*a^2*b^2 - b \\
& ^4)*c^2)/(a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^5*c^5 - (16*a^6 - a^4*b^2)*c^4 \\
& - 12*(2*a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^6*b^2 + a^4*b^4)*c^2 - 4*(a^ \\
& 9 - 3*a^7*b^2 + 2*a^5*b^4)*c))*\sin(x) - \sqrt{2}*((a^4*b^5 - a^2*b^7 + 4*a^3 \\
& *b*c^5 + (20*a^4*b - a^2*b^3)*c^4 + (28*a^5*b - 13*a^3*b^3)*c^3 + (12*a^6*b \\
& - 27*a^4*b^3 + 2*a^2*b^5)*c^2 - (7*a^5*b^3 - 9*a^3*b^5)*c)*\sqrt{((b^6 - 4*a \\
& *b^4*c + 4*a*b^2*c^3 + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2)/(a^8*b^2 - 2*a^6* \\
& b^4 + a^4*b^6 - 4*a^5*c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*(2*a^7 - a^5*b^2)*c \\
& ^3 - 2*(8*a^8 - 11*a^6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^7*b^2 + 2*a^5*b^4) \\
& *c))*\cos(x) + (b^7 - 7*a*b^5*c - 4*a*b*c^5 - (12*a^2*b - b^3)*c^4 - (8*a^3* \\
& b - 11*a*b^3)*c^3 + 2*(7*a^2*b^3 - b^5)*c^2)*\cos(x))*\sqrt{-(b^4 - 4*a*b^2*c \\
& + 2*a*c^3 + (2*a^2 - b^2)*c^2 - (a^4*b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - \\
& a^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)*c)*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 \\
& + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2)/(a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^ \\
& 5*c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*(2*a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^ \\
& 6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^7*b^2 + 2*a^5*b^4)*c)))/(a^4*b^2 - a^2* \\
& b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)*c)) - 2*(b \\
& ^4*c^2 - 2*a*b^2*c^3 - b^2*c^4)*\sin(x)) - \sqrt{2}*a*\sqrt{-(b^4 - 4*a*b^2*c \\
& + 2*a*c^3 + (2*a^2 - b^2)*c^2 + (a^4*b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a \\
& ^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)*c)*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 \\
& + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2)/(a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^5 \\
& *c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*(2*a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^ \\
& 6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^7*b^2 + 2*a^5*b^4)*c)))/(a^4*b^2 - a^2* \\
& b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)*c))*\log(-4* \\
& b^3*c^3 + 8*a*b*c^4 + 4*b*c^5 - 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2 \\
& *a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*\sqrt{((b^6 - 4*a*b^4*c + 4* \\
& a*b^2*c^3 + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2)/(a^8*b^2 - 2*a^6*b^4 + a^4*b^ \\
& ^6 - 4*a^5*c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*(2*a^7 - a^5*b^2)*c^3 - 2*(8*a \\
& ^8 - 11*a^6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^7*b^2 + 2*a^5*b^4)*c))*\sin(x) \\
& - \sqrt{2}*((a^4*b^5 - a^2*b^7 + 4*a^3*b*c^5 + (20*a^4*b - a^2*b^3)*c^4 + ( \\
& 28*a^5*b - 13*a^3*b^3)*c^3 + (12*a^6*b - 27*a^4*b^3 + 2*a^2*b^5)*c^2 - (7*a \\
& ^5*b^3 - 9*a^3*b^5)*c)*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 + b^2*c^4 + 2*(2 \\
& *a^2*b^2 - b^4)*c^2)/(a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^5*c^5 - (16*a^6 - \\
& a^4*b^2)*c^4 - 12*(2*a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^6*b^2 + a^4*b^4) \\
& *c^2 - 4*(a^9 - 3*a^7*b^2 + 2*a^5*b^4)*c))*\cos(x) - (b^7 - 7*a*b^5*c - 4*a* \\
& b*c^5 - (12*a^2*b - b^3)*c^4 - (8*a^3*b - 11*a*b^3)*c^3 + 2*(7*a^2*b^3 - b^ \\
& 5)*c^2)*\cos(x))*\sqrt{-(b^4 - 4*a*b^2*c + 2*a*c^3 + (2*a^2 - b^2)*c^2 + (a^4 \\
& *b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)*c^2 - 2*(2*a^5 - 3*a^3*b^2)* \\
& c)*\sqrt{((b^6 - 4*a*b^4*c + 4*a*b^2*c^3 + b^2*c^4 + 2*(2*a^2*b^2 - b^4)*c^2) \\
& / (a^8*b^2 - 2*a^6*b^4 + a^4*b^6 - 4*a^5*c^5 - (16*a^6 - a^4*b^2)*c^4 - 12*( \\
& 2*a^7 - a^5*b^2)*c^3 - 2*(8*a^8 - 11*a^6*b^2 + a^4*b^4)*c^2 - 4*(a^9 - 3*a^ \\
& 7*b^2 + 2*a^5*b^4)*c)))/(a^4*b^2 - a^2*b^4 - 4*a^3*c^3 - (8*a^4 - a^2*b^2)* \\
& c^2 - 2*(2*a^5 - 3*a^3*b^2)*c)) - 2*(b^4*c^2 - 2*a*b^2*c^3 - b^2*c^4)*\sin(x) \\
& )) - 2*\log(1/2*\cos(x) + 1/2) + 2*\log(-1/2*\cos(x) + 1/2))/a
\end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.36, size = 849, normalized size = 3.48

$$\frac{4 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2} c}{(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} + \frac{2 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2} b^2}{a(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} + \frac{8a}{(4ca - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 
$$\begin{aligned} & -4/(4*a*c-b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *(-4*a*c+b^2)^{(1/2)}*c+2/a/(4*a*c-b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *(-4*a*c+b^2)^{(1/2)}*b^2+8/(4*a*c-b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *c*b-2/a/(4*a*c-b^2)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *b^3-4/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *(-4*a*c+b^2)^{(1/2)}*c+2/a/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *b^2-8/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *c*b+2/a/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)+4*a^2})^{(1/2)}) \\ & *b^3+1/a*\ln(\tan(1/2*x)) \end{aligned}$$

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Timed out

**mupad [B]** time = 26.52, size = 11540, normalized size = 47.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)\*(a + c\*sin(x)^2 + b\*sin(x))),x)

[Out] 
$$\begin{aligned} & \operatorname{atan}\left(\left(\left(8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3\right)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3\right)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3\right)^{(1/2)}\right) / \left(2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)\right)^{(1/2)} \\ & * \left(\left(8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3\right)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3\right)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3\right)^{(1/2)}\right) / \left(2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)\right)^{(1/2)} \\ & * \left(\left(8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3\right)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3\right)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3\right)^{(1/2)}\right) / \left(2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)\right)^{(1/2)} \end{aligned}$$

$$\begin{aligned}
&^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3)^{1/2} + b^4c^2 - 6ab^2c^3 \\
&+ b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^2c^2 + 8ab^4c + 2ab^2c^2(- \\
&(4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 1 \\
&6a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a \\
&^4b^2c^2))^{1/2} * ((8a^2c^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3) \\
&^{1/2} + b^4c^2 - 6ab^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^ \\
&2c^2 + 8ab^4c + 2ab^2c^2(-4ac - b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 \\
&+ 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b \\
&b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^6c - 51 \\
&2ab^6 + 544a^3b^4 - 64a^5b^2 + 6144a^3c^4 + 12288a^4c^3 + 6400a^ \\
&5c^2 + 512ab^4c^2 + 4608a^2b^4c - 3776a^4b^2c - 3584a^2b^2c^3 \\
&- 13312a^3b^2c^2) - 128a^2b^5 + 96a^4b^3 - 512a^3b^3c^3 + 800a^3b \\
&^3c - 1152a^4b^2c^2 + 128a^2b^3c^2 - 384a^5b^3c) - \tan(x/2) * (256a^5c \\
&c - 512b^6 + 416a^2b^4 - 64a^4b^2 + 3072a^2c^4 + 5632a^3c^3 + 2816 \\
&a^4c^2 + 512b^4c^2 - 2816ab^2c^3 - 2368a^3b^2c - 8576a^2b^2c^2 \\
&+ 3840ab^4c) + 256ab^5 - 128a^3b^3 - 256ab^3c^2 + 1024a^2b^3c^3 \\
&- 1568a^2b^3c + 2176a^3b^3c^2 + 512a^4b^3c) - 128b^5 + \tan(x/2) * (96a \\
&ab^4 - 1536a^4c - 256b^4c - 1024a^2c^3 + 448a^3c^2 + 256b^2c^3 + \\
&1408ab^2c^2 - 512a^2b^2c) + 32a^2b^3 + 128b^3c^2 - 1312a^2b^3c^ \\
&2 - 640ab^3c^3 + 864ab^3c - 128a^3b^3c) + \tan(x/2) * (640a^3c^3 + 32b^4 \\
&+ 768c^4 + 64a^2c^2 - 256b^2c^2 - 128ab^2c) + 128b^3c^3 - 96b^3c \\
&+ 320ab^3c^2) * i + ((8a^2c^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3) \\
&^{1/2} + b^4c^2 - 6ab^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^ \\
&2c^2 + 8ab^4c + 2ab^2c^2(-4ac - b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 \\
&+ 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b \\
&b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (640a^3c^3 + 32 \\
&b^4 + 768c^4 + 64a^2c^2 - 256b^2c^2 - 128ab^2c) - ((8a^2c^4 - b^ \\
&6 + 8a^3c^3 - b^3(-4ac - b^2)^3)^{1/2} + b^4c^2 - 6ab^2c^3 + b^2c^ \\
&2(-4ac - b^2)^3)^{1/2} - 18a^2b^2c^2 + 8ab^4c + 2ab^2c^2(-4ac \\
&- b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c \\
&^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^ \\
&c^2))^{1/2} * ((8a^2c^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3)^{1/2} \\
&+ b^4c^2 - 6ab^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^2c^2 + \\
&8ab^4c + 2ab^2c^2(-4ac - b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 + 16a \\
&^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 \\
&- 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^5c - 512b^6 + \\
&416a^2b^4 - 64a^4b^2 + 3072a^2c^4 + 5632a^3c^3 + 2816a^4c^2 + 51 \\
&2b^4c^2 - 2816ab^2c^3 - 2368a^3b^2c - 8576a^2b^2c^2 + 3840ab^4 \\
&c) + ((8a^2c^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3)^{1/2} + b^4c^ \\
&2 - 6ab^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^2c^2 + 8ab^4 \\
&c + 2ab^2c^2(-4ac - b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + \\
&32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3 \\
&b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^6c - 512ab^6 + 544a \\
&^3b^4 - 64a^5b^2 + 6144a^3c^4 + 12288a^4c^3 + 6400a^5c^2 + 512ab \\
&^4c^2 + 4608a^2b^4c - 3776a^4b^2c - 3584a^2b^2c^3 - 13312a^3b^2 \\
&c^2) - 128a^2b^5 + 96a^4b^3 - 512a^3b^3c^3 + 800a^3b^3c - 1152a^4 \\
&b^2c^2 + 128a^2b^3c^2 - 384a^5b^3c) - 256ab^5 + 128a^3b^3 + 256ab \\
&^3c^2 - 1024a^2b^3c^3 + 1568a^2b^3c - 2176a^3b^3c^2 - 512a^4b^3c) - \\
&128b^5 + \tan(x/2) * (96ab^4 - 1536a^4c - 256b^4c - 1024a^2c^3 + 448a \\
&a^3c^2 + 256b^2c^3 + 1408ab^2c^2 - 512a^2b^2c) + 32a^2b^3 + 128b \\
&b^3c^2 - 1312a^2b^3c^2 - 640ab^3c^3 + 864ab^3c - 128a^3b^3c) + 128b \\
&c^3 - 96b^3c + 320ab^3c^2) * i) / (((8a^2c^4 - b^6 + 8a^3c^3 - b^3(- \\
&4ac - b^2)^3)^{1/2} + b^4c^2 - 6ab^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - ( \\
&/2) - 18a^2b^2c^2 + 8ab^4c + 2ab^2c^2(-4ac - b^2)^3)^{1/2} / (2(a^ \\
&4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a \\
&^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * \\
&(640a^3c^3 + 32b^4 + 768c^4 + 64a^2c^2 - 256b^2c^2 - 128ab^2c) - ( \\
&(8a^2c^4 - b^6 + 8a^3c^3 - b^3(-4ac - b^2)^3)^{1/2} + b^4c^2 - 6a \\
&b^2c^3 + b^2c^2(-4ac - b^2)^3)^{1/2} - 18a^2b^2c^2 + 8ab^4c + 2
\end{aligned}$$

$$\begin{aligned}
& a*b*c*(-(4*a*c - b^2)^3)^{(1/2)} / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2))^{(1/2)} * (((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * (\tan(x/2) * (256*a^5*c - 512*b^6 + 416*a^2*b^4 - 64*a^4*b^2 + 3072*a^2*c^4 + 5632*a^3*c^3 + 2816*a^4*c^2 + 512*b^4*c^2 - 2816*a*b^2*c^3 - 2368*a^3*b^2*c - 8576*a^2*b^2*c^2 + 3840*a*b^4*c) + ((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * (\tan(x/2) * (256*a^6*c - 512*a*b^6 + 544*a^3*b^4 - 64*a^5*b^2 + 6144*a^3*c^4 + 12288*a^4*c^3 + 6400*a^5*c^2 + 512*a*b^4*c^2 + 4608*a^2*b^4*c - 3776*a^4*b^2*c - 3584*a^2*b^2*c^3 - 13312*a^3*b^2*c^2) - 128*a^2*b^5 + 96*a^4*b^3 - 512*a^3*b*c^3 + 800*a^3*b^3*c - 1152*a^4*b*c^2 + 128*a^2*b^3*c^2 - 384*a^5*b*c) - 256*a*b^5 + 128*a^3*b^3 + 256*a*b^3*c^2 - 1024*a^2*b*c^3 + 1568*a^2*b^3*c - 2176*a^3*b*c^2 - 512*a^4*b*c) - 128*b^5 + \tan(x/2) * (96*a*b^4 - 1536*a*c^4 - 256*b^4*c - 1024*a^2*c^3 + 448*a^3*c^2 + 256*b^2*c^3 + 1408*a*b^2*c^2 - 512*a^2*b^2*c) + 32*a^2*b^3 + 128*b^3*c^2 - 1312*a^2*b*c^2 - 640*a*b*c^3 + 864*a*b^3*c - 128*a^3*b*c) + 128*b*c^3 - 96*b^3*c + 320*a*b*c^2) - ((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * (((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * (((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * (\tan(x/2) * (256*a^6*c - 512*a*b^6 + 544*a^3*b^4 - 64*a^5*b^2 + 6144*a^3*c^4 + 12288*a^4*c^3 + 6400*a^5*c^2 + 512*a*b^4*c^2 + 4608*a^2*b^4*c - 3776*a^4*b^2*c - 3584*a^2*b^2*c^3 - 13312*a^3*b^2*c^2) - 128*a^2*b^5 + 96*a^4*b^3 - 512*a^3*b*c^3 + 800*a^3*b^3*c - 1152*a^4*b*c^2 + 128*a^2*b^3*c^2 - 384*a^5*b*c) - \tan(x/2) * (256*a^5*c - 512*b^6 + 416*a^2*b^4 - 64*a^4*b^2 + 3072*a^2*c^4 + 5632*a^3*c^3 + 2816*a^4*c^2 + 512*b^4*c^2 - 2816*a*b^2*c^3 - 2368*a^3*b^2*c - 8576*a^2*b^2*c^2 + 3840*a*b^4*c) + 256*a*b^5 - 128*a^3*b^3 - 256*a*b^3*c^2 + 1024*a^2*b*c^3 - 1568*a^2*b^3*c + 2176*a^3*b*c^2 + 512*a^4*b*c) - 128*b^5 + \tan(x/2) * (96*a*b^4 - 1536*a*c^4 - 256*b^4*c - 1024*a^2*c^3 + 448*a^3*c^2 + 256*b^2*c^3 + 1408*a*b^2*c^2 - 512*a^2*b^2*c) + 32*a^2*b^3 + 128*b^3*c^2 - 1312*a^2*b*c^2 - 640*a*b*c^3 + 864*a*b^3*c - 128*a^3*b*c) + \tan(x/2) * (640*a*c^3 + 32*b^4 + 768*c^4 + 64*a^2*c^2 - 256*b^2*c^2 - 128*a*b^2*c) + 128*b*c^3 - 96*b^3*c + 320*a*b*c^2) + 256*c^3 * \tan(x/2) + 64*b*c^2) * ((8*a^2*c^4 - b^6 + 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2 - 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 18*a^2*b^2*c^2 + 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{(1/2)} * 2i + \operatorname{atan}(((b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(
\end{aligned}$$

$$\begin{aligned}
& - (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2} / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * ((- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * ((- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (256ab^5 - \tan(x/2) * (256a^5c - 512b^6 + 416a^2b^4 - 64a^4b^2 + 3072a^2c^4 + 5632a^3c^3 + 2816a^4c^2 + 512b^4c^2 - 2816ab^2c^3 - 2368a^3b^2c - 8576a^2b^2c^2 + 3840ab^4c) + (- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^6c - 512ab^6 + 544a^3b^4 - 64a^5b^2 + 6144a^3c^4 + 12288a^4c^3 + 6400a^5c^2 + 512ab^4c^2 + 4608a^2b^4c - 3776a^4b^2c - 3584a^2b^2c^3 - 13312a^3b^2c^2) - 128a^2b^5 + 96a^4b^3 - 512a^3b^3c^3 + 800a^3b^3c - 1152a^4b^3c^2 + 128a^2b^3c^2 - 384a^5b^3c) - 128a^3b^3 - 256ab^3c^2 + 1024a^2b^3c^3 - 1568a^2b^3c + 2176a^3b^3c^2 + 512a^4b^3c) - 128b^5 + \tan(x/2) * (96ab^4 - 1536a^4c^4 - 256b^4c - 1024a^2c^3 + 448a^3c^2 + 256b^2c^3 + 1408ab^2c^2 - 512a^2b^2c) + 32a^2b^3 + 128b^3c^2 - 1312a^2b^3c^2 - 640ab^3c^3 + 864ab^3c - 128a^3b^3c) + \tan(x/2) * (640a^3c^3 + 32b^4 + 768c^4 + 64a^2c^2 - 256b^2c^2 - 128ab^2c) + 128b^3c^3 - 96b^3c + 320abc^2) * i + (- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (640a^3c^3 + 32b^4 + 768c^4 + 64a^2c^2 - 256b^2c^2 - 128ab^2c) - (- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^5c - 512b^6 + 416a^2b^4 - 64a^4b^2 + 3072a^2c^4 + 5632a^3c^3 + 2816a^4c^2 + 512b^4c^2 - 2816ab^2c^3 - 2368a^3b^2c - 8576a^2b^2c^2 + 3840ab^4c) - 256ab^5 + (- (b^6 - 8a^2c^4 - 8a^3c^3 - b^3 * (- (4ac - b^2)^3)^{1/2} - b^4c^2 + 6ab^2c^3 + b^2c^2 * (- (4ac - b^2)^3)^{1/2} + 18a^2b^2c^2 - 8ab^4c + 2abc * (- (4ac - b^2)^3)^{1/2}) / (2(a^4b^4 - a^2b^6 + 16a^4c^4 + 32a^5c^3 + 16a^6c^2 + 10a^3b^4c - 8a^5b^2c + a^2b^4c^2 - 8a^3b^2c^3 - 32a^4b^2c^2))^{1/2} * (\tan(x/2) * (256a^6c - 512ab^6 + 544a^3b^4 - 64a^5b^2 + 6144a^3c^4 + 12288a^4c^3 + 6400a^5c^2 + 512ab^4c^2 + 4608a^2b^4c - 3776a^4b^2c - 3584a^2b^2c^3 - 13312a^3b^2c^2) - 128a^2b^5 + 96a^4b^3 - 512a^3b^3c^3 + 800a^3b^3c - 1152a^4b^3c^2 + 128a^2b^3c^2 - 384a^5b^3c) + 128a^3b^3 + 256ab^3c^2 - 1024a^2b^3c^3 + 1568a^2b^3c - 2176a^3b^3c^2 - 512a^4b^3c) - 128b^5 + \tan(x/2) * (96ab^4 - 1536a^4c^4 - 256b^4c - 1024a^2c^3 + 448a^3c^2 + 256b^2c^3 + 1408ab^2c^2 - 512a^2b^2c) + 32a^2b^3 + 128b^3c^2 - 1312a^2b^3c^2 - 640ab^3c^3 + 864ab^3c - 128a^3b^3c) + 128b^3c^3 - 96b^3c +
\end{aligned}$$

$$\begin{aligned}
& 320*a*b*c^2)*i)/(256*c^3*\tan(x/2) + 64*b*c^2 - ((b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*((-b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*((b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*((b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*((256*a*b^5 - \tan(x/2)*(256*a^5*c - 512*b^6 + 416*a^2*b^4 - 64*a^4*b^2 + 3072*a^2*c^4 + 5632*a^3*c^3 + 2816*a^4*c^2 + 512*b^4*c^2 - 2816*a*b^2*c^3 - 2368*a^3*b^2*c - 8576*a^2*b^2*c^2 + 3840*a*b^4*c) + (-b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*(\tan(x/2)*(256*a^6*c - 512*a*b^6 + 544*a^3*b^4 - 64*a^5*b^2 + 6144*a^3*c^4 + 12288*a^4*c^3 + 6400*a^5*c^2 + 512*a*b^4*c^2 + 4608*a^2*b^4*c - 3776*a^4*b^2*c - 3584*a^2*b^2*c^3 - 13312*a^3*b^2*c^2) - 128*a^2*b^5 + 96*a^4*b^3 - 512*a^3*b*c^3 + 800*a^3*b^3*c - 1152*a^4*b*c^2 + 128*a^2*b^3*c^2 - 384*a^5*b*c) - 128*a^3*b^3 - 256*a*b^3*c^2 + 1024*a^2*b*c^3 - 1568*a^2*b^3*c + 2176*a^3*b*c^2 + 512*a^4*b*c) - 128*b^5 + \tan(x/2)*(96*a*b^4 - 1536*a*c^4 - 256*b^4*c - 1024*a^2*c^3 + 448*a^3*c^2 + 256*b^2*c^3 + 1408*a*b^2*c^2 - 512*a^2*b^2*c) + 32*a^2*b^3 + 128*b^3*c^2 - 1312*a^2*b*c^2 - 640*a*b*c^3 + 864*a*b^3*c - 128*a^3*b*c) + \tan(x/2)*(640*a*c^3 + 32*b^4 + 768*c^4 + 64*a^2*c^2 - 256*b^2*c^2 - 128*a*b^2*c) + 128*b*c^3 - 96*b^3*c + 320*a*b*c^2) + (-b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*(\tan(x/2)*(640*a*c^3 + 32*b^4 + 768*c^4 + 64*a^2*c^2 - 256*b^2*c^2 - 128*a*b^2*c) - (-b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*((b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*(\tan(x/2)*(256*a^5*c - 512*b^6 + 416*a^2*b^4 - 64*a^4*b^2 + 3072*a^2*c^4 + 5632*a^3*c^3 + 2816*a^4*c^2 + 512*b^4*c^2 - 2816*a*b^2*c^3 - 2368*a^3*b^2*c - 8576*a^2*b^2*c^2 + 3840*a*b^4*c) - 256*a*b^5 + (-b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{1/2} - b^4*c^2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 18*a^2*b^2*c^2 - 8*a*b^4*c + 2*a*b*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 + 32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3*b^2*c^3 - 32*a^4*b^2*c^2)))^{1/2})*(\tan(x/2)*(256*a^6*c - 512*a*b^6 + 544*a^3*b^4 - 64*a^5*b^2 + 6144*a^3*c^4 + 12288*a^4*c^3 + 6400*a^5*c^2 + 512*a*b^4*c^2 + 4608*a^2*b^4*c - 3776*a^4*b^2*c - 3584*a^2*b^2*c^3 - 13312*a^3*b^2*c^2) - 128*a^2*b^5 + 96*a^4*b^3 - 512*a^3*b*c^3 + 800*a^3*b^3*c - 1152*a^4*b*c^2 + 128*a^2*b^3*c^2 - 384*a^5*b*c) + 128*a^3*b^3 + 256*a*b^3*c^2 - 1024*a^2*b*c^3 + 1568*a^2*b^3*c - 2176*a^3*b*c^2 - 512*a^4*b*c) - 128*b^5 + \tan(x/2)*(96*a*b^4 - 1536*a*c^4 - 256*b^4*c - 1024*a^2*c^3 + 448*a^3*c^2 + 256*b^2*c^3 + 1408
\end{aligned}$$



```
*a*b^2*c^2 - 512*a^2*b^2*c) + 32*a^2*b^3 + 128*b^3*c^2 - 1312*a^2*b*c^2 - 6
40*a*b*c^3 + 864*a*b^3*c - 128*a^3*b*c) + 128*b*c^3 - 96*b^3*c + 320*a*b*c^
2)))*(-(b^6 - 8*a^2*c^4 - 8*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^
2 + 6*a*b^2*c^3 + b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^2 - 8*a*b^4
*c + 2*a*b*c*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 - a^2*b^6 + 16*a^4*c^4 +
32*a^5*c^3 + 16*a^6*c^2 + 10*a^3*b^4*c - 8*a^5*b^2*c + a^2*b^4*c^2 - 8*a^3
*b^2*c^3 - 32*a^4*b^2*c^2)))^(1/2)*2i + log(tan(x/2))/a
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Integral(csc(x)/(a + b\*sin(x) + c\*sin(x)\*\*2), x)

### 3.7 $\int \frac{\csc^2(x)}{a+b \sin(x)+c \sin^2(x)} dx$

**Optimal.** Leaf size=271

$$\frac{\sqrt{2} bc \left( \frac{b^2-2ac}{b\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a^2\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} bc \left( 1 - \frac{b^2-2ac}{b\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a^2\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + b \tan(x)$$

[Out]  $b \operatorname{arctanh}(\cos(x))/a^2 - \cot(x)/a + b c \operatorname{arctan}(1/2*(2*c+(b-(-4*a*c+b^2)^{(1/2)}))*\tan(1/2*x))*2^{(1/2)}/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}*(1+(-2*a*c+b^2)/b/(-4*a*c+b^2)^{(1/2)})/a^2/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}+b*c \operatorname{arctan}(1/2*(2*c+(b+(-4*a*c+b^2)^{(1/2)}))*\tan(1/2*x))*2^{(1/2)}/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}*(1+(2*a*c-b^2)/b/(-4*a*c+b^2)^{(1/2)})/a^2/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.90, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3256, 3770, 3767, 8, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} bc \left( \frac{b^2-2ac}{b\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a^2\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} bc \left( 1 - \frac{b^2-2ac}{b\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{a^2\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + b \tan(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[x]^2/(a + b*\text{Sin}[x] + c*\text{Sin}[x]^2), x]$

[Out]  $(\text{Sqrt}[2]*b*c*(1 + (b^2 - 2*a*c)/(b*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(2*c + (b - \text{Sqrt}[b^2 - 4*a*c])*\text{Tan}[x/2])/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) - b*\text{Sqrt}[b^2 - 4*a*c]])]/(a^2*\text{Sqrt}[b^2 - 2*c*(a + c) - b*\text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*b*c*(1 - (b^2 - 2*a*c)/(b*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(2*c + (b + \text{Sqrt}[b^2 - 4*a*c])*\text{Tan}[x/2])/(\text{Sqrt}[2]*\text{Sqrt}[b^2 - 2*c*(a + c) + b*\text{Sqrt}[b^2 - 4*a*c]])]/(a^2*\text{Sqrt}[b^2 - 2*c*(a + c) + b*\text{Sqrt}[b^2 - 4*a*c]]) + (b*\text{ArcTanh}[\text{Cos}[x]])/a^2 - \text{Cot}[x]/a$

#### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

#### Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 2660

$\text{Int}[(a_) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{-1}, x\_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[\dots]$

$a^2 - b^2, 0]$

### Rule 3256

`Int[sin[(d_.) + (e_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_.)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]^(n2_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] / ; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegersQ[m, n, p]`

### Rule 3292

`Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_.)])/((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_.)] + (c_.)*sin[(d_.) + (e_.)*(x_.)]^2), x_Symbol] := Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]] / ; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]`

### Rule 3767

`Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] / ; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

### Rule 3770

`Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] / ; FreeQ[{c, d}, x]`

### Rubi steps

$$\begin{aligned}
 \int \frac{\csc^2(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( -\frac{b \csc(x)}{a^2} + \frac{\csc^2(x)}{a} + \frac{b^2 \left(1 - \frac{ac}{b^2}\right) + bc \sin(x)}{a^2 (a + b \sin(x) + c \sin^2(x))} \right) dx \\
 &= \frac{\int \frac{b^2 \left(1 - \frac{ac}{b^2}\right) + bc \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{a^2} + \frac{\int \csc^2(x) dx}{a} - \frac{b \int \csc(x) dx}{a^2} \\
 &= \frac{b \tanh^{-1}(\cos(x))}{a^2} - \frac{\text{Subst}\left(\int 1 dx, x, \cot(x)\right)}{a} + \frac{\left(c \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2x}}{a^2} \\
 &= \frac{b \tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} + \frac{\left(2c \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})^2}\right)}{a^2} \\
 &= \frac{b \tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} - \frac{\left(4c \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{4 \left(4c^2 - (b + \sqrt{b^2 - 4ac})^2\right) - x^2}\right)}{a^2} \\
 &= \frac{\sqrt{2} c \left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}}\right)}{a^2 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} c \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{2c - (b + \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}}\right)}{a^2 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}}
 \end{aligned}$$

**Mathematica [C]** time = 1.27, size = 388, normalized size = 1.43

$$\frac{\csc^2(x)(-2a - 2b \sin(x) + c \cos(2x) - c) \left( \frac{2c(b\sqrt{4ac-b^2} + 2iac - ib^2) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b - i\sqrt{4ac-b^2})}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2} - 2c(a+c)+b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}}\sqrt{-ib\sqrt{4ac-b^2} - 2c(a+c)+b^2}} \right) + \frac{2ic(ib\sqrt{4ac-b^2} + 2ac - b^2) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b + i\sqrt{4ac-b^2})}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2} - 2c(a+c)+b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}}\sqrt{-ib\sqrt{4ac-b^2} - 2c(a+c)+b^2}} \right)}{4a^2(a \csc^2(x) + b \csc(x) + c)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out] (Csc[x]^2\*(-2\*a - c + c\*Cos[2\*x] - 2\*b\*Sin[x]))\*((-2\*c\*((-I)\*b^2 + (2\*I)\*a\*c + b\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]]) + ((2\*I)\*c\*(-b^2 + 2\*a\*c + I\*b\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]]) + a\*Cot[x/2] - 2\*b\*Log[Cos[x/2]] + 2\*b\*Log[Sin[x/2]] - a\*Tan[x/2))/(4\*a^2\*(c + b\*Csc[x] + a\*Csc[x]^2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="fricas")

[Out] Timed out

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.37, size = 1087, normalized size = 4.01

$$\frac{\tan\left(\frac{x}{2}\right)}{2a} + \frac{6 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2} bc}{a(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}} - \frac{2 \arctan\left(\frac{-2a \tan\left(\frac{x}{2}\right) + \sqrt{-4ca + b^2} - b}{\sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}\right) \sqrt{-4ca + b^2} b^3}{a^2(4ca - b^2) \sqrt{4ca - 2b^2 + 2b\sqrt{-4ca + b^2} + 4a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2/(a+b\*sin(x)+c\*sin(x)^2), x)

[Out] 1/2/a\*tan(1/2\*x)+6/a/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*(-4\*a\*c+b^2)^(1/2)\*b\*c-2/a^2/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*(-4\*a\*c+b^2)^(1/2)\*b^3+8/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*c^2-10/a/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)

$$\begin{aligned} & 2)+4*a^2)^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)}-b)/(4*c*a-2*b^2+ \\ & 2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*b^2*c+2/a^2/(4*a*c-b^2)/(4*c*a-2*b^2+2 \\ & *b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((-2*a*\tan(1/2*x)+(-4*a*c+b^2)^{(1/2)} \\ & -b)/(4*c*a-2*b^2+2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*b^4+6/a/(4*a*c-b^2) \\ & /(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+ \\ & (-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*(-4*a \\ & *c+b^2)^{(1/2)}*b*c-2/a^2/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a \\ & ^2)^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4 \\ & *a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*b^3-8/(4*a*c-b^2)/(4*c*a-2 \\ & *b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2*a*\tan(1/2*x)+b+(-4*a*c+b \\ & ^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)})*c^2+10/a/(4*a* \\ & c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*\arctan((2*a*\tan(1/2 \\ & *x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)} \\ & )*b^2*c-2/a^2/(4*a*c-b^2)/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{(1/2)}+4*a^2)^{(1/2)}*a \\ & rctan((2*a*\tan(1/2*x)+b+(-4*a*c+b^2)^{(1/2)})/(4*c*a-2*b^2-2*b*(-4*a*c+b^2)^{( \\ & 1/2)}+4*a^2)^{(1/2)})*b^4-1/2/a/\tan(1/2*x)-1/a^2*b*\ln(\tan(1/2*x)) \end{aligned}$$

**maxima** [F(-1)]    time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Timed out

**mupad** [B]    time = 25.68, size = 16102, normalized size = 59.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^2\*(a + c\*sin(x)^2 + b\*sin(x))),x)

$$\begin{aligned} & [Out] \tan(x/2)/(2*a) - 1/(2*a*\tan(x/2)) - \operatorname{atan}(\frac{((-b^8 + 8*a^3*c^5 + 8*a^4*c^4 \\ & + b^5*(-4*a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 3 \\ & 3*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - b^3*c^2*(-4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c \\ & + 3*a^2*b*c^2*(-4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c^3*(-4*a*c - b^2)^3)^{(1/2)} - 4*a*b^3*c*(-4*a*c - b^2)^3)^{(1/2)}}{(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c \\ & ^4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8 \\ & *a^5*b^2*c^3 - 32*a^6*b^2*c^2))^{(1/2)}*((32*(4*a^5*b^4 - 8*a^3*b^6 + 16*a^5 \\ & *c^4 + 20*a^6*c^3 + 4*a^7*c^2 + 53*a^4*b^4*c - 17*a^6*b^2*c + 8*a^3*b^4*c^2 \\ & - 36*a^4*b^2*c^3 - 89*a^5*b^2*c^2)))/a^3 - ((32*(4*a^5*b^5 - 3*a^7*b^3 + 16 \\ & *a^6*b*c^3 - 25*a^6*b^3*c + 36*a^7*b*c^2 - 4*a^5*b^3*c^2 + 12*a^8*b*c)))/a^3 \\ & - (32*\tan(x/2)*(8*a^9*c - 16*a^4*b^6 + 17*a^6*b^4 - 2*a^8*b^2 + 192*a^6*c^4 \\ & + 384*a^7*c^3 + 200*a^8*c^2 + 144*a^5*b^4*c - 118*a^7*b^2*c + 16*a^4*b^4*c^2 \\ & - 112*a^5*b^2*c^3 - 416*a^6*b^2*c^2))/a^3)*((-b^8 + 8*a^3*c^5 + 8*a^4*c^4 \\ & ^4 + b^5*(-4*a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 \\ & + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - b^3*c^2*(-4*a*c - b^2)^3)^{(1/2)} - 10*a \\ & *b^6*c + 3*a^2*b*c^2*(-4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c^3*(-4*a*c - b^2)^3 \\ & )^{(1/2)} - 4*a*b^3*c*(-4*a*c - b^2)^3)^{(1/2)}}{(2*(a^6*b^4 - a^4*b^6 + 16*a^6 \\ & *c^4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 \\ & - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2))^{(1/2)} + (32*\tan(x/2)*(13*a^4*b^5 - 16*a \\ & ^2*b^7 - 2*a^6*b^3 + 128*a^3*b^5*c + 128*a^4*b*c^4 + 240*a^5*b*c^3 - 78*a^5 \\ & *b^3*c + 104*a^6*b*c^2 + 16*a^2*b^5*c^2 - 96*a^3*b^3*c^3 - 316*a^4*b^3*c^2 \\ & + 8*a^7*b*c))/a^3} + (32*(a^3*b^5 - 4*a*b^7 + 4*a*b^5*c^2 + 31*a^2*b^5*c + \\ & 28*a^3*b*c^4 + 35*a^4*b*c^3 - 5*a^4*b^3*c + 4*a^5*b*c^2 - 24*a^2*b^3*c^3 - \\ & 68*a^3*b^3*c^2))/a^3 + (32*\tan(x/2)*(3*a^2*b^6 + 80*a^3*c^5 + 80*a^4*c^4 + \\ & 2*a^5*c^3 + 16*a*b^4*c^3 - 18*a^3*b^4*c - 88*a^2*b^2*c^4 + 116*a^2*b^4*c^2 \\ & - 224*a^3*b^2*c^3 + 23*a^4*b^2*c^2 - 16*a*b^6*c))/a^3)*((-b^8 + 8*a^3*c^5 + \\ & 8*a^4*c^4 + b^5*(-4*a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2* \end{aligned}$$

$$\begin{aligned}
& b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} \\
& - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)} \\
& / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& + (32(3b^6c + 4a^2c^5 + a^3c^4 - 4b^4c^3 + 12ab^2c^4 - 15ab^4c^2 + 14a^2b^2c^3))/a^3 \\
& + (32\tan(x/2)(8b^5c^2 - 8b^3c^4 - b^7 - 32ab^3c^3 + 12a^2b^4c + 2a^3b^2c^3 - 9a^2b^3c^2 + 16ab^4c^5 + 6ab^5c^2))/a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} \\
& - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)} \\
& / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} * i \\
& + ((32(3b^6c + 4a^2c^5 + a^3c^4 - 4b^4c^3 + 12ab^2c^4 - 15ab^4c^2 + 14a^2b^2c^3))/a^3 - ((32(a^3b^5 - 4ab^7 + 4ab^5c^2 + 31a^2b^5c + 28a^3b^3c^4 + 35a^4b^3c^3 - 5a^4b^3c^3 + 4a^5b^3c^2 - 24a^2b^3c^3 - 68a^3b^3c^2))/a^3 \\
& - (-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)}) / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& * ((32(4a^5b^4 - 8a^3b^6 + 16a^5c^4 + 20a^6c^3 + 4a^7c^2 + 53a^4b^4c - 17a^6b^2c + 8a^3b^4c^2 - 36a^4b^2c^3 - 89a^5b^2c^2))/a^3 + ((32(4a^5b^5 - 3a^7b^3 + 16a^6b^3c - 25a^6b^3c + 36a^7b^3c^2 - 4a^5b^3c^2 + 12a^8b^3c))/a^3 - (32\tan(x/2)(8a^9c - 16a^4b^6 + 17a^6b^4 - 2a^8b^2 + 192a^6c^4 + 384a^7c^3 + 200a^8c^2 + 144a^5b^4c - 118a^7b^2c + 16a^4b^4c^2 - 112a^5b^2c^3 - 416a^6b^2c^2))/a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)}) / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& + (32\tan(x/2)(13a^4b^5 - 16a^2b^7 - 2a^6b^3 + 128a^3b^5c + 128a^4b^3c^4 + 240a^5b^3c^3 - 78a^5b^3c + 104a^6b^3c^2 + 16a^2b^5c^2 - 96a^3b^3c^3 - 316a^4b^3c^2 + 8a^7b^3c))/a^3 + (32\tan(x/2)(3a^2b^6 + 80a^3c^5 + 80a^4c^4 + 2a^5c^3 + 16ab^4c^3 - 18a^3b^4c - 88a^2b^2c^4 + 116a^2b^4c^2 - 224a^3b^2c^3 + 23a^4b^2c^2 - 16ab^6c))/a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)}) / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& + (32\tan(x/2)(8b^5c^2 - 8b^3c^4 - b^7 - 32ab^3c^3 + 12a^2b^4c + 2a^3b^2c^3 - 9a^2b^3c^2 + 16ab^4c^5 + 6ab^5c^2))/a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)}) / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} * i \\
& / ((((-b^8 + 8a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& + 2ab^3c^3(-4ac - b^2)^3)^{(1/2)} - 4ab^3c^3(-4ac - b^2)^3)^{(1/2)}) / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& *a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2))^{1/2} \\
& ((1/2)*((32*(4*a^5*b^4 - 8*a^3*b^6 + 16*a^5*c^4 + 20*a^6*c^3 + 4*a^7*c^2 + 5 \\
& 3*a^4*b^4*c - 17*a^6*b^2*c + 8*a^3*b^4*c^2 - 36*a^4*b^2*c^3 - 89*a^5*b^2*c^2 \\
& 2))/a^3 - ((32*(4*a^5*b^5 - 3*a^7*b^3 + 16*a^6*b*c^3 - 25*a^6*b^3*c + 36*a^7 \\
& 7*b*c^2 - 4*a^5*b^3*c^2 + 12*a^8*b*c))/a^3 - (32*\tan(x/2)*(8*a^9*c - 16*a^4 \\
& *b^6 + 17*a^6*b^4 - 2*a^8*b^2 + 192*a^6*c^4 + 384*a^7*c^3 + 200*a^8*c^2 + 1 \\
& 44*a^5*b^4*c - 118*a^7*b^2*c + 16*a^4*b^4*c^2 - 112*a^5*b^2*c^3 - 416*a^6*b^2 \\
& ^2*c^2))/a^3)*(-(b^8 + 8*a^3*c^5 + 8*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^{1/2} \\
& - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 \\
& - b^3*c^2*(-(4*a*c - b^2)^3)^{1/2} - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b \\
& ^2)^3)^{1/2} + 2*a*b*c^3*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^ \\
& ^2)^3)^{1/2}))/((2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16*a^8*c^2 + \\
& 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2) \\
& ))^{1/2} + (32*\tan(x/2)*(13*a^4*b^5 - 16*a^2*b^7 - 2*a^6*b^3 + 128*a^3*b^5*c \\
& c + 128*a^4*b*c^4 + 240*a^5*b*c^3 - 78*a^5*b^3*c + 104*a^6*b*c^2 + 16*a^2*b^5 \\
& ^5*c^2 - 96*a^3*b^3*c^3 - 316*a^4*b^3*c^2 + 8*a^7*b*c))/a^3 + (32*(a^3*b^5 \\
& - 4*a*b^7 + 4*a*b^5*c^2 + 31*a^2*b^5*c + 28*a^3*b*c^4 + 35*a^4*b*c^3 - 5*a^4 \\
& ^4*b^3*c + 4*a^5*b*c^2 - 24*a^2*b^3*c^3 - 68*a^3*b^3*c^2))/a^3 + (32*\tan(x/ \\
& 2)*(3*a^2*b^6 + 80*a^3*c^5 + 80*a^4*c^4 + 2*a^5*c^3 + 16*a*b^4*c^3 - 18*a^3 \\
& *b^4*c - 88*a^2*b^2*c^4 + 116*a^2*b^4*c^2 - 224*a^3*b^2*c^3 + 23*a^4*b^2*c^2 \\
& ^2 - 16*a*b^6*c))/a^3)*(-(b^8 + 8*a^3*c^5 + 8*a^4*c^4 + b^5*(-(4*a*c - b^2)^ \\
& ^3)^{1/2} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3 \\
& *b^2*c^3 - b^3*c^2*(-(4*a*c - b^2)^3)^{1/2} - 10*a*b^6*c + 3*a^2*b*c^2*(-(4 \\
& *a*c - b^2)^3)^{1/2} + 2*a*b*c^3*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4 \\
& a*c - b^2)^3)^{1/2}))/((2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16*a \\
& ^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6 \\
& ^6*b^2*c^2))^{1/2} + (32*(3*b^6*c + 4*a^2*c^5 + a^3*c^4 - 4*b^4*c^3 + 12*a*b^2 \\
& ^2*c^4 - 15*a*b^4*c^2 + 14*a^2*b^2*c^3))/a^3 + (32*\tan(x/2)*(8*b^5*c^2 - 8*b \\
& ^3*c^4 - b^7 - 32*a*b^3*c^3 + 12*a^2*b*c^4 + 2*a^3*b*c^3 - 9*a^2*b^3*c^2 + \\
& 16*a*b*c^5 + 6*a*b^5*c))/a^3)*(-(b^8 + 8*a^3*c^5 + 8*a^4*c^4 + b^5*(-(4*a*c \\
& - b^2)^3)^{1/2} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 \\
& - 38*a^3*b^2*c^3 - b^3*c^2*(-(4*a*c - b^2)^3)^{1/2} - 10*a*b^6*c + 3*a^2*b \\
& ^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 2*a*b*c^3*(-(4*a*c - b^2)^3)^{1/2} - 4*a*b^3 \\
& ^3*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^ \\
& ^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - \\
& 32*a^6*b^2*c^2))^{1/2} - ((32*(3*b^6*c + 4*a^2*c^5 + a^3*c^4 - 4*b^4*c^3 \\
& + 12*a*b^2*c^4 - 15*a*b^4*c^2 + 14*a^2*b^2*c^3))/a^3 - ((32*(a^3*b^5 - 4*a* \\
& b^7 + 4*a*b^5*c^2 + 31*a^2*b^5*c + 28*a^3*b*c^4 + 35*a^4*b*c^3 - 5*a^4*b^3* \\
& c + 4*a^5*b*c^2 - 24*a^2*b^3*c^3 - 68*a^3*b^3*c^2))/a^3 - ((b^8 + 8*a^3*c^ \\
& ^5 + 8*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^{1/2} - b^6*c^2 + 8*a*b^4*c^3 - 18*a \\
& ^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - b^3*c^2*(-(4*a*c - b^2)^3)^{ \\
& ^{1/2} - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 2*a*b*c^3*(-(4 \\
& *a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^6*b^4 - a \\
& ^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a \\
& ^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2))^{1/2})*((32*(4*a^5*b^4 - 8*a^ \\
& ^3*b^6 + 16*a^5*c^4 + 20*a^6*c^3 + 4*a^7*c^2 + 53*a^4*b^4*c - 17*a^6*b^2*c + \\
& 8*a^3*b^4*c^2 - 36*a^4*b^2*c^3 - 89*a^5*b^2*c^2))/a^3 + ((32*(4*a^5*b^5 - \\
& 3*a^7*b^3 + 16*a^6*b*c^3 - 25*a^6*b^3*c + 36*a^7*b*c^2 - 4*a^5*b^3*c^2 + 12 \\
& *a^8*b*c))/a^3 - (32*\tan(x/2)*(8*a^9*c - 16*a^4*b^6 + 17*a^6*b^4 - 2*a^8*b^ \\
& ^2 + 192*a^6*c^4 + 384*a^7*c^3 + 200*a^8*c^2 + 144*a^5*b^4*c - 118*a^7*b^2*c \\
& + 16*a^4*b^4*c^2 - 112*a^5*b^2*c^3 - 416*a^6*b^2*c^2))/a^3)*(-(b^8 + 8*a^3 \\
& ^3*c^5 + 8*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^{1/2} - b^6*c^2 + 8*a*b^4*c^3 - 1 \\
& 8*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - b^3*c^2*(-(4*a*c - b^2)^3 \\
& )^{1/2} - 10*a*b^6*c + 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{1/2} + 2*a*b*c^3*(-( \\
& 4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c*(-(4*a*c - b^2)^3)^{1/2}))/((2*(a^6*b^4 - a \\
& ^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c \\
& + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2))^{1/2} + (32*\tan(x/2)*(13 \\
& a^4*b^5 - 16*a^2*b^7 - 2*a^6*b^3 + 128*a^3*b^5*c + 128*a^4*b*c^4 + 240*a^5 \\
& ^5*b*c^3 - 78*a^5*b^3*c + 104*a^6*b*c^2 + 16*a^2*b^5*c^2 - 96*a^3*b^3*c^3 - 31
\end{aligned}$$

$$\begin{aligned}
& (6a^4b^3c^2 + 8a^7b^3c)/a^3 + (32\tan(x/2)*(3a^2b^6 + 80a^3c^5 + 80a^4c^4 + 2a^5c^3 + 16ab^4c^3 - 18a^3b^4c - 88a^2b^2c^4 + 116a^2b^4c^2 - 224a^3b^2c^3 + 23a^4b^2c^2 - 16ab^6c))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 + b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c + 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} + 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} - 4ab^3c*(-(4ac - b^2)^3)^{1/2})/(2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2)))^{1/2} + (32\tan(x/2)*(8b^5c^2 - 8b^3c^4 - b^7 - 32ab^3c^3 + 12a^2b^4c + 2a^3b^3c^3 - 9a^2b^3c^2 + 16ab^5c^5 + 6ab^5c))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 + b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c + 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} + 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} - 4ab^3c*(-(4ac - b^2)^3)^{1/2})/(2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2)))^{1/2} + (64*(4b^5c^5 - b^3c^3 + ab^4c^4))/a^3 + (64\tan(x/2)*(8c^6 - 4b^2c^4))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 + b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c + 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} + 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} - 4ab^3c*(-(4ac - b^2)^3)^{1/2})/(2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2)))^{1/2} * 2i - \operatorname{atan}(\frac{(-(b^8 + 8a^3c^5 + 8a^4c^4 - b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c - 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} - 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} + 4ab^3c*(-(4ac - b^2)^3)^{1/2})}{2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))} \\
& - ((32*(4a^5b^4 - 8a^3b^6 + 16a^5c^4 + 20a^6c^3 + 4a^7c^2 + 53a^4b^4c - 17a^6b^2c + 8a^3b^4c^2 - 36a^4b^2c^3 - 89a^5b^2c^2))/a^3 - (32*(4a^5b^5 - 3a^7b^3 + 16a^6b^3c - 25a^6b^3c + 36a^7b^3c^2 - 4a^5b^3c^2 + 12a^8b^3c))/a^3 - (32\tan(x/2)*(8a^9c - 16a^4b^6 + 17a^6b^4 - 2a^8b^2 + 192a^6c^4 + 384a^7c^3 + 200a^8c^2 + 144a^5b^4c - 118a^7b^2c + 16a^4b^4c^2 - 112a^5b^2c^3 - 416a^6b^2c^2))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 - b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c - 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} - 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} + 4ab^3c*(-(4ac - b^2)^3)^{1/2})/(2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2)))^{1/2} + (32\tan(x/2)*(13a^4b^5 - 16a^2b^7 - 2a^6b^3 + 128a^3b^5c + 128a^4b^3c^4 + 240a^5b^3c^3 - 78a^5b^3c + 104a^6b^3c^2 + 16a^2b^5c^2 - 96a^3b^3c^3 - 316a^4b^3c^2 + 8a^7b^3c))/a^3 + (32*(a^3b^5 - 4ab^7 + 4ab^5c^2 + 31a^2b^5c + 28a^3b^4c + 35a^4b^3c^3 - 5a^4b^3c + 4a^5b^3c^2 - 24a^2b^3c^3 - 68a^3b^3c^2))/a^3 + (32\tan(x/2)*(3a^2b^6 + 80a^3c^5 + 80a^4c^4 + 2a^5c^3 + 16ab^4c^3 - 18a^3b^4c - 88a^2b^2c^4 + 116a^2b^4c^2 - 224a^3b^2c^3 + 23a^4b^2c^2 - 16ab^6c))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 - b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2*(-(4ac - b^2)^3)^{1/2} - 10ab^6c - 3a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} - 2ab^3c^3*(-(4ac - b^2)^3)^{1/2} + 4ab^3c*(-(4ac - b^2)^3)^{1/2})/(2*(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2)))^{1/2} + (32*(3b^6c + 4a^2c^5 + a^3c^4 - 4b^4c^3 + 12ab^2c^4 - 15ab^4c^2 + 14a^2b^2c^3))/a^3 + (32\tan(x/2)*(8b^5c^2 - 8b^3c^4 - b^7 - 32ab^3c^3 + 12a^2b^4c + 2a^3b^3c^3 - 9a^2b^3c^2 + 16ab^5c^5 + 6ab^5c))/a^3 * (-(b^8 + 8a^3c^5 + 8a^4c^4 - b^5*(-(4ac - b^2)^3)^{1/2} - b^6c^2
\end{aligned}$$



$$\begin{aligned}
& + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + b^3*c^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} \\
& ) - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)} \\
& )/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4 \\
& *c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^6*b^2*c^2)))^{(1/2)}*i \\
& + ((32*(3*b^6*c + 4*a^2*c^5 + a^3*c^4 - 4*b^4*c^3 + 12*a*b^2*c^4 - 15*a*b^ \\
& 4*c^2 + 14*a^2*b^2*c^3))/a^3 - ((32*(a^3*b^5 - 4*a*b^7 + 4*a*b^5*c^2 + 31*a \\
& ^2*b^5*c + 28*a^3*b*c^4 + 35*a^4*b*c^3 - 5*a^4*b^3*c + 4*a^5*b*c^2 - 24*a^2 \\
& *b^3*c^3 - 68*a^3*b^3*c^2))/a^3 - (- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - b^5*(- (4 \\
& *a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4* \\
& c^2 - 38*a^3*b^2*c^3 + b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^ \\
& 2*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} + 4*a \\
& *b^3*c*(- (4*a*c - b^2)^3)^{(1/2)))/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^ \\
& 7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c \\
& ^3 - 32*a^6*b^2*c^2)))^{(1/2)}*((32*(4*a^5*b^4 - 8*a^3*b^6 + 16*a^5*c^4 + 20* \\
& a^6*c^3 + 4*a^7*c^2 + 53*a^4*b^4*c - 17*a^6*b^2*c + 8*a^3*b^4*c^2 - 36*a^4* \\
& b^2*c^3 - 89*a^5*b^2*c^2))/a^3 + ((32*(4*a^5*b^5 - 3*a^7*b^3 + 16*a^6*b*c^3 \\
& - 25*a^6*b^3*c + 36*a^7*b*c^2 - 4*a^5*b^3*c^2 + 12*a^8*b*c))/a^3 - (32*tan \\
& (x/2)*(8*a^9*c - 16*a^4*b^6 + 17*a^6*b^4 - 2*a^8*b^2 + 192*a^6*c^4 + 384*a^ \\
& 7*c^3 + 200*a^8*c^2 + 144*a^5*b^4*c - 118*a^7*b^2*c + 16*a^4*b^4*c^2 - 112* \\
& a^5*b^2*c^3 - 416*a^6*b^2*c^2))/a^3)*(- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - b^5*(- \\
& - (4*a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b \\
& ^4*c^2 - 38*a^3*b^2*c^3 + b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3 \\
& *a^2*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} + \\
& 4*a*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)))/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32 \\
& *a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^ \\
& 2*c^3 - 32*a^6*b^2*c^2)))^{(1/2)} + (32*tan(x/2)*(13*a^4*b^5 - 16*a^2*b^7 - 2 \\
& *a^6*b^3 + 128*a^3*b^5*c + 128*a^4*b*c^4 + 240*a^5*b*c^3 - 78*a^5*b^3*c + 1 \\
& 04*a^6*b*c^2 + 16*a^2*b^5*c^2 - 96*a^3*b^3*c^3 - 316*a^4*b^3*c^2 + 8*a^7*b* \\
& c))/a^3) + (32*tan(x/2)*(3*a^2*b^6 + 80*a^3*c^5 + 80*a^4*c^4 + 2*a^5*c^3 + \\
& 16*a*b^4*c^3 - 18*a^3*b^4*c - 88*a^2*b^2*c^4 + 116*a^2*b^4*c^2 - 224*a^3*b^ \\
& 2*c^3 + 23*a^4*b^2*c^2 - 16*a*b^6*c))/a^3)*(- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - \\
& b^5*(- (4*a*c - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33 \\
& *a^2*b^4*c^2 - 38*a^3*b^2*c^3 + b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6 \\
& *c - 3*a^2*b*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1 \\
& /2)} + 4*a*b^3*c*(- (4*a*c - b^2)^3)^{(1/2)))/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^ \\
& 4 + 32*a^7*c^3 + 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8* \\
& a^5*b^2*c^3 - 32*a^6*b^2*c^2)))^{(1/2)} + (32*tan(x/2)*(8*b^5*c^2 - 8*b^3*c^4 \\
& - b^7 - 32*a*b^3*c^3 + 12*a^2*b*c^4 + 2*a^3*b*c^3 - 9*a^2*b^3*c^2 + 16*a*b \\
& *c^5 + 6*a*b^5*c))/a^3)*(- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - b^5*(- (4*a*c - b^2 \\
& )^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a \\
& ^3*b^2*c^3 + b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(- \\
& (4*a*c - b^2)^3)^{(1/2)))/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + 16 \\
& *a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32*a^ \\
& 6*b^2*c^2)))^{(1/2)}*i)/(((((- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - b^5*(- (4*a*c - b \\
& ^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38 \\
& *a^3*b^2*c^3 + b^3*c^2*(- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c^3*(- (4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(- \\
& - (4*a*c - b^2)^3)^{(1/2)))/(2*(a^6*b^4 - a^4*b^6 + 16*a^6*c^4 + 32*a^7*c^3 + \\
& 16*a^8*c^2 + 10*a^5*b^4*c - 8*a^7*b^2*c + a^4*b^4*c^2 - 8*a^5*b^2*c^3 - 32* \\
& a^6*b^2*c^2)))^{(1/2)}*((32*(4*a^5*b^4 - 8*a^3*b^6 + 16*a^5*c^4 + 20*a^6*c^3 \\
& + 4*a^7*c^2 + 53*a^4*b^4*c - 17*a^6*b^2*c + 8*a^3*b^4*c^2 - 36*a^4*b^2*c^3 \\
& - 89*a^5*b^2*c^2))/a^3 - ((32*(4*a^5*b^5 - 3*a^7*b^3 + 16*a^6*b*c^3 - 25*a^ \\
& 6*b^3*c + 36*a^7*b*c^2 - 4*a^5*b^3*c^2 + 12*a^8*b*c))/a^3 - (32*tan(x/2)*(8 \\
& *a^9*c - 16*a^4*b^6 + 17*a^6*b^4 - 2*a^8*b^2 + 192*a^6*c^4 + 384*a^7*c^3 + \\
& 200*a^8*c^2 + 144*a^5*b^4*c - 118*a^7*b^2*c + 16*a^4*b^4*c^2 - 112*a^5*b^2* \\
& c^3 - 416*a^6*b^2*c^2))/a^3)*(- (b^8 + 8*a^3*c^5 + 8*a^4*c^4 - b^5*(- (4*a*c \\
& - b^2)^3)^{(1/2)} - b^6*c^2 + 8*a*b^4*c^3 - 18*a^2*b^2*c^4 + 33*a^2*b^4*c^2 -
\end{aligned}$$

$$\begin{aligned}
& 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} \\
& / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& + (32\tan(x/2)(13a^4b^5 - 16a^2b^7 - 2a^6b^3 + 128a^3b^5c + 128a^4b^3c^4 + 240a^5b^2c^3 - 78a^5b^3c + 104a^6b^2c^2 + 16a^2b^5c^2 - 96a^3b^3c^3 - 316a^4b^3c^2 + 8a^7b^3c)) / a^3 \\
& + (32(a^3b^5 - 4ab^7 + 4ab^5c^2 + 31a^2b^5c + 28a^3b^3c^4 + 35a^4b^3c^3 - 5a^4b^3c + 4a^5b^3c^2 - 24a^2b^3c^3 - 68a^3b^3c^2)) / a^3 \\
& + (32\tan(x/2)(3a^2b^6 + 80a^3c^5 + 80a^4c^4 + 2a^5c^3 + 16ab^4c^3 - 18a^3b^4c - 88a^2b^2c^4 + 116a^2b^4c^2 - 224a^3b^2c^3 + 23a^4b^2c^2 - 16ab^6c)) / a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 - b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& + (32(3b^6c + 4a^2c^5 + a^3c^4 - 4b^4c^3 + 12ab^2c^4 - 15ab^4c^2 + 14a^2b^2c^3)) / a^3 + (32\tan(x/2)(8b^5c^2 - 8b^3c^4 - b^7 - 32ab^3c^3 + 12a^2b^3c^4 + 2a^3b^3c^3 - 9a^2b^3c^2 + 16ab^3c^5 + 6ab^5c)) / a^3 \\
& * (-b^8 + 8a^3c^5 + 8a^4c^4 - b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} \\
& - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} \\
& - ((32(3b^6c + 4a^2c^5 + a^3c^4 - 4b^4c^3 + 12ab^2c^4 - 15ab^4c^2 + 14a^2b^2c^3)) / a^3 - ((32(a^3b^5 - 4ab^7 + 4ab^5c^2 + 31a^2b^5c + 28a^3b^3c^4 + 35a^4b^3c^3 - 5a^4b^3c + 4a^5b^3c^2 - 24a^2b^3c^3 - 68a^3b^3c^2)) / a^3 - (-b^8 + 8a^3c^5 + 8a^4c^4 - b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} * ((32(4a^5b^4 - 8a^3b^6 + 16a^5c^4 + 20a^6c^3 + 4a^7c^2 + 53a^4b^4c - 17a^6b^2c + 8a^3b^4c^2 - 36a^4b^2c^3 - 89a^5b^2c^2)) / a^3 + ((32(4a^5b^5 - 3a^7b^3 + 16a^6b^3c - 25a^6b^3c + 36a^7b^3c^2 - 4a^5b^3c^2 + 12a^8b^3c)) / a^3 - (32\tan(x/2)(8a^9c - 16a^4b^6 + 17a^6b^4 - 2a^8b^2 + 192a^6c^4 + 384a^7c^3 + 200a^8c^2 + 144a^5b^4c - 118a^7b^2c + 16a^4b^4c^2 - 112a^5b^2c^3 - 416a^6b^2c^2)) / a^3) * (-b^8 + 8a^3c^5 + 8a^4c^4 - b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)} + (32\tan(x/2)(13a^4b^5 - 16a^2b^7 - 2a^6b^3 + 128a^3b^5c + 128a^4b^3c^4 + 240a^5b^2c^3 - 78a^5b^3c + 104a^6b^2c^2 + 16a^2b^5c^2 - 96a^3b^3c^3 - 316a^4b^3c^2 + 8a^7b^3c)) / a^3 + (32\tan(x/2)(3a^2b^6 + 80a^3c^5 + 80a^4c^4 + 2a^5c^3 + 16ab^4c^3 - 18a^3b^4c - 88a^2b^2c^4 + 116a^2b^4c^2 - 224a^3b^2c^3 + 23a^4b^2c^2 - 16ab^6c)) / a^3 * (-b^8 + 8a^3c^5 + 8a^4c^4 - b^5(-4ac - b^2)^3)^{(1/2)} - b^6c^2 + 8ab^4c^3 - 18a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + b^3c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^6c - 3a^2b^2c^2(-4ac - b^2)^3)^{(1/2)} - 2abc^3(-4ac - b^2)^3)^{(1/2)} + 4ab^3c^2(-4ac - b^2)^3)^{(1/2)} / (2(a^6b^4 - a^4b^6 + 16a^6c^4 + 32a^7c^3 + 16a^8c^2 + 10a^5b^4c - 8a^7b^2c + a^4b^4c^2 - 8a^5b^2c^3 - 32a^6b^2c^2))^{(1/2)}
\end{aligned}$$

) + (32\*tan(x/2)\*(8\*b^5\*c^2 - 8\*b^3\*c^4 - b^7 - 32\*a\*b^3\*c^3 + 12\*a^2\*b\*c^4 + 2\*a^3\*b\*c^3 - 9\*a^2\*b^3\*c^2 + 16\*a\*b\*c^5 + 6\*a\*b^5\*c))/a^3\*(-(b^8 + 8\*a^3\*c^5 + 8\*a^4\*c^4 - b^5\*(-(4\*a\*c - b^2)^3)^(1/2) - b^6\*c^2 + 8\*a\*b^4\*c^3 - 18\*a^2\*b^2\*c^4 + 33\*a^2\*b^4\*c^2 - 38\*a^3\*b^2\*c^3 + b^3\*c^2\*(-(4\*a\*c - b^2)^3)^(1/2) - 10\*a\*b^6\*c - 3\*a^2\*b\*c^2\*(-(4\*a\*c - b^2)^3)^(1/2) - 2\*a\*b\*c^3\*(-(4\*a\*c - b^2)^3)^(1/2) + 4\*a\*b^3\*c\*(-(4\*a\*c - b^2)^3)^(1/2))/(2\*(a^6\*b^4 - a^4\*b^6 + 16\*a^6\*c^4 + 32\*a^7\*c^3 + 16\*a^8\*c^2 + 10\*a^5\*b^4\*c - 8\*a^7\*b^2\*c + a^4\*b^4\*c^2 - 8\*a^5\*b^2\*c^3 - 32\*a^6\*b^2\*c^2)))^(1/2) + (64\*(4\*b\*c^5 - b^3\*c^3 + a\*b\*c^4))/a^3 + (64\*tan(x/2)\*(8\*c^6 - 4\*b^2\*c^4))/a^3)\*(-(b^8 + 8\*a^3\*c^5 + 8\*a^4\*c^4 - b^5\*(-(4\*a\*c - b^2)^3)^(1/2) - b^6\*c^2 + 8\*a\*b^4\*c^3 - 18\*a^2\*b^2\*c^4 + 33\*a^2\*b^4\*c^2 - 38\*a^3\*b^2\*c^3 + b^3\*c^2\*(-(4\*a\*c - b^2)^3)^(1/2) - 10\*a\*b^6\*c - 3\*a^2\*b\*c^2\*(-(4\*a\*c - b^2)^3)^(1/2) - 2\*a\*b\*c^3\*(-(4\*a\*c - b^2)^3)^(1/2) + 4\*a\*b^3\*c\*(-(4\*a\*c - b^2)^3)^(1/2))/(2\*(a^6\*b^4 - a^4\*b^6 + 16\*a^6\*c^4 + 32\*a^7\*c^3 + 16\*a^8\*c^2 + 10\*a^5\*b^4\*c - 8\*a^7\*b^2\*c + a^4\*b^4\*c^2 - 8\*a^5\*b^2\*c^3 - 32\*a^6\*b^2\*c^2)))^(1/2)\*2i - (b\*log(tan(x/2)))/a^2

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)\*\*2/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Integral(csc(x)\*\*2/(a + b\*sin(x) + c\*sin(x)\*\*2), x)

$$3.8 \quad \int \frac{\csc^3(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=331

$$\frac{(b^2 - ac) \tanh^{-1}(\cos(x))}{a^3} + \frac{\sqrt{2} c \left( \sqrt{b^2 - 4ac} (b^2 - ac) - 3abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) (b - \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} \right)}{a^3 \sqrt{b^2 - 4ac} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} + \frac{\sqrt{2} c \left( -\sqrt{b^2 - 4ac} (b^2 - ac) - 3abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) (b + \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} \right)}{a^3 \sqrt{b^2 - 4ac} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}}$$

[Out]  $-1/2 \cdot \operatorname{arctanh}(\cos(x)) / a - (-a \cdot c + b^2) \cdot \operatorname{arctanh}(\cos(x)) / a^3 + b \cdot \cot(x) / a^2 - 1/2 \cdot \cot(x) \cdot \csc(x) / a - c \cdot \arctan(1/2 \cdot (2 \cdot c + (b - (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot \tan(1/2 \cdot x)) \cdot 2^{1/2} / (b^2 - 2 \cdot c \cdot (a + c) - b \cdot (-4 \cdot a \cdot c + b^2)^{1/2})^{1/2} \cdot 2^{1/2} \cdot (b^3 - 3 \cdot a \cdot b \cdot c + (-a \cdot c + b^2) \cdot (-4 \cdot a \cdot c + b^2)^{1/2}) / a^3 / (-4 \cdot a \cdot c + b^2)^{1/2} / (b^2 - 2 \cdot c \cdot (a + c) - b \cdot (-4 \cdot a \cdot c + b^2)^{1/2})^{1/2} + c \cdot \arctan(1/2 \cdot (2 \cdot c + (b + (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot \tan(1/2 \cdot x)) \cdot 2^{1/2} / (b^2 - 2 \cdot c \cdot (a + c) + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2})^{1/2} \cdot 2^{1/2} \cdot (b^3 - 3 \cdot a \cdot b \cdot c - (-a \cdot c + b^2) \cdot (-4 \cdot a \cdot c + b^2)^{1/2}) / a^3 / (-4 \cdot a \cdot c + b^2)^{1/2} / (b^2 - 2 \cdot c \cdot (a + c) + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2})^{1/2}$

**Rubi [A]** time = 3.21, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3256, 3770, 3767, 8, 3768, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} c \left( \sqrt{b^2 - 4ac} (b^2 - ac) - 3abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) (b - \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} \right)}{a^3 \sqrt{b^2 - 4ac} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} + \frac{\sqrt{2} c \left( -\sqrt{b^2 - 4ac} (b^2 - ac) - 3abc + b^3 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right) (b + \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}} \right)}{a^3 \sqrt{b^2 - 4ac} \sqrt{-b \sqrt{b^2 - 4ac} - 2c(a+c) + b^2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^3/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out]  $-((\sqrt{2} \cdot c \cdot (b^3 - 3 \cdot a \cdot b \cdot c + \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot (b^2 - a \cdot c)) \cdot \operatorname{ArcTan}[(2 \cdot c + (b - \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \tan[x/2]) / (\sqrt{2} \cdot \sqrt{b^2 - 2 \cdot c \cdot (a + c) - b \cdot \sqrt{b^2 - 4 \cdot a \cdot c}})]) / (a^3 \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b^2 - 2 \cdot c \cdot (a + c) - b \cdot \sqrt{b^2 - 4 \cdot a \cdot c}})) + (\sqrt{2} \cdot c \cdot (b^3 - 3 \cdot a \cdot b \cdot c - \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot (b^2 - a \cdot c)) \cdot \operatorname{ArcTan}[(2 \cdot c + (b + \sqrt{b^2 - 4 \cdot a \cdot c}) \cdot \tan[x/2]) / (\sqrt{2} \cdot \sqrt{b^2 - 2 \cdot c \cdot (a + c) + b \cdot \sqrt{b^2 - 4 \cdot a \cdot c}})]) / (a^3 \cdot \sqrt{b^2 - 4 \cdot a \cdot c} \cdot \sqrt{b^2 - 2 \cdot c \cdot (a + c) + b \cdot \sqrt{b^2 - 4 \cdot a \cdot c}}) - \operatorname{ArcTanh}[\cos[x]] / (2 \cdot a) - ((b^2 - a \cdot c) \cdot \operatorname{ArcTanh}[\cos[x]]) / a^3 + (b \cdot \cot[x]) / a^2 - (\cot[x] \cdot \csc[x]) / (2 \cdot a)$

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 3256

```
Int[sin[(d_) + (e_)*(x_)]^(m_)*((a_) + (b_)*sin[(d_) + (e_)*(x_)]^(n_) + (c_)*sin[(d_) + (e_)*(x_)]^(n2_))^(p_), x_Symbol] := Int[ExpandTrig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegersQ[m, n, p]
```

### Rule 3292

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])/((a_) + (b_)*sin[(d_) + (e_)*(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2), x_Symbol] := Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3767

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

### Rule 3768

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

### Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( \frac{(b^2 - ac) \csc(x)}{a^3} - \frac{b \csc^2(x)}{a^2} + \frac{\csc^3(x)}{a} + \frac{-b^3 \left(1 - \frac{2ac}{b^2}\right) - b^2 c \left(1 - \frac{ac}{b^2}\right) \sin(x)}{a^3 (a + b \sin(x) + c \sin^2(x))} \right) dx \\
&= \frac{\int \frac{-b^3 \left(1 - \frac{2ac}{b^2}\right) - b^2 c \left(1 - \frac{ac}{b^2}\right) \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{a^3} + \frac{\int \csc^3(x) dx}{a} - \frac{b \int \csc^2(x) dx}{a^2} + \frac{(b^2 - ac) \int \csc(x) dx}{a^3} \\
&= -\frac{(b^2 - ac) \tanh^{-1}(\cos(x))}{a^3} - \frac{\cot(x) \csc(x)}{2a} + \frac{\int \csc(x) dx}{2a} + \frac{b \operatorname{Subst}(\int 1 dx, x, \cos(x))}{a^2} \\
&= -\frac{\tanh^{-1}(\cos(x))}{2a} - \frac{(b^2 - ac) \tanh^{-1}(\cos(x))}{a^3} + \frac{b \cot(x)}{a^2} - \frac{\cot(x) \csc(x)}{2a} + \frac{(2c(b^2 - ac) \tan^{-1}(\cos(x)))}{a^2} \\
&= -\frac{\tanh^{-1}(\cos(x))}{2a} - \frac{(b^2 - ac) \tanh^{-1}(\cos(x))}{a^3} + \frac{b \cot(x)}{a^2} - \frac{\cot(x) \csc(x)}{2a} - \frac{(4c(b^2 - ac) \tan^{-1}(\cos(x)))}{a^2} \\
&= -\frac{\sqrt{2} c \left( b^3 - 3abc + \sqrt{b^2 - 4ac} (b^2 - ac) \right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{a^3 \sqrt{b^2 - 4ac} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} c}{a^2}
\end{aligned}$$

**Mathematica** [C] time = 1.62, size = 481, normalized size = 1.45

$$\csc^2(x)(-2a - 2b \sin(x) + c \cos(2x) - c) \left( -4(a^2 - 2ac + 2b^2) \log\left(\sin\left(\frac{x}{2}\right)\right) + 4(a^2 - 2ac + 2b^2) \log\left(\cos\left(\frac{x}{2}\right)\right) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^3/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out] (Csc[x]^2\*(-2\*a - c + c\*Cos[2\*x] - 2\*b\*Sin[x])\*((8\*c\*((-I)\*b^3 + (3\*I)\*a\*b\*c + b^2\*Sqrt[-b^2 + 4\*a\*c] - a\*c\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])]/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])))/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]]) + (8\*c\*(I\*b^3 - (3\*I)\*a\*b\*c + b^2\*Sqrt[-b^2 + 4\*a\*c] - a\*c\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])]/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]]))/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]]) - 4\*a\*b\*Cot[x/2] + a^2\*Csc[x/2]^2 + 4\*(a^2 + 2\*b^2 - 2\*a\*c)\*Log[Cos[x/2]] - 4\*(a^2 + 2\*b^2 - 2\*a\*c)\*Log[Sin[x/2]] - a^2\*Sec[x/2]^2 + 4\*a\*b\*Tan[x/2))/(16\*a^3\*(c + b\*Csc[x] + a\*Csc[x]^2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.39, size = 1369, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 
$$\frac{1}{8} \frac{1}{a} \tan\left(\frac{1}{2}x\right)^2 - \frac{1}{2} \frac{1}{a^2} \tan\left(\frac{1}{2}x\right) * b + \frac{4}{a} \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * (-4ac + b^2)^{1/2} * c^2 - 8/a^2 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * (-4ac + b^2)^{1/2} * b^2 * c + 2/a^3 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * (-4ac + b^2)^{1/2} * b^4 - 16/a \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b * c^2 + 12/a^2 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b^3 * c - 2/a^3 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{-2a \tan\left(\frac{1}{2}x\right) + (-4ac + b^2)^{1/2} - b}{(4ca - 2b^2 + 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b^5 + 4/a \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{2a \tan\left(\frac{1}{2}x\right) + b + (-4ac + b^2)^{1/2}}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * (-4ac + b^2)^{1/2} * c^2 - 8/a^2 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{2a \tan\left(\frac{1}{2}x\right) + b + (-4ac + b^2)^{1/2}}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b^4 + 16/a \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{2a \tan\left(\frac{1}{2}x\right) + b + (-4ac + b^2)^{1/2}}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b * c^2 - 12/a^2 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{2a \tan\left(\frac{1}{2}x\right) + b + (-4ac + b^2)^{1/2}}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b^3 * c + 2/a^3 \frac{1}{(4ac - b^2)} \frac{1}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}} \arctan\left(\frac{2a \tan\left(\frac{1}{2}x\right) + b + (-4ac + b^2)^{1/2}}{(4ca - 2b^2 - 2b(-4ac + b^2)^{1/2} + 4a^2)^{1/2}}\right) * b^5 - 1/8 \frac{1}{a} \tan\left(\frac{1}{2}x\right)^2 + 1/2 \frac{1}{a} \ln\left(\tan\left(\frac{1}{2}x\right)\right) - 1/a^2 \ln\left(\tan\left(\frac{1}{2}x\right)\right) * c + 1/a^3 \ln\left(\tan\left(\frac{1}{2}x\right)\right) * b^2 + 1/2 * b/a^2 / \tan\left(\frac{1}{2}x\right)$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 24.32, size = 21909, normalized size = 66.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(\sin(x)^3*(a + c*\sin(x)^2 + b*\sin(x))),x)$

[Out]  $\text{atan}\left(\frac{\left(\left(8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3\right)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3\right)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3\right)^{1/2} + 3a^2b^4c^4(-4ac - b^2)^3\left(\frac{1}{2}\right) + 4a^3b^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) - 10a^2b^3c^2(-4ac - b^2)^3\left(\frac{1}{2}\right) + 6ab^5c(-4ac - b^2)^3\left(\frac{1}{2}\right)}{2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)\right)^{1/2}}\left(\left(8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3\right)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3\right)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) + 3a^2b^4c^4(-4ac - b^2)^3\left(\frac{1}{2}\right) + 4a^3b^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) - 10a^2b^3c^2(-4ac - b^2)^3\left(\frac{1}{2}\right) + 6ab^5c(-4ac - b^2)^3\left(\frac{1}{2}\right)}{2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)\right)^{1/2}}\left(\left(8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3\right)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3\right)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) + 3a^2b^4c^4(-4ac - b^2)^3\left(\frac{1}{2}\right) + 4a^3b^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) - 10a^2b^3c^2(-4ac - b^2)^3\left(\frac{1}{2}\right) + 6ab^5c(-4ac - b^2)^3\left(\frac{1}{2}\right)}{2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)\right)^{1/2}}\left(\left(16(4a^7b^5 - 16a^5b^7 + 3a^9b^3 + 122a^6b^5c + 96a^7b^3c^4 + 160a^8b^3c^3 - 17a^8b^3c + 4a^9b^3c^2 + 16a^5b^5c^2 - 88a^6b^3c^3 - 272a^7b^3c^2 - 12a^{10}b^3c)\right)/a^6 + \left(\left(16(8a^8b^5 - 6a^{10}b^3 + 32a^9b^3c^3 - 50a^9b^3c + 72a^{10}b^3c^2 - 8a^8b^3c^2 + 24a^{11}b^3c)\right)/a^6 - (16\tan(x/2)*(16a^{12}c - 32a^7b^6 + 34a^9b^4 - 4a^{11}b^2 + 384a^9c^4 + 768a^{10}c^3 + 400a^{11}c^2 + 288a^8b^4c - 236a^{10}b^2c + 32a^7b^4c^2 - 224a^8b^2c^3 - 832a^9b^2c^2)\right)/a^6)\right)^{1/2}}\left(\left(8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3\right)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3\right)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) + 3a^2b^4c^4(-4ac - b^2)^3\left(\frac{1}{2}\right) + 4a^3b^3c^3(-4ac - b^2)^3\left(\frac{1}{2}\right) - 10a^2b^3c^2(-4ac - b^2)^3\left(\frac{1}{2}\right) + 6ab^5c(-4ac - b^2)^3\left(\frac{1}{2}\right)}{2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)\right)^{1/2}} + (16\tan(x/2)*(8a^{11}c - 32a^4b^8 + 18a^6b^6 + 5a^8b^4 - 2a^{10}b^2 - 192a^7c^5 - 288a^8c^4 - 48a^9c^3 + 56a^{10}c^2 + 288a^5b^6c - 118a^7b^4c - 34a^9b^2c + 32a^4b^6c^2 - 224a^5b^4c^3 + 432a^6b^2c^4 - 864a^6b^4c^2 + 968a^7b^2c^3 + 196a^8b^2c^2))/a^6) + (16(8a^2b^9 + 2a^4b^7 - a^6b^5 - 78a^3b^7c + 104a^5b^5c^5 - 18a^5b^5c + 114a^6b^3c^4 - 36a^7b^3c^3 + 6a^7b^3c - 8a^8b^3c^2 - 8a^2b^7c^2 + 64a^3b^5c^3 - 152a^4b^3c^4 + 256a^4b^5c^2 - 318a^5b^3c^3 + 49a^6b^3c^2))/a^6 + (16\tan(x/2)*(2a^3b^8 - 4a^5b^6 + 96a^5c^6 + 96a^6c^5 + 20a^7c^4 + 16a^8c^3 + 32a^2b^8c - 24a^4b^6c + 28a^6b^4c - 32a^2b^6c^3 + 224a^3b^4c^4 - 288a^3b^6c^2 - 400a^4b^2c^5 + 824a^4b^4c^3 - 768a^5b^2c^4 + 92a^5b^4c^2 - 116a^6b^2c^3 - 52a^7b^2c^2))/a^6) + (16(6b^9c - 8b^7c^3 + 48ab^5c^4 - 48ab^7c^2 + 3a^2b^7c + 48a^3b^5c^6 + 26a^4b^3c^5 - 21a^5b^3c^4 - 80a^2b^3c^5 + 122a^2b^5c^3 - 108a^3b^3c^4 - 21a^3b^5c^2 + 42a^4b^3c^3))/a^6 - (16\tan(x/2)*(2b^{10} + a^2b^8 - 48a^3c^7 - 24a^4c^6 + 12a^5c^5 + 2$



$$\begin{aligned}
& *a^6*c^4 + 16*b^6*c^4 - 16*b^8*c^2 - 80*a*b^4*c^5 + 112*a*b^6*c^3 - 8*a^3*b^6*c \\
& + 96*a^2*b^2*c^6 - 232*a^2*b^4*c^4 + 48*a^2*b^6*c^2 + 152*a^3*b^2*c^5 \\
& - 24*a^3*b^4*c^3 - 36*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 - 18*a^5*b^8*c)) / a^6 * i - ((8*a^4*c^6 - b^10 + 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + b^8*c^2 - 10*a*b^6*c^3 + 33*a^2*b^4*c^4 - 52*a^2*b^6*c^2 - 38*a^3*b^2*c^5 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^10*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} * (((8*a^4*c^6 - b^10 + 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + b^8*c^2 - 10*a*b^6*c^3 + 33*a^2*b^4*c^4 - 52*a^2*b^6*c^2 - 38*a^3*b^2*c^5 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^10*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} * ((16*(8*a^2*b^9 + 2*a^4*b^7 - a^6*b^5 - 78*a^3*b^7*c + 104*a^5*b*c^5 - 18*a^5*b^5*c + 114*a^6*b*c^4 - 36*a^7*b*c^3 + 6*a^7*b^3*c - 8*a^8*b*c^2 - 8*a^2*b^7*c^2 + 64*a^3*b^5*c^3 - 152*a^4*b^3*c^4 + 256*a^4*b^5*c^2 - 318*a^5*b^3*c^3 + 49*a^6*b^3*c^2)) / a^6 - ((8*a^4*c^6 - b^10 + 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + b^8*c^2 - 10*a*b^6*c^3 + 33*a^2*b^4*c^4 - 52*a^2*b^6*c^2 - 38*a^3*b^2*c^5 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^10*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} * ((16*(4*a^7*b^5 - 16*a^5*b^7 + 3*a^9*b^3 + 122*a^6*b^5*c + 96*a^7*b*c^4 + 160*a^8*b*c^3 - 17*a^8*b^3*c + 4*a^9*b*c^2 + 16*a^5*b^5*c^2 - 88*a^6*b^3*c^3 - 272*a^7*b^3*c^2 - 12*a^10*b*c)) / a^6 - ((16*(8*a^8*b^5 - 6*a^10*b^3 + 32*a^9*b*c^3 - 50*a^9*b^3*c + 72*a^10*b*c^2 - 8*a^8*b^3*c^2 + 24*a^11*b*c)) / a^6 - (16*tan(x/2)*(16*a^12*c - 32*a^7*b^6 + 34*a^9*b^4 - 4*a^11*b^2 + 384*a^9*c^4 + 768*a^10*c^3 + 400*a^11*c^2 + 288*a^8*b^4*c - 236*a^10*b^2*c + 32*a^7*b^4*c^2 - 224*a^8*b^2*c^3 - 832*a^9*b^2*c^2)) / a^6) * ((8*a^4*c^6 - b^10 + 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} + b^8*c^2 - 10*a*b^6*c^3 + 33*a^2*b^4*c^4 - 52*a^2*b^6*c^2 - 38*a^3*b^2*c^5 + 96*a^3*b^4*c^3 - 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^10*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (16*tan(x/2)*(8*a^11*c - 32*a^4*b^8 + 18*a^6*b^6 + 5*a^8*b^4 - 2*a^10*b^2 - 192*a^7*c^5 - 288*a^8*c^4 - 48*a^9*c^3 + 56*a^10*c^2 + 288*a^5*b^6*c - 118*a^7*b^4*c - 34*a^9*b^2*c + 32*a^4*b^6*c^2 - 224*a^5*b^4*c^3 + 432*a^6*b^2*c^4 - 864*a^6*b^4*c^2 + 968*a^7*b^2*c^3 + 196*a^8*b^2*c^2)) / a^6) + (16*tan(x/2)*(2*a^3*b^8 - 4*a^5*b^6 + 96*a^5*c^6 + 96*a^6*c^5 + 20*a^7*c^4 + 16*a^8*c^3 + 32*a^2*b^8*c - 24*a^4*b^6*c + 28*a^6*b^4*c - 32*a^2*b^6*c^3 + 224*a^3*b^4*c^4 - 288*a^3*b^6*c^2 - 400*a^4*b^2*c^5 + 824*a^4*b^4*c^3 - 768*a^5*b^2*c^4 + 92*a^5*b^4*c^2 - 116*a^6*b^2*c^3 - 52*a^7*b^2*c^2)) / a^6) - (16*(6*b^9*c - 8*b^7*c^3 + 48*a*b^5*c^4 - 48*a*b^7*c^2 + 3*a^2*b^7*c + 48*a^3*b*c^6 + 26*a^4*b*c^5 - 21*a^5*b*c^4 - 80*a^2*b^3*c^5 + 122*a^2*b^5*c^3 - 108*a^3*b^3*c^4 - 21*a^3*b^5*c^2 + 42*a^4*b^3*c^3)) / a^6 + (16*tan(x/2)*(2*b^10 + a^2*b^8 - 48*a^3*c^7 - 24*a^4*c^6 + 12*a^5*c^5 + 2*a^6*c^4 + 16*b^6*c^4 - 16*b^8*c^2 - 80*a*b^4*c^5 + 112*a*b^6*c^3 - 8*a^3*b^6*c + 96*a^2*b^2*c^6 - 232*a^2*b^4*c^4 + 48*a^2*b^6*c^2 + 152*a^3*b^2*c^5 - 24*a^3*b^4*c^3 - 36*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 - 18*a*b^8*c)) / a^6) * i) / (((8*a^4*c^6 - b^10 + 8*a^5*c^5 - b^7*(-(4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} + b^8 c^2 - 10 a b^6 c^3 + 33 a^2 b^4 c^4 - 52 a^2 b^6 c^2 \\
& - 38 a^3 b^2 c^5 + 96 a^3 b^4 c^3 - 66 a^4 b^2 c^4 + b^5 c^2 (- (4 a c - b^2) \\
& )^3)^{(1/2)} + 12 a b^8 c - 4 a b^3 c^3 (- (4 a c - b^2) ^3)^{(1/2)} + 3 a^2 b c^4 \\
& (- (4 a c - b^2) ^3)^{(1/2)} + 4 a^3 b c^3 (- (4 a c - b^2) ^3)^{(1/2)} - 10 a^2 b^3 c^2 \\
& (- (4 a c - b^2) ^3)^{(1/2)} + 6 a b^5 c (- (4 a c - b^2) ^3)^{(1/2)} / (2 (a^8 b^4 - a^6 b^6 \\
& + 16 a^8 c^4 + 32 a^9 c^3 + 16 a^{10} c^2 + 10 a^7 b^4 c - 8 a^9 b^2 c + a^6 b^4 c^2 - 8 a^7 b^2 c^3 \\
& - 32 a^8 b^2 c^2))^{(1/2)} * (((8 a^4 c^6 - b^{10} + 8 a^5 c^5 - b^7 (- (4 a c - b^2) ^3)^{(1/2)} \\
& + b^8 c^2 - 10 a b^6 c^3 + 33 a^2 b^4 c^4 - 52 a^2 b^6 c^2 - 38 a^3 b^2 c^5 + 96 a^3 b^4 c^3 - \\
& 66 a^4 b^2 c^4 + b^5 c^2 (- (4 a c - b^2) ^3)^{(1/2)} + 12 a b^8 c - 4 a b^3 c^3 (- (4 a c - b^2) ^3)^{(1/2)} \\
& + 3 a^2 b c^4 (- (4 a c - b^2) ^3)^{(1/2)} + 4 a^3 b c^3 (- (4 a c - b^2) ^3)^{(1/2)} - 10 a^2 b^3 c^2 \\
& (- (4 a c - b^2) ^3)^{(1/2)} + 6 a b^5 c (- (4 a c - b^2) ^3)^{(1/2)} / (2 (a^8 b^4 - a^6 b^6 + 16 a^8 c^4 + 32 \\
& a^9 c^3 + 16 a^{10} c^2 + 10 a^7 b^4 c - 8 a^9 b^2 c + a^6 b^4 c^2 - 8 a^7 b^2 c^3 - 32 a^8 b^2 c^2))^{(1/2)} * (((8 a^4 c^6 - b^{10} + 8 a^5 c^5 - b^7 (- (4 a c - b^2) ^3)^{(1/2)} \\
& + b^8 c^2 - 10 a b^6 c^3 + 33 a^2 b^4 c^4 - 52 a^2 b^6 c^2 - 38 a^3 b^2 c^5 + 96 a^3 b^4 c^3 - 66 a^4 b^2 c^4 + b^5 c^2 (- (4 a c - b^2) ^3)^{(1/2)} \\
& + 12 a b^8 c - 4 a b^3 c^3 (- (4 a c - b^2) ^3)^{(1/2)} + 3 a^2 b c^4 (- (4 a c - b^2) ^3)^{(1/2)} + 4 a^3 b c^3 (- (4 a c - b^2) ^3)^{(1/2)} - 10 a^2 b^3 c^2 \\
& (- (4 a c - b^2) ^3)^{(1/2)} + 6 a b^5 c (- (4 a c - b^2) ^3)^{(1/2)} / (2 (a^8 b^4 - a^6 b^6 + 16 a^8 c^4 + 32 a^9 c^3 + 16 a^{10} c^2 + 10 a^7 b^4 c - 8 a^9 b^2 c + a^6 b^4 c^2 - 8 a^7 b^2 c^3 - 32 a^8 b^2 c^2))^{(1/2)} * ((16 (4 a^7 b^5 - 16 a^5 b^7 + 3 a^9 b^3 + 122 a^6 b^5 c + 96 a^7 b^3 c^4 + 160 a^8 b^3 c^3 - 17 a^8 b^3 c + 4 a^9 b^3 c^2 + 16 a^5 b^5 c^2 - 88 a^6 b^3 c^3 - 272 a^7 b^3 c^2 - 12 a^{10} b^3 c)) / a^6 + ((16 (8 a^8 b^5 - 6 a^{10} b^3 + 32 a^9 b^3 c^3 - 50 a^9 b^3 c + 72 a^{10} b^3 c^2 - 8 a^8 b^3 c^2 + 24 a^{11} b^3 c)) / a^6 - (16 \tan(x/2) (16 a^{12} c - 32 a^7 b^6 + 34 a^9 b^4 - 4 a^{11} b^2 + 384 a^9 c^4 + 768 a^{10} c^3 + 400 a^{11} c^2 + 288 a^8 b^4 c - 236 a^{10} b^2 c + 32 a^7 b^4 c^2 - 224 a^8 b^2 c^3 - 832 a^9 b^2 c^2)) / a^6) * ((8 a^4 c^6 - b^{10} + 8 a^5 c^5 - b^7 (- (4 a c - b^2) ^3)^{(1/2)} + b^8 c^2 - 10 a b^6 c^3 + 33 a^2 b^4 c^4 - 52 a^2 b^6 c^2 - 38 a^3 b^2 c^5 + 96 a^3 b^4 c^3 - 66 a^4 b^2 c^4 + b^5 c^2 (- (4 a c - b^2) ^3)^{(1/2)} + 12 a b^8 c - 4 a b^3 c^3 (- (4 a c - b^2) ^3)^{(1/2)} + 3 a^2 b c^4 (- (4 a c - b^2) ^3)^{(1/2)} + 4 a^3 b c^3 (- (4 a c - b^2) ^3)^{(1/2)} - 10 a^2 b^3 c^2 (- (4 a c - b^2) ^3)^{(1/2)} + 6 a b^5 c (- (4 a c - b^2) ^3)^{(1/2)} / (2 (a^8 b^4 - a^6 b^6 + 16 a^8 c^4 + 32 a^9 c^3 + 16 a^{10} c^2 + 10 a^7 b^4 c - 8 a^9 b^2 c + a^6 b^4 c^2 - 8 a^7 b^2 c^3 - 32 a^8 b^2 c^2))^{(1/2)} + (16 \tan(x/2) (8 a^{11} c - 32 a^4 b^8 + 18 a^6 b^6 + 5 a^8 b^4 - 2 a^{10} b^2 - 192 a^7 c^5 - 288 a^8 c^4 - 48 a^9 c^3 + 56 a^{10} c^2 + 288 a^5 b^6 c - 118 a^7 b^4 c - 34 a^9 b^2 c + 32 a^4 b^6 c^2 - 224 a^5 b^4 c^3 + 432 a^6 b^2 c^4 - 864 a^6 b^4 c^2 + 968 a^7 b^2 c^3 + 196 a^8 b^2 c^2)) / a^6) + (16 (8 a^2 b^9 + 2 a^4 b^7 - a^6 b^5 - 78 a^3 b^7 c + 104 a^5 b^3 c^5 - 18 a^5 b^5 c + 114 a^6 b^3 c^4 - 36 a^7 b^3 c^3 + 6 a^7 b^3 c - 8 a^8 b^3 c^2 - 8 a^2 b^7 c^2 + 64 a^3 b^5 c^3 - 152 a^4 b^3 c^4 + 256 a^4 b^5 c^2 - 318 a^5 b^3 c^3 + 49 a^6 b^3 c^2)) / a^6 + (16 \tan(x/2) (2 a^3 b^8 - 4 a^5 b^6 + 96 a^5 c^6 + 96 a^6 c^5 + 20 a^7 c^4 + 16 a^8 c^3 + 32 a^2 b^8 c - 24 a^4 b^6 c + 28 a^6 b^4 c - 32 a^2 b^6 c^3 + 224 a^3 b^4 c^4 - 288 a^3 b^6 c^2 - 400 a^4 b^2 c^5 + 824 a^4 b^4 c^3 - 768 a^5 b^2 c^4 + 92 a^5 b^4 c^2 - 116 a^6 b^2 c^3 - 52 a^7 b^2 c^2)) / a^6) + (16 (6 b^9 c - 8 b^7 c^3 + 48 a b^5 c^4 - 48 a b^7 c^2 + 3 a^2 b^7 c + 48 a^3 b^3 c^6 + 26 a^4 b^3 c^5 - 21 a^5 b^3 c^4 - 80 a^2 b^3 c^5 + 122 a^2 b^5 c^3 - 108 a^3 b^3 c^4 - 21 a^3 b^5 c^2 + 42 a^4 b^3 c^3)) / a^6 - (16 \tan(x/2) (2 b^{10} + a^2 b^8 - 48 a^3 c^7 - 24 a^4 c^6 + 12 a^5 c^5 + 2 a^6 c^4 + 16 b^6 c^4 - 16 b^8 c^2 - 80 a b^4 c^5 + 12 a b^6 c^3 - 8 a^3 b^6 c + 96 a^2 b^2 c^6 - 232 a^2 b^4 c^4 + 48 a^2 b^6 c^2 + 152 a^3 b^2 c^5 - 24 a^3 b^4 c^3 - 36 a^4 b^2 c^4 + 20 a^4 b^4 c^2 - 16 a^5 b^2 c^3 - 18 a b^8 c)) / a^6) + ((8 a^4 c^6 - b^{10} + 8 a^5 c^5 - b^7 (- (4 a c - b^2) ^3)^{(1/2)} + b^8 c^2 - 10 a b^6 c^3 + 33 a^2 b^4 c^4 - 52 a^2 b^6 c^2 - 38 a^3 b^2 c^5 + 96 a^3 b^4 c^3 - 66 a^4 b^2 c^4 + b^5 c^2 (- (4 a c - b^2) ^3)^{(1/2)} + 12 a b^8 c - 4 a b^3 c^3 (- (4 a c - b^2) ^3)^{(1/2)} + 3 a^2 b c^4 (- (4 a c - b^2) ^3)^{(1/2)} + 4 a^3 b c^3 (- (4 a c - b^2) ^3)^{(1/2)} -
\end{aligned}$$

$$\begin{aligned}
& 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2} \\
& ) / (2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{1/2} \\
& * (((8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} + 12ab^8c - 4 \\
& ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^4c(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} \\
& + 6ab^5c(-4ac - b^2)^3)^{1/2}) / (2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{1/2} \\
& * ((16(8a^2b^9 + 2a^4b^7 - a^6b^5 - 78a^3b^7c + 104a^5b^5c^5 - 18a^5b^5c + 114a^6b^3c^4 - 36a^7b^3c^3 + 6a^7b^3c - 8a^8b^3c^2 - 8a^2b^7c^2 + 64a^3b^5c^3 - 152a^4b^3c^4 + 256a^4b^5c^2 - 318a^5b^3c^3 + 49a^6b^3c^2)) / a^6 - ((8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^4c(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2}) / (2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{1/2} * ((16(4a^7b^5 - 16a^5b^7 + 3a^9b^3 + 122a^6b^5c + 96a^7b^3c^4 + 160a^8b^3c^3 - 17a^8b^3c + 4a^9b^3c^2 + 16a^5b^5c^2 - 88a^6b^3c^3 - 272a^7b^3c^2 - 12a^{10}b^3c)) / a^6 - ((16(8a^8b^5 - 6a^{10}b^3 + 32a^9b^3c^3 - 50a^9b^3c + 72a^{10}b^3c^2 - 8a^8b^3c^2 + 24a^{11}b^3c)) / a^6 - (16 \tan(x/2) * (16a^{12}c - 32a^7b^6 + 34a^9b^4 - 4a^{11}b^2 + 384a^9c^4 + 768a^{10}c^3 + 400a^{11}c^2 + 288a^8b^4c - 236a^{10}b^2c + 32a^7b^4c^2 - 224a^8b^2c^3 - 832a^9b^2c^2)) / a^6 * ((8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^4c(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2}) / (2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{1/2} + (16 \tan(x/2) * (8a^{11}c - 32a^4b^8 + 18a^6b^6 + 5a^8b^4 - 2a^{10}b^2 - 192a^7c^5 - 288a^8c^4 - 48a^9c^3 + 56a^{10}c^2 + 288a^5b^6c - 118a^7b^4c - 34a^9b^2c + 32a^4b^6c^2 - 224a^5b^4c^3 + 432a^6b^2c^4 - 864a^6b^4c^2 + 968a^7b^2c^3 + 196a^8b^2c^2)) / a^6) + (16 \tan(x/2) * (2a^3b^8 - 4a^5b^6 + 96a^5c^6 + 96a^6c^5 + 20a^7c^4 + 16a^8c^3 + 32a^2b^8c - 24a^4b^6c + 28a^6b^4c - 32a^2b^6c^3 + 224a^3b^4c^4 - 288a^3b^6c^2 - 400a^4b^2c^5 + 824a^4b^4c^3 - 768a^5b^2c^4 + 92a^5b^4c^2 - 116a^6b^2c^3 - 52a^7b^2c^2)) / a^6) - (16(6b^9c - 8b^7c^3 + 48ab^5c^4 - 48ab^7c^2 + 3a^2b^7c + 48a^3b^6c + 26a^4b^5c - 21a^5b^4c - 80a^2b^3c^5 + 122a^2b^5c^3 - 108a^3b^3c^4 - 21a^3b^5c^2 + 42a^4b^3c^3)) / a^6 + (16 \tan(x/2) * (2b^{10} + a^2b^8 - 48a^3c^7 - 24a^4c^6 + 12a^5c^5 + 2a^6c^4 + 16b^6c^4 - 16b^8c^2 - 80ab^4c^5 + 112ab^6c^3 - 8a^3b^6c + 96a^2b^2c^6 - 232a^2b^4c^4 + 48a^2b^6c^2 + 152a^3b^2c^5 - 24a^3b^4c^3 - 36a^4b^2c^4 + 20a^4b^4c^2 - 16a^5b^2c^3 - 18ab^8c)) / a^6) - (32(8b^3c^6 - 2b^5c^4 + 6ab^3c^5 + 2a^3b^3c^5 - a^2b^3c^4 - 8ab^3c^7)) / a^6 - (32 \tan(x/2) * (4a^3c^6 + 16b^2c^7 - 8b^4c^5 + 16ab^2c^6 - 4a^2b^2c^5)) / a^6) * ((8a^4c^6 - b^{10} + 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} + b^8c^2 - 10ab^6c^3 + 33a^2b^4c^4 - 52a^2b^6c^2 - 38a^3b^2c^5 + 96a^3b^4c^3 - 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} + 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^4c(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2}) / (2(a^8b^4 - a^6b^6 + 16a^8c^4 +
\end{aligned}$$

$$\begin{aligned}
& 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{(1/2)} * 2i - \operatorname{atan}\left(\frac{(((((16(4a^7b^5 - 16a^5b^7 + 3a^9b^3 + 122a^6b^5c + 96a^7b^3c^4 + 160a^8b^3c^3 - 17a^8b^3c + 4a^9b^3c^2 + 16a^5b^5c^2 - 88a^6b^3c^3 - 272a^7b^3c^2 - 12a^{10}b^3c)))/a^6 + ((16(8a^8b^5 - 6a^{10}b^3 + 32a^9b^3c^3 - 50a^9b^3c + 72a^{10}b^3c^2 - 8a^8b^3c^2 + 24a^{11}b^3c)))/a^6 - (16\tan(x/2)*(16a^{12}c - 32a^7b^6 + 34a^9b^4 - 4a^{11}b^2 + 384a^9c^4 + 768a^{10}c^3 + 400a^{11}c^2 + 288a^8b^4c - 236a^{10}b^2c + 32a^7b^4c^2 - 224a^8b^2c^3 - 832a^9b^2c^2))/a^6)*(-(b^{10} - 8a^4c^6 - 8a^5c^5 - b^7*(-(4ac - b^2)^3))^{(1/2)} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2*(-(4ac - b^2)^3))^{(1/2)} - 12ab^8c - 4ab^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^4*(-(4ac - b^2)^3)^{(1/2)} + 4a^3b^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1/2)} + 6ab^5c*(-(4ac - b^2)^3)^{(1/2)})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{(1/2)} + (16\tan(x/2)*(8a^{11}c - 32a^4b^8 + 18a^6b^6 + 5a^8b^4 - 2a^{10}b^2 - 192a^7c^5 - 288a^8c^4 - 48a^9c^3 + 56a^{10}c^2 + 288a^5b^6c - 118a^7b^4c - 34a^9b^2c + 32a^4b^6c^2 - 224a^5b^4c^3 + 432a^6b^2c^4 - 864a^6b^4c^2 + 968a^7b^2c^3 + 196a^8b^2c^2))/a^6)*(-(b^{10} - 8a^4c^6 - 8a^5c^5 - b^7*(-(4ac - b^2)^3))^{(1/2)} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2*(-(4ac - b^2)^3))^{(1/2)} - 12ab^8c - 4ab^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^4*(-(4ac - b^2)^3)^{(1/2)} + 4a^3b^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1/2)} + 6ab^5c*(-(4ac - b^2)^3)^{(1/2)})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{(1/2)} + (16(8a^2b^9 + 2a^4b^7 - a^6b^5 - 78a^3b^7c + 104a^5b^5c - 18a^5b^5c + 114a^6b^3c^4 - 36a^7b^3c^3 + 6a^7b^3c - 8a^8b^3c^2 - 8a^2b^7c^2 + 64a^3b^5c^3 - 152a^4b^3c^4 + 256a^4b^5c^2 - 318a^5b^3c^3 + 49a^6b^3c^2))/a^6 + (16\tan(x/2)*(2a^3b^8 - 4a^5b^6 + 96a^5c^6 + 96a^6c^5 + 20a^7c^4 + 16a^8c^3 + 32a^2b^8c - 24a^4b^6c + 28a^6b^4c - 32a^2b^6c^3 + 224a^3b^4c^4 - 288a^3b^6c^2 - 400a^4b^2c^5 + 824a^4b^4c^3 - 768a^5b^2c^4 + 92a^5b^4c^2 - 116a^6b^2c^3 - 52a^7b^2c^2))/a^6)*(-(b^{10} - 8a^4c^6 - 8a^5c^5 - b^7*(-(4ac - b^2)^3))^{(1/2)} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2*(-(4ac - b^2)^3))^{(1/2)} - 12ab^8c - 4ab^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^4*(-(4ac - b^2)^3)^{(1/2)} + 4a^3b^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1/2)} + 6ab^5c*(-(4ac - b^2)^3)^{(1/2)})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{(1/2)} + (16(6b^9c - 8b^7c^3 + 48ab^5c^4 - 48ab^7c^2 + 3a^2b^7c + 48a^3b^3c^6 + 26a^4b^3c^5 - 21a^5b^3c^4 - 80a^2b^3c^5 + 122a^2b^5c^3 - 108a^3b^3c^4 - 21a^3b^5c^2 + 42a^4b^3c^3))/a^6 - (16\tan(x/2)*(2b^{10} + a^2b^8 - 48a^3c^7 - 24a^4c^6 + 12a^5c^5 + 2a^6c^4 + 16b^6c^4 - 16b^8c^2 - 80ab^4c^5 + 112ab^6c^3 - 8a^3b^6c + 96a^2b^2c^6 - 232a^2b^4c^4 + 48a^2b^6c^2 + 152a^3b^2c^5 - 24a^3b^4c^3 - 36a^4b^2c^4 + 20a^4b^4c^2 - 16a^5b^2c^3 - 18ab^8c))/a^6)*(-(b^{10} - 8a^4c^6 - 8a^5c^5 - b^7*(-(4ac - b^2)^3))^{(1/2)} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2*(-(4ac - b^2)^3))^{(1/2)} - 12ab^8c - 4ab^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^4*(-(4ac - b^2)^3)^{(1/2)} + 4a^3b^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 10a^2b^3c^2*(-(4ac - b^2)^3)^{(1/2)} + 6ab^5c*(-(4ac - b^2)^3)^{(1/2)})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2))^{(1/2)} * i - (((16(8a^2b^9 + 2a^4b^7 - a^6b^5 - 78a^3b^7c + 104a^5b^5c - 18a^5b^5c + 114a^6b^3c^4 - 36a^7b^3c^3 + 6a^7b^3c - 8a^8b^3c^2 - 8a^2b^7c^2 +
\end{aligned}$$

$$\begin{aligned}
& 64a^3b^5c^3 - 152a^4b^3c^4 + 256a^4b^5c^2 - 318a^5b^3c^3 + 49a^6b^3c^2)/a^6 - ((16(4a^7b^5 - 16a^5b^7 + 3a^9b^3 + 122a^6b^5c + 96a^7b^3c^4 + 160a^8b^3c^3 - 17a^8b^3c + 4a^9b^3c^2 + 16a^5b^5c^2 - 88a^6b^3c^3 - 272a^7b^3c^2 - 12a^{10}b^3c))/a^6 - ((16(8a^8b^5 - 6a^{10}b^3 + 32a^9b^3c - 50a^9b^3c + 72a^{10}b^3c^2 - 8a^8b^3c^2 + 24a^{11}b^3c))/a^6 - (16\tan(x/2)(16a^{12}c - 32a^7b^6 + 34a^9b^4 - 4a^{11}b^2 + 384a^9c^4 + 768a^{10}c^3 + 400a^{11}c^2 + 288a^8b^4c - 236a^{10}b^2c + 32a^7b^4c^2 - 224a^8b^2c^3 - 832a^9b^2c^2))/a^6)*(- (b^{10} - 8a^4c^6 - 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} - 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^3c^4(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)))^{1/2} + (16\tan(x/2)(8a^{11}c - 32a^4b^8 + 18a^6b^6 + 5a^8b^4 - 2a^{10}b^2 - 192a^7c^5 - 288a^8c^4 - 48a^9c^3 + 56a^{10}c^2 + 288a^5b^6c - 118a^7b^4c - 34a^9b^2c + 32a^4b^6c^2 - 224a^5b^4c^3 + 432a^6b^2c^4 - 864a^6b^4c^2 + 968a^7b^2c^3 + 196a^8b^2c^2))/a^6)*(- (b^{10} - 8a^4c^6 - 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} - 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^3c^4(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)))^{1/2} + (16\tan(x/2)(2a^3b^8 - 4a^5b^6 + 96a^5c^6 + 96a^6c^5 + 20a^7c^4 + 16a^8c^3 + 32a^2b^8c - 24a^4b^6c + 28a^6b^4c - 32a^2b^6c^3 + 224a^3b^4c^4 - 288a^3b^6c^2 - 400a^4b^2c^5 + 824a^4b^4c^3 - 768a^5b^2c^4 + 92a^5b^4c^2 - 116a^6b^2c^3 - 52a^7b^2c^2))/a^6)*(- (b^{10} - 8a^4c^6 - 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} - 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^3c^4(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)))^{1/2} - (16(6b^9c - 8b^7c^3 + 48ab^5c^4 - 48ab^7c^2 + 3a^2b^7c + 48a^3b^3c^6 + 26a^4b^3c^5 - 21a^5b^3c^4 - 80a^2b^3c^5 + 122a^2b^5c^3 - 108a^3b^3c^4 - 21a^3b^5c^2 + 42a^4b^3c^3))/a^6 + (16\tan(x/2)(2b^{10} + a^2b^8 - 48a^3c^7 - 24a^4c^6 + 12a^5c^5 + 2a^6c^4 + 16b^6c^4 - 16b^8c^2 - 80ab^4c^5 + 112ab^6c^3 - 8a^3b^6c + 96a^2b^2c^6 - 232a^2b^4c^4 + 48a^2b^6c^2 + 152a^3b^2c^5 - 24a^3b^4c^3 - 36a^4b^2c^4 + 20a^4b^4c^2 - 16a^5b^2c^3 - 18ab^8c))/a^6)*(- (b^{10} - 8a^4c^6 - 8a^5c^5 - b^7(-4ac - b^2)^3)^{1/2} - b^8c^2 + 10ab^6c^3 - 33a^2b^4c^4 + 52a^2b^6c^2 + 38a^3b^2c^5 - 96a^3b^4c^3 + 66a^4b^2c^4 + b^5c^2(-4ac - b^2)^3)^{1/2} - 12ab^8c - 4ab^3c^3(-4ac - b^2)^3)^{1/2} + 3a^2b^3c^4(-4ac - b^2)^3)^{1/2} + 4a^3b^3c^3(-4ac - b^2)^3)^{1/2} - 10a^2b^3c^2(-4ac - b^2)^3)^{1/2} + 6ab^5c(-4ac - b^2)^3)^{1/2})/(2(a^8b^4 - a^6b^6 + 16a^8c^4 + 32a^9c^3 + 16a^{10}c^2 + 10a^7b^4c - 8a^9b^2c + a^6b^4c^2 - 8a^7b^2c^3 - 32a^8b^2c^2)))^{1/2} * i) / ((32(8b^3c^6 - 2b^5c^4 + 6ab^3c^5 + 2a^3b^3c^5 - a^2b^3c^4 - 8ab^3c^7))/a^6 - (((16(4a^7b^5 - 16a^5b^7 + 3a^9b^3 + 122a^6b^5c + 96a^7b^3c^4 + 160a^8b^3c^3 - 17a^8b^3c + 4a^9b^3c^2 + 16a^5b^5c^2 - 88a^6b^3c^3 - 272a^7b^3c^2 - 12a^{10}b^3c))/a^6 + ((16(8a^8b^5 - 6a^{10}b^3 + 32a^9b^3c - 50a^9b^3c + 72a^{10}b^3c^2 - 8a^8b^3c^2 + 24a^{11}b^3c))/a^6 - (16\tan(x/2)(16a^{12}c - 32a^7b^6 + 34a^9b^4 -
\end{aligned}$$

$$\begin{aligned}
& 4*a^{11}*b^2 + 384*a^9*c^4 + 768*a^{10}*c^3 + 400*a^{11}*c^2 + 288*a^8*b^4*c - 23 \\
& 6*a^{10}*b^2*c + 32*a^7*b^4*c^2 - 224*a^8*b^2*c^3 - 832*a^9*b^2*c^2)/a^6)*(- \\
& (b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10 \\
& *a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4* \\
& c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a* \\
& b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4 \\
& *a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 \\
& + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8* \\
& a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (16*\tan(x/2)*(8*a^{11}*c - 32*a^4*b^8 \\
& + 18*a^6*b^6 + 5*a^8*b^4 - 2*a^{10}*b^2 - 192*a^7*c^5 - 288*a^8*c^4 - 48*a^9 \\
& *c^3 + 56*a^{10}*c^2 + 288*a^5*b^6*c - 118*a^7*b^4*c - 34*a^9*b^2*c + 32*a^4* \\
& b^6*c^2 - 224*a^5*b^4*c^3 + 432*a^6*b^2*c^4 - 864*a^6*b^4*c^2 + 968*a^7*b^2 \\
& *c^3 + 196*a^8*b^2*c^2))/a^6)*(-(b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 \\
& + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b* \\
& c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^ \\
& 2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2 \\
& *(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c \\
& - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (16 \\
& *(8*a^2*b^9 + 2*a^4*b^7 - a^6*b^5 - 78*a^3*b^7*c + 104*a^5*b*c^5 - 18*a^5*b \\
& ^5*c + 114*a^6*b*c^4 - 36*a^7*b*c^3 + 6*a^7*b^3*c - 8*a^8*b*c^2 - 8*a^2*b^7 \\
& *c^2 + 64*a^3*b^5*c^3 - 152*a^4*b^3*c^4 + 256*a^4*b^5*c^2 - 318*a^5*b^3*c^3 \\
& + 49*a^6*b^3*c^2))/a^6 + (16*\tan(x/2)*(2*a^3*b^8 - 4*a^5*b^6 + 96*a^5*c^6 \\
& + 96*a^6*c^5 + 20*a^7*c^4 + 16*a^8*c^3 + 32*a^2*b^8*c - 24*a^4*b^6*c + 28*a \\
& ^6*b^4*c - 32*a^2*b^6*c^3 + 224*a^3*b^4*c^4 - 288*a^3*b^6*c^2 - 400*a^4*b^2 \\
& *c^5 + 824*a^4*b^4*c^3 - 768*a^5*b^2*c^4 + 92*a^5*b^4*c^2 - 116*a^6*b^2*c^3 \\
& - 52*a^7*b^2*c^2))/a^6)*(-(b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 + 3 \\
& 8*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3 \\
& )^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*( \\
& -(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3 \\
& *c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(a^8 \\
& *b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c - 8*a \\
& ^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (16*(6*b \\
& ^9*c - 8*b^7*c^3 + 48*a*b^5*c^4 - 48*a*b^7*c^2 + 3*a^2*b^7*c + 48*a^3*b*c^6 \\
& + 26*a^4*b*c^5 - 21*a^5*b*c^4 - 80*a^2*b^3*c^5 + 122*a^2*b^5*c^3 - 108*a^3 \\
& *b^3*c^4 - 21*a^3*b^5*c^2 + 42*a^4*b^3*c^3))/a^6 - (16*\tan(x/2)*(2*b^{10} + a \\
& ^2*b^8 - 48*a^3*c^7 - 24*a^4*c^6 + 12*a^5*c^5 + 2*a^6*c^4 + 16*b^6*c^4 - 16 \\
& *b^8*c^2 - 80*a*b^4*c^5 + 112*a*b^6*c^3 - 8*a^3*b^6*c + 96*a^2*b^2*c^6 - 23 \\
& 2*a^2*b^4*c^4 + 48*a^2*b^6*c^2 + 152*a^3*b^2*c^5 - 24*a^3*b^4*c^3 - 36*a^4* \\
& b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 - 18*a*b^8*c))/a^6)*(-(b^{10} - 8*a \\
& ^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 \\
& - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^ \\
& 4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-( \\
& 4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^ \\
& 5*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c \\
& ^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 \\
& - 32*a^8*b^2*c^2)))^{(1/2)} - (((16*(8*a^2*b^9 + 2*a^4*b^7 - a^6*b^5 - 78*a^ \\
& 3*b^7*c + 104*a^5*b*c^5 - 18*a^5*b^5*c + 114*a^6*b*c^4 - 36*a^7*b*c^3 + 6*a \\
& ^7*b^3*c - 8*a^8*b*c^2 - 8*a^2*b^7*c^2 + 64*a^3*b^5*c^3 - 152*a^4*b^3*c^4 + \\
& 256*a^4*b^5*c^2 - 318*a^5*b^3*c^3 + 49*a^6*b^3*c^2))/a^6 - ((16*(4*a^7*b^5 \\
& - 16*a^5*b^7 + 3*a^9*b^3 + 122*a^6*b^5*c + 96*a^7*b*c^4 + 160*a^8*b*c^3 - \\
& 17*a^8*b^3*c + 4*a^9*b*c^2 + 16*a^5*b^5*c^2 - 88*a^6*b^3*c^3 - 272*a^7*b^3* \\
& c^2 - 12*a^{10}*b*c))/a^6 - ((16*(8*a^8*b^5 - 6*a^{10}*b^3 + 32*a^9*b*c^3 - 50* \\
& a^9*b^3*c + 72*a^{10}*b*c^2 - 8*a^8*b^3*c^2 + 24*a^{11}*b*c))/a^6 - (16*\tan(x/2) \\
& )*(16*a^{12}*c - 32*a^7*b^6 + 34*a^9*b^4 - 4*a^{11}*b^2 + 384*a^9*c^4 + 768*a^{11}
\end{aligned}$$

$$\begin{aligned}
& 0*c^3 + 400*a^{11}*c^2 + 288*a^8*b^4*c - 236*a^{10}*b^2*c + 32*a^7*b^4*c^2 - 22 \\
& 4*a^8*b^2*c^3 - 832*a^9*b^2*c^2)/a^6)*(-(b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7 \\
& 7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a \\
& ^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4 \\
& 4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& ) - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1 \\
& /2)))/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a \\
& ^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1 \\
& /2)} + (16*\tan(x/2)*(8*a^{11}*c - 32*a^4*b^8 + 18*a^6*b^6 + 5*a^8*b^4 - 2*a^{10} \\
& *b^2 - 192*a^7*c^5 - 288*a^8*c^4 - 48*a^9*c^3 + 56*a^{10}*c^2 + 288*a^5*b^6*c \\
& - 118*a^7*b^4*c - 34*a^9*b^2*c + 32*a^4*b^6*c^2 - 224*a^5*b^4*c^3 + 432*a^6 \\
& *b^2*c^4 - 864*a^6*b^4*c^2 + 968*a^7*b^2*c^3 + 196*a^8*b^2*c^2))/a^6)*(-(b \\
& ^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a \\
& *b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 \\
& + 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3 \\
& *c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a \\
& ^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + \\
& 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7 \\
& *b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (16*\tan(x/2)*(2*a^3*b^8 - 4*a^5*b^6 + \\
& 96*a^5*c^6 + 96*a^6*c^5 + 20*a^7*c^4 + 16*a^8*c^3 + 32*a^2*b^8*c - 24*a^4* \\
& b^6*c + 28*a^6*b^4*c - 32*a^2*b^6*c^3 + 224*a^3*b^4*c^4 - 288*a^3*b^6*c^2 - \\
& 400*a^4*b^2*c^5 + 824*a^4*b^4*c^3 - 768*a^5*b^2*c^4 + 92*a^5*b^4*c^2 - 116 \\
& *a^6*b^2*c^3 - 52*a^7*b^2*c^2))/a^6)*(-(b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7* \\
& -(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2 \\
& *b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3 \\
& *a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1 \\
& /2)))/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7 \\
& *b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} \\
& ) - (16*(6*b^9*c - 8*b^7*c^3 + 48*a*b^5*c^4 - 48*a*b^7*c^2 + 3*a^2*b^7*c + \\
& 48*a^3*b*c^6 + 26*a^4*b*c^5 - 21*a^5*b*c^4 - 80*a^2*b^3*c^5 + 122*a^2*b^5*c \\
& ^3 - 108*a^3*b^3*c^4 - 21*a^3*b^5*c^2 + 42*a^4*b^3*c^3))/a^6 + (16*\tan(x/2) \\
& *(2*b^{10} + a^2*b^8 - 48*a^3*c^7 - 24*a^4*c^6 + 12*a^5*c^5 + 2*a^6*c^4 + 16* \\
& b^6*c^4 - 16*b^8*c^2 - 80*a*b^4*c^5 + 112*a*b^6*c^3 - 8*a^3*b^6*c + 96*a^2* \\
& b^2*c^6 - 232*a^2*b^4*c^4 + 48*a^2*b^6*c^2 + 152*a^3*b^2*c^5 - 24*a^3*b^4*c \\
& ^3 - 36*a^4*b^2*c^4 + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 - 18*a*b^8*c))/a^6)* \\
& -(b^{10} - 8*a^4*c^6 - 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 1 \\
& 0*a*b^6*c^3 - 33*a^2*b^4*c^4 + 52*a^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4 \\
& *c^3 + 66*a^4*b^2*c^4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a \\
& *b^3*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 4*a^3*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 6*a*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^ \\
& 4 + 32*a^9*c^3 + 16*a^{10}*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8 \\
& *a^7*b^2*c^3 - 32*a^8*b^2*c^2)))^{(1/2)} + (32*\tan(x/2)*(4*a^3*c^6 + 16*b^2*c \\
& ^7 - 8*b^4*c^5 + 16*a*b^2*c^6 - 4*a^2*b^2*c^5))/a^6)*(-(b^{10} - 8*a^4*c^6 - \\
& 8*a^5*c^5 - b^7*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c^2 + 10*a*b^6*c^3 - 33*a^2 \\
& *b^4*c^4 + 52*a^2*b^6*c^2 + 38*a^3*b^2*c^5 - 96*a^3*b^4*c^3 + 66*a^4*b^2*c^ \\
& 4 + b^5*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^8*c - 4*a*b^3*c^3*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 3*a^2*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a^3*b*c^3*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 10*a^2*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^5*c*(-(4 \\
& *a*c - b^2)^3)^{(1/2)))/(2*(a^8*b^4 - a^6*b^6 + 16*a^8*c^4 + 32*a^9*c^3 + 16* \\
& a^{10}*c^2 + 10*a^7*b^4*c - 8*a^9*b^2*c + a^6*b^4*c^2 - 8*a^7*b^2*c^3 - 32*a^ \\
& 8*b^2*c^2)))^{(1/2)}*2i + \tan(x/2)^2/(8*a) + (\log(\tan(x/2))*(a^2 - 2*a*c + 2* \\
& b^2))/(2*a^3) - (b*\tan(x/2))/(2*a^2) - (a/2 - 2*b*\tan(x/2))/(4*a^2*\tan(x/2) \\
& ^2)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x) + c*sin(x)**2), x)
```



$$3.9 \quad \int \frac{\cos^3(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=76

$$\frac{(b^2 - 2c(a + c)) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} + \frac{b \log(a + b \sin(x) + c \sin^2(x))}{2c^2} - \frac{\sin(x)}{c}$$

[Out] 1/2\*b\*ln(a+b\*sin(x)+c\*sin(x)^2)/c^2-sin(x)/c+(b^2-2\*c\*(a+c))\*arctanh((b+2\*c\*sin(x))/(-4\*a\*c+b^2)^(1/2))/c^2/(-4\*a\*c+b^2)^(1/2)

**Rubi [A]** time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {3258, 1657, 634, 618, 206, 628}

$$\frac{(b^2 - 2c(a + c)) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} + \frac{b \log(a + b \sin(x) + c \sin^2(x))}{2c^2} - \frac{\sin(x)}{c}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] ((b^2 - 2\*c\*(a + c))\*ArcTanh[(b + 2\*c\*Sin[x])/Sqrt[b^2 - 4\*a\*c]])/(c^2\*Sqrt[b^2 - 4\*a\*c]) + (b\*Log[a + b\*Sin[x] + c\*Sin[x]^2])/(2\*c^2) - Sin[x]/c

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 3258

Int[cos[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n\_.) + (c\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n2\_.))^p, x\_Symbol]

```
] := Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^(m - 1)/2*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \text{Subst} \left( \int \frac{1 - x^2}{a + bx + cx^2} dx, x, \sin(x) \right) \\
&= \text{Subst} \left( \int \left( \frac{1}{c} + \frac{a + c + bx}{c(a + bx + cx^2)} \right) dx, x, \sin(x) \right) \\
&= -\frac{\sin(x)}{c} + \frac{\text{Subst} \left( \int \frac{a + c + bx}{a + bx + cx^2} dx, x, \sin(x) \right)}{c} \\
&= -\frac{\sin(x)}{c} + \frac{b \text{Subst} \left( \int \frac{b + 2cx}{a + bx + cx^2} dx, x, \sin(x) \right)}{2c^2} - \frac{(b^2 - 2c(a + c)) \text{Subst} \left( \int \frac{1}{a + bx + cx^2} dx, x, \sin(x) \right)}{2c^2} \\
&= \frac{b \log(a + b \sin(x) + c \sin^2(x))}{2c^2} - \frac{\sin(x)}{c} + \frac{(b^2 - 2c(a + c)) \text{Subst} \left( \int \frac{1}{b^2 - 4ac - x^2} dx, x, \sin(x) \right)}{c^2} \\
&= \frac{(b^2 - 2c(a + c)) \tanh^{-1} \left( \frac{b + 2c \sin(x)}{\sqrt{b^2 - 4ac}} \right)}{c^2 \sqrt{b^2 - 4ac}} + \frac{b \log(a + b \sin(x) + c \sin^2(x))}{2c^2} - \frac{\sin(x)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 73, normalized size = 0.96

$$\frac{2(b^2 - 2c(a + c)) \tanh^{-1} \left( \frac{b + 2c \sin(x)}{\sqrt{b^2 - 4ac}} \right) + b \log(a + b \sin(x) + c \sin^2(x)) - 2c \sin(x)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*SIN[x] + c*SIN[x]^2), x]
```

```
[Out] ((2*(b^2 - 2*c*(a + c))*ArcTanh[(b + 2*c*SIN[x])/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + b*Log[a + b*SIN[x] + c*SIN[x]^2] - 2*c*SIN[x])/(2*c^2)
```

**fricas [A]** time = 1.04, size = 276, normalized size = 3.63

$$\left[ \frac{(b^2 - 2ac - 2c^2) \sqrt{b^2 - 4ac} \log \left( -\frac{2c^2 \cos(x)^2 - 2bc \sin(x) - b^2 + 2ac - 2c^2 + \sqrt{b^2 - 4ac} (2c \sin(x) + b)}{c \cos(x)^2 - b \sin(x) - a - c} \right) - (b^3 - 4abc) \log(-c \cos(x)^2 + b \sin(x) + a + c)}{2(b^2 c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)+c*sin(x)^2), x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c - 2*c^2)*sqrt(b^2 - 4*a*c)*log(-(2*c^2*cos(x)^2 - 2*b*c*sin(x) - b^2 + 2*a*c - 2*c^2 + sqrt(b^2 - 4*a*c)*(2*c*sin(x) + b))/(c*cos(x)^2 - b*sin(x) - a - c)) - (b^3 - 4*a*b*c)*log(-c*cos(x)^2 + b*sin(x) + a + c) + 2*(b^2*c - 4*a*c^2)*sin(x))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2 - 2*a*c - 2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*sin(x) + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(-c*cos(x)^2 + b*sin(x) + a + c) - 2*(b^2*c - 4*a*c^2)*sin(x))/(b^2*c^2 - 4*a*c^3)]
```

**giac** [A] time = 0.29, size = 78, normalized size = 1.03

$$\frac{b \log(c \sin(x)^2 + b \sin(x) + a)}{2c^2} - \frac{\sin(x)}{c} - \frac{(b^2 - 2ac - 2c^2) \arctan\left(\frac{2c \sin(x) + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] 1/2\*b\*log(c\*sin(x)^2 + b\*sin(x) + a)/c^2 - sin(x)/c - (b^2 - 2\*a\*c - 2\*c^2)\*arctan((2\*c\*sin(x) + b)/sqrt(-b^2 + 4\*a\*c))/(sqrt(-b^2 + 4\*a\*c)\*c^2)

**maple** [B] time = 0.29, size = 143, normalized size = 1.88

$$-\frac{\sin(x)}{c} + \frac{b \ln(a + b \sin(x) + c(\sin^2(x)))}{2c^2} + \frac{2 \arctan\left(\frac{b+2c \sin(x)}{\sqrt{4ca-b^2}}\right) a}{c \sqrt{4ca-b^2}} + \frac{2 \arctan\left(\frac{b+2c \sin(x)}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}} - \frac{\arctan\left(\frac{b+2c \sin(x)}{\sqrt{4ca-b^2}}\right) b^2}{c^2 \sqrt{4ca-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] -sin(x)/c+1/2\*b\*ln(a+b\*sin(x)+c\*sin(x)^2)/c^2+2/c/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))\*a+2/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))-1/c^2/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))\*b^2

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 0.21, size = 229, normalized size = 3.01

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2c \sin(x)}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{\sin(x)}{c} - \frac{b^3 \ln(c \sin(x)^2 + b \sin(x) + a)}{2(4ac^3 - b^2c^2)} - \frac{b^2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2c \sin(x)}{\sqrt{4ac-b^2}}\right)}{c^2 \sqrt{4ac-b^2}} + \frac{2a \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2c \sin(x)}{\sqrt{4ac-b^2}}\right)}{c^2 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(a + c\*sin(x)^2 + b\*sin(x)),x)

[Out] (2\*atan(b/(4\*a\*c - b^2)^(1/2) + (2\*c\*sin(x))/(4\*a\*c - b^2)^(1/2)))/(4\*a\*c - b^2)^(1/2) - sin(x)/c - (b^3\*log(a + c\*sin(x)^2 + b\*sin(x)))/(2\*(4\*a\*c^3 - b^2\*c^2)) - (b^2\*atan(b/(4\*a\*c - b^2)^(1/2) + (2\*c\*sin(x))/(4\*a\*c - b^2)^(1/2)))/(c^2\*(4\*a\*c - b^2)^(1/2)) + (2\*a\*atan(b/(4\*a\*c - b^2)^(1/2) + (2\*c\*sin(x))/(4\*a\*c - b^2)^(1/2)))/(c\*(4\*a\*c - b^2)^(1/2)) + (2\*a\*b\*c\*log(a + c\*sin(x)^2 + b\*sin(x)))/(4\*a\*c^3 - b^2\*c^2)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)\*\*3/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

$$3.10 \quad \int \frac{\cos^2(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=230

$$\frac{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2} \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{c\sqrt{b^2-4ac}} + \frac{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2} \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{c\sqrt{b^2-4ac}}$$

[Out]  $-x/c - \arctan\left(\frac{1}{2} \frac{2c + (b - (-4ac + b^2)^{1/2}) \tan(x/2)}{(b^2 - 2c(a+c) - b(-4ac + b^2)^{1/2})^{1/2}}\right) \frac{2^{1/2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}{(b^2 - 2c(a+c) - b(-4ac + b^2)^{1/2})^{1/2}} + \arctan\left(\frac{1}{2} \frac{2c + (b + (-4ac + b^2)^{1/2}) \tan(x/2)}{(b^2 - 2c(a+c) + b(-4ac + b^2)^{1/2})^{1/2}}\right) \frac{2^{1/2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}{(b^2 - 2c(a+c) + b(-4ac + b^2)^{1/2})^{1/2}}$

Rubi [A] time = 0.59, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {3266, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2} \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{c\sqrt{b^2-4ac}} + \frac{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2} \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out]  $-(x/c) - (\text{Sqrt}[2] \text{Sqrt}[b^2 - 2c(a+c) - b\text{Sqrt}[b^2 - 4ac]] \text{ArcTan}[(2c + (b - \text{Sqrt}[b^2 - 4ac]) \text{Tan}[x/2]) / (\text{Sqrt}[2] \text{Sqrt}[b^2 - 2c(a+c) - b\text{Sqrt}[b^2 - 4ac]])]) / (c \text{Sqrt}[b^2 - 4ac]) + (\text{Sqrt}[2] \text{Sqrt}[b^2 - 2c(a+c) + b\text{Sqrt}[b^2 - 4ac]] \text{ArcTan}[(2c + (b + \text{Sqrt}[b^2 - 4ac]) \text{Tan}[x/2]) / (\text{Sqrt}[2] \text{Sqrt}[b^2 - 2c(a+c) + b\text{Sqrt}[b^2 - 4ac]])]) / (c \text{Sqrt}[b^2 - 4ac])$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

#### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + dx)/2], x]}, Dist[(2e)/d, Subst[Int[1/(a + 2be\*x + ae^2\*x^2), x], x, Tan[(c + dx)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3266

Int[cos[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n\_.) + (c\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]^(n2\_.))^p, x\_Symbol] := Int[ExpandTrig[(1 - sin[d + ex]^2)^(m/2)\*(a + b\*sin[d + ex]^n + c\*sin[d + ex]^(2n))]^p, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2n] && IntegerQ[m/2] &

& NeQ[b^2 - 4\*a\*c, 0] && IntegersQ[n, p]

### Rule 3292

Int[((A\_) + (B\_)\*sin[(d\_) + (e\_)\*(x\_)])/((a\_) + (b\_)\*sin[(d\_) + (e\_)\*  
\*(x\_) + (c\_)\*sin[(d\_) + (e\_)\*(x\_)^2]), x\_Symbol] :> Module[{q = Rt[b^2  
- 4\*a\*c, 2]}, Dist[B + (b\*B - 2\*A\*c)/q, Int[1/(b + q + 2\*c\*Sin[d + e\*x]), x  
], x] + Dist[B - (b\*B - 2\*A\*c)/q, Int[1/(b - q + 2\*c\*Sin[d + e\*x]), x], x]  
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( -\frac{1}{c} + \frac{a \left(1 + \frac{c}{a}\right) + b \sin(x)}{c \left(a + b \sin(x) + c \sin^2(x)\right)} \right) dx \\ &= -\frac{x}{c} + \frac{\int \frac{a \left(1 + \frac{c}{a}\right) + b \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx}{c} \\ &= -\frac{x}{c} + \frac{\left(b - \frac{b^2 - 2c(a+c)}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{c} + \frac{\left(b + \frac{b^2 - 2c(a+c)}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx}{c} \\ &= -\frac{x}{c} + \frac{\left(2 \left(b - \frac{b^2 - 2c(a+c)}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b - \sqrt{b^2 - 4ac} + 4cx + \left(b - \sqrt{b^2 - 4ac}\right)x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{c} \\ &= -\frac{x}{c} - \frac{\left(4 \left(b - \frac{b^2 - 2c(a+c)}{\sqrt{b^2 - 4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{-8 \left(b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}\right) - x^2} dx, x, 4c + 2 \left(b - \frac{b^2 - 2c(a+c)}{\sqrt{b^2 - 4ac}}\right)\right)}{c} \\ &= -\frac{x}{c} - \frac{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{2c + \left(b - \sqrt{b^2 - 4ac}\right) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4ac}}}\right)}{c \sqrt{b^2 - 4ac}} + \dots \end{aligned}$$

**Mathematica [C]** time = 0.48, size = 314, normalized size = 1.37

$$\frac{\left(b \sqrt{4ac - b^2} - 2ic(a+c) + ib^2\right) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b - i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{-ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} + \frac{\left(b \sqrt{4ac - b^2} + 2ic(a+c) - ib^2\right) \tan^{-1}\left(\frac{2c + \tan\left(\frac{x}{2}\right)(b + i \sqrt{4ac - b^2})}{\sqrt{2} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}}\right)}{\sqrt{2ac - \frac{b^2}{2}} \sqrt{ib \sqrt{4ac - b^2} - 2c(a+c) + b^2}} - x$$

c

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out] (-x + ((I\*b^2 - (2\*I)\*c\*(a + c) + b\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b - I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) - I\*b\*Sqrt[-b^2 + 4\*a\*c]]) + (((-I)\*b^2 + (2\*I)\*c\*(a + c) + b\*Sqrt[-b^2 + 4\*a\*c])\*ArcTan[(2\*c + (b + I\*Sqrt[-b^2 + 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])])/(Sqrt[-1/2\*b^2 + 2\*a\*c]\*Sqrt[b^2 - 2\*c\*(a + c) + I\*b\*Sqrt[-b^2 + 4\*a\*c]])/c

**fricas** [B] time = 0.89, size = 971, normalized size = 4.22

$$\sqrt{2}c\sqrt{-\frac{b^2-2ac-2c^2+(b^2c^2-4ac^3)\sqrt{\frac{b^2}{b^2c^4-4ac^5}}}{b^2c^2-4ac^3}}\log\left(\sqrt{2}(b^2c^3-4ac^4)\sqrt{\frac{b^2}{b^2c^4-4ac^5}}\sqrt{-\frac{b^2-2ac-2c^2+(b^2c^2-4ac^3)\sqrt{\frac{b^2}{b^2c^4-4ac^5}}}{b^2c^2-4ac^3}}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] 1/4\*(sqrt(2)\*c\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*log(sqrt(2)\*(b^2\*c^3 - 4\*a\*c^4)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*cos(x) + b^2\*sin(x) + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sin(x) + 2\*b\*c) - sqrt(2)\*c\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*log(sqrt(2)\*(b^2\*c^3 - 4\*a\*c^4)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*cos(x) - b^2\*sin(x) - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sin(x) - 2\*b\*c) - sqrt(2)\*c\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*log(sqrt(2)\*(b^2\*c^3 - 4\*a\*c^4)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*cos(x) + b^2\*sin(x) - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sin(x) + 2\*b\*c) + sqrt(2)\*c\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*log(sqrt(2)\*(b^2\*c^3 - 4\*a\*c^4)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sqrt(-(b^2 - 2\*a\*c - 2\*c^2 - (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5)))/(b^2\*c^2 - 4\*a\*c^3))\*cos(x) - b^2\*sin(x) + (b^2\*c^2 - 4\*a\*c^3)\*sqrt(b^2/(b^2\*c^4 - 4\*a\*c^5))\*sin(x) - 2\*b\*c) - 4\*x)/c

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.34, size = 1246, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 2\*a/c/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*(-4\*a\*c+b^2)^(1/2)\*b-2/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*b\*(-4\*a\*c+b^2)^(1/2)-8\*a^2/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))+2\*a/c/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2))\*b^2-8\*a/(4\*a\*c-b^2)/(4\*c\*a-2\*b^2+2\*b\*(-4\*a\*c+b^2)^(1/2)+4\*a^2)^(1/2)\*arctan((-2\*a\*tan(1/2\*x)+(-4\*a\*c+b^2)^(1/2)-b)/(4\*c\*a-2\*b^2+2\*b\*(-4



$$\begin{aligned}
& 2) * (32768 * a * b^5 - 32768 * a^3 * b^3 + 65536 * a^2 * b * c^3 - 196608 * a^2 * b^3 * c + 2293 \\
& 76 * a^3 * b * c^2 - 32768 * a * b * c^4 + 131072 * a^4 * b * c) + 32768 * a * c^5 - 24576 * a^5 * c \\
& - 8192 * a^2 * b^4 + 8192 * a^4 * b^2 + 172032 * a^2 * c^4 + 221184 * a^3 * c^3 + 57344 * a^4 \\
& * c^2 - 57344 * a * b^2 * c^3 + 16384 * a^3 * b^2 * c - 147456 * a^2 * b^2 * c^2 + 24576 * a * b^4 \\
& * c) + 8192 * a^2 * b * c^2 + 32768 * a * b * c^3 - 24576 * a * b^3 * c - 49152 * a^3 * b * c) * i - \\
& (- (8 * a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a \\
& * b^2 * c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * (24576 * a^4 * b - \tan(x/2) \\
& * (81920 * a * b^4 + 139264 * a * c^4 + 196608 * a^4 * c + 24576 * a^5 - 98304 * a^3 * b^2 \\
& + 425984 * a^2 * c^3 + 458752 * a^3 * c^2 - 212992 * a * b^2 * c^2 - 327680 * a^2 * b^2 * c) - \\
& 32768 * a^2 * b^3 + (- (8 * a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 \\
& - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * (3 \\
& 2768 * a * c^5 - (- (8 * a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * \\
& b^2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * ((- (8 * \\
& a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * \\
& c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * ((- (8 * a * c^3 + b * (- (4 * a * c \\
& - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 \\
& + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * (\tan(x/2) * (524288 * a^2 * c^7 + 1179648 * a^3 * c^6 \\
& + 851968 * a^4 * c^5 + 196608 * a^5 * c^4 - 131072 * a * b^2 * c^6 + 139264 * a * b^4 * c^4 - \\
& 16384 * a * b^6 * c^2 - 851968 * a^2 * b^2 * c^5 + 147456 * a^2 * b^4 * c^3 - 540672 * a^3 * b^2 \\
& * c^4 + 16384 * a^3 * b^4 * c^2 - 114688 * a^4 * b^2 * c^3) - 32768 * a * b^3 * c^5 + 24576 * a * \\
& b^5 * c^3 + 131072 * a^2 * b * c^6 + 163840 * a^3 * b * c^5 + 98304 * a^4 * b * c^4 - 139264 * a^ \\
& 2 * b^3 * c^4 - 24576 * a^3 * b^3 * c^3) + \tan(x/2) * (32768 * a * b^5 * c^2 - 32768 * a * b^3 * c^ \\
& 4 + 131072 * a^2 * b * c^5 + 262144 * a^3 * b * c^4 + 131072 * a^4 * b * c^3 - 196608 * a^2 * b^3 \\
& * c^3 - 32768 * a^3 * b^3 * c^2) - 131072 * a^2 * c^6 - 163840 * a^3 * c^5 + 65536 * a^4 * c^4 \\
& + 98304 * a^5 * c^3 + 32768 * a * b^2 * c^5 - 32768 * a * b^4 * c^3 + 172032 * a^2 * b^2 * c^4 + \\
& 24576 * a^2 * b^4 * c^2 - 114688 * a^3 * b^2 * c^3 - 24576 * a^4 * b^2 * c^2) + \tan(x/2) * (13 \\
& 1072 * a * c^6 - 16384 * a * b^6 + 16384 * a^3 * b^4 + 983040 * a^2 * c^5 + 1654784 * a^3 * c^4 \\
& + 950272 * a^4 * c^3 + 147456 * a^5 * c^2 - 344064 * a * b^2 * c^4 + 229376 * a * b^4 * c^2 + \\
& 131072 * a^2 * b^4 * c - 98304 * a^4 * b^2 * c - 1228800 * a^2 * b^2 * c^3 - 540672 * a^3 * b^2 * c \\
& ^2) - 57344 * a * b^3 * c^3 + 139264 * a^2 * b * c^4 + 114688 * a^3 * b * c^3 - 24576 * a^3 * b^3 \\
& * c + 73728 * a^4 * b * c^2 - 106496 * a^2 * b^3 * c^2 + 32768 * a * b * c^5 + 24576 * a * b^5 * c) \\
& - \tan(x/2) * (32768 * a * b^5 - 32768 * a^3 * b^3 + 65536 * a^2 * b * c^3 - 196608 * a^2 * b^3 * \\
& c + 229376 * a^3 * b * c^2 - 32768 * a * b * c^4 + 131072 * a^4 * b * c) - 24576 * a^5 * c - 8192 \\
& * a^2 * b^4 + 8192 * a^4 * b^2 + 172032 * a^2 * c^4 + 221184 * a^3 * c^3 + 57344 * a^4 * c^2 - \\
& 57344 * a * b^2 * c^3 + 16384 * a^3 * b^2 * c - 147456 * a^2 * b^2 * c^2 + 24576 * a * b^4 * c) - \\
& 8192 * a^2 * b * c^2 - 32768 * a * b * c^3 + 24576 * a * b^3 * c + 49152 * a^3 * b * c) * i) / ((- (8 * a \\
& * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) \\
& ) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * (\tan(x/2) * (81920 * a * b^4 + \\
& 139264 * a * c^4 + 196608 * a^4 * c + 24576 * a^5 - 98304 * a^3 * b^2 + 425984 * a^2 * c^3 + \\
& 458752 * a^3 * c^2 - 212992 * a * b^2 * c^2 - 327680 * a^2 * b^2 * c) - 24576 * a^4 * b + 32768 \\
& * a^2 * b^3 + (- (8 * a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^ \\
& 2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * ((- (8 * a * \\
& c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) \\
& ) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * ((- (8 * a * c^3 + b * (- (4 * a * c - \\
& b^2)^3)^{(1/2)} + b^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 + \\
& b^4 * c^2 - 8 * a * b^2 * c^3)))^{(1/2)} * ((- (8 * a * c^3 + b * (- (4 * a * c - b^2)^3)^{(1/2)} + b \\
& ^4 + 8 * a^2 * c^2 - 2 * b^2 * c^2 - 6 * a * b^2 * c) / (2 * (16 * a^2 * c^4 + b^4 * c^2 - 8 * a * b^2 * \\
& c^3)))^{(1/2)} * (\tan(x/2) * (524288 * a^2 * c^7 + 1179648 * a^3 * c^6 + 851968 * a^4 * c^5 + \\
& 196608 * a^5 * c^4 - 131072 * a * b^2 * c^6 + 139264 * a * b^4 * c^4 - 16384 * a * b^6 * c^2 - 8 \\
& 51968 * a^2 * b^2 * c^5 + 147456 * a^2 * b^4 * c^3 - 540672 * a^3 * b^2 * c^4 + 16384 * a^3 * b^4 \\
& * c^2 - 114688 * a^4 * b^2 * c^3) - 32768 * a * b^3 * c^5 + 24576 * a * b^5 * c^3 + 131072 * a^2 \\
& * b * c^6 + 163840 * a^3 * b * c^5 + 98304 * a^4 * b * c^4 - 139264 * a^2 * b^3 * c^4 - 24576 * a^ \\
& 3 * b^3 * c^3) - \tan(x/2) * (32768 * a * b^5 * c^2 - 32768 * a * b^3 * c^4 + 131072 * a^2 * b * c^5 \\
& + 262144 * a^3 * b * c^4 + 131072 * a^4 * b * c^3 - 196608 * a^2 * b^3 * c^3 - 32768 * a^3 * b^3 \\
& * c^2) + 131072 * a^2 * c^6 + 163840 * a^3 * c^5 - 65536 * a^4 * c^4 - 98304 * a^5 * c^3 - 3 \\
& 2768 * a * b^2 * c^5 + 32768 * a * b^4 * c^3 - 172032 * a^2 * b^2 * c^4 - 24576 * a^2 * b^4 * c^2 + \\
& 114688 * a^3 * b^2 * c^3 + 24576 * a^4 * b^2 * c^2) + \tan(x/2) * (131072 * a * c^6 - 16384 * a \\
& * b^6 + 16384 * a^3 * b^4 + 983040 * a^2 * c^5 + 1654784 * a^3 * c^4 + 950272 * a^4 * c^3 + \\
& 147456 * a^5 * c^2 - 344064 * a * b^2 * c^4 + 229376 * a * b^4 * c^2 + 131072 * a^2 * b^4 * c - 9
\end{aligned}$$



$$\begin{aligned}
& 8304a^4b^2c - 1228800a^2b^2c^3 - 540672a^3b^2c^2) - 57344a^4b^3c^3 + 139264a^2b^3c^4 + 114688a^3b^3c^3 - 24576a^3b^3c + 73728a^4b^3c^2 \\
& - 106496a^2b^3c^2 + 32768a^4b^3c^5 + 24576a^4b^5c) - \tan(x/2) * (32768a^4b^5 - 32768a^3b^3 + 65536a^2b^3c^3 - 196608a^2b^3c + 229376a^3b^3c^2 \\
& - 32768a^4b^3c^4 + 131072a^4b^3c) + 32768a^4c^5 - 24576a^5c - 8192a^2b^4 + 8192a^4b^2 + 172032a^2c^4 + 221184a^3c^3 + 57344a^4c^2 - 57344 \\
& a^4b^2c^3 + 16384a^3b^2c - 147456a^2b^2c^2 + 24576a^4b^4c) + 8192a^2b^2c^2 + 32768a^4b^3c - 24576a^4b^3c - 49152a^3b^3c) + (- (8a^4c^3 + b^4 \\
& (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * (24576a^4b - \tan(x/2) * (81920a^4b^4 \\
& + 139264a^4c^4 + 196608a^4c + 24576a^5 - 98304a^3b^2 + 425984a^2c^3 + 458752a^3c^2 - 212992a^4b^2c^2 - 327680a^2b^2c) - 32768a^2b^3 \\
& + (- (8a^4c^3 + b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * (32768a^4c^5 - (- \\
& (8a^4c^3 + b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * ((- (8a^4c^3 + b^4 * (- (4a^4c \\
& - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * ((- (8a^4c^3 + b^4 * (- (4a^4c - b^2)^3)^{1/2} \\
& ) + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * (\tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4c^5 \\
& + 196608a^5c^4 - 131072a^4b^2c^6 + 139264a^4b^4c^4 - 16384a^4b^6c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 \\
& - 114688a^4b^2c^3) - 32768a^4b^3c^5 + 24576a^4b^5c^3 + 131072a^2b^3c^6 + 163840a^3b^3c^5 + 98304a^4b^3c^4 - 139264a^2b^3c^4 - 245 \\
& 76a^3b^3c^3) + \tan(x/2) * (32768a^4b^5c^2 - 32768a^4b^3c^4 + 131072a^2b^3c^5 + 262144a^3b^3c^4 + 131072a^4b^3c^3 - 196608a^2b^3c^3 - 32768a^3b^3c^2) \\
& - 131072a^2c^6 - 163840a^3c^5 + 65536a^4c^4 + 98304a^5c^3 + 32768a^4b^2c^5 - 32768a^4b^4c^3 + 172032a^2b^2c^4 + 24576a^2b^4c^2 - 114688a^3b^2c^3 \\
& - 24576a^4b^2c^2) + \tan(x/2) * (131072a^4c^6 - 16384a^4b^6 + 16384a^3b^4 + 983040a^2c^5 + 1654784a^3c^4 + 950272a^4c^3 + 147456a^5c^2 - 344064a^4b^2c^4 \\
& + 229376a^4b^4c^2 + 131072a^2b^4c - 98304a^4b^2c - 1228800a^2b^2c^3 - 540672a^3b^2c^2) - 57344a^4b^3c^3 + 139264a^2b^3c^4 + 114688a^3b^3c^3 - 24576a^3b^3c \\
& + 73728a^4b^3c^2 - 106496a^2b^3c^2 + 32768a^4b^3c^5 + 24576a^4b^5c) - \tan(x/2) * (32768a^4b^5 - 32768a^3b^3 + 65536a^2b^3c^3 - 196608a^2b^3c + 229376a^3b^3c^2 \\
& - 32768a^4b^3c^4 + 131072a^4b^3c) - 24576a^5c - 8192a^2b^4 + 8192a^4b^2 + 172032a^2c^4 + 221184a^3c^3 + 57344a^4c^2 - 57344a^4b^2c^3 + 16384a^3b^2c \\
& - 147456a^2b^2c^2 + 24576a^4b^4c) - 8192a^2b^2c^2 - 32768a^4b^3c + 24576a^4b^3c + 49152a^3b^3c) + 49152a^4c^3 + 147456a^3c + 49152a^4 + 2 * \tan(x/2) * (32768a^3b \\
& - 32768a^4b^3 + 32768a^4b^3c^2 + 65536a^2b^3c) - 49152a^2b^2 + 147456a^2c^2 - 49152a^4b^2c) * (- (8a^4c^3 + b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 \\
& - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * 2i + \operatorname{atan}((( - (8a^4c^3 - b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 \\
& + b^4c^2 - 8a^4b^2c^3)))^{1/2} * (\tan(x/2) * (81920a^4b^4 + 139264a^4c^4 + 196608a^4c + 24576a^5 - 98304a^3b^2 + 425984a^2c^3 + 458752a^3c^2 - 212992a^4b^2c^2 \\
& - 327680a^2b^2c) - 24576a^4b + 32768a^2b^3 + (- (8a^4c^3 - b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * ((- (8a^4c^3 - b^4 * (- (4a^4c \\
& - b^2)^3)^{1/2} + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * ((- (8a^4c^3 - b^4 * (- (4a^4c - b^2)^3)^{1/2} \\
& ) + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * ((- (8a^4c^3 - b^4 * (- (4a^4c - b^2)^3)^{1/2} + b^4 + 8a^2c^2 \\
& - 2b^2c^2 - 6a^4b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^4b^2c^3)))^{1/2} * (\tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4c^5 + 196608a^5c^4 - 131072a^4b^2c^6 \\
& + 139264a^4b^4c^4 - 16384a^4b^6c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 - 114688a^4b^2c^3) - 32768a^4b^3c^5 + 24576a^4b^5c^3 \\
& + 131072a^2b^3c^6 + 163840a^3b^3c^5 + 98304a^4b^3c^4 - 139264a^2b^3c^4 - 24576a^3b^3c^3) -
\end{aligned}$$



$$\begin{aligned}
& 6a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 - 114688a^4b^2c^3 \\
& ) - 32768a^2b^3c^5 + 24576a^2b^5c^3 + 131072a^2b^3c^6 + 163840a^3b^3c^5 \\
& + 98304a^4b^3c^4 - 139264a^2b^3c^4 - 24576a^3b^3c^3) - \tan(x/2) * (32768a^2b^5c^2 - 32768a^2b^3c^4 + 131072a^2b^3c^5 + 262144a^3b^3c^4 + 131072a^4b^3c^3 - 196608a^2b^3c^3 - 32768a^3b^3c^2) + 131072a^2c^6 + 163840a^3c^5 - 65536a^4c^4 - 98304a^5c^3 - 32768a^2b^2c^5 + 32768a^2b^4c^3 - 172032a^2b^2c^4 - 24576a^2b^4c^2 + 114688a^3b^2c^3 + 24576a^4b^2c^2) + \tan(x/2) * (131072a^2c^6 - 16384a^2b^6 + 16384a^3b^4 + 983040a^2c^5 + 1654784a^3c^4 + 950272a^4c^3 + 147456a^5c^2 - 344064a^2b^2c^4 + 229376a^2b^4c^2 + 131072a^2b^4c - 98304a^4b^2c - 1228800a^2b^2c^3 - 540672a^3b^2c^2) - 57344a^2b^3c^3 + 139264a^2b^3c^4 + 114688a^3b^3c^3 - 24576a^3b^3c + 73728a^4b^3c^2 - 106496a^2b^3c^2 + 32768a^2b^3c^5 + 24576a^2b^5c) - \tan(x/2) * (32768a^2b^5 - 32768a^3b^3 + 65536a^2b^3c^3 - 196608a^2b^3c + 229376a^3b^3c^2 - 32768a^2b^3c^4 + 131072a^4b^3c) + 32768a^2c^5 - 24576a^5c - 8192a^2b^4 + 8192a^4b^2 + 172032a^2c^4 + 221184a^3c^3 + 57344a^4c^2 - 57344a^2b^2c^3 + 16384a^3b^2c - 147456a^2b^2c^2 + 24576a^2b^4c) + 8192a^2b^2c^2 + 32768a^2b^3c^3 - 24576a^2b^3c - 49152a^3b^3c) + ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * (24576a^4b - \tan(x/2) * (81920a^2b^4 + 139264a^2c^4 + 196608a^4c + 24576a^5 - 98304a^3b^2 + 425984a^2c^3 + 458752a^3c^2 - 212992a^2b^2c^2 - 327680a^2b^2c) - 32768a^2b^3 + ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * (32768a^2c^5 - ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) + b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * (tan(x/2) * (524288a^2c^7 + 1179648a^3c^6 + 851968a^4c^5 + 196608a^5c^4 - 131072a^2b^2c^6 + 139264a^2b^4c^4 - 16384a^2b^6c^2 - 851968a^2b^2c^5 + 147456a^2b^4c^3 - 540672a^3b^2c^4 + 16384a^3b^4c^2 - 114688a^4b^2c^3) - 32768a^2b^3c^5 + 24576a^2b^5c^3 + 131072a^2b^3c^6 + 163840a^3b^3c^5 + 98304a^4b^3c^4 - 139264a^2b^3c^4 - 24576a^3b^3c^3) + \tan(x/2) * (32768a^2b^5c^2 - 32768a^2b^3c^4 + 131072a^2b^3c^5 + 262144a^3b^3c^4 + 131072a^4b^3c^3 - 196608a^2b^3c^3 - 32768a^3b^3c^2) - 131072a^2c^6 - 163840a^3c^5 + 65536a^4c^4 + 98304a^5c^3 + 32768a^2b^2c^5 - 32768a^2b^4c^3 + 172032a^2b^2c^4 + 24576a^2b^4c^2 - 114688a^3b^2c^3 - 24576a^4b^2c^2) + \tan(x/2) * (131072a^2c^6 - 16384a^2b^6 + 16384a^3b^4 + 983040a^2c^5 + 1654784a^3c^4 + 950272a^4c^3 + 147456a^5c^2 - 344064a^2b^2c^4 + 229376a^2b^4c^2 + 131072a^2b^4c - 98304a^4b^2c - 1228800a^2b^2c^3 - 540672a^3b^2c^2) - 57344a^2b^3c^3 + 139264a^2b^3c^4 + 114688a^3b^3c^3 - 24576a^3b^3c + 73728a^4b^3c^2 - 106496a^2b^3c^2 + 32768a^2b^3c^5 + 24576a^2b^5c) - \tan(x/2) * (32768a^2b^5 - 32768a^3b^3 + 65536a^2b^3c^3 - 196608a^2b^3c + 229376a^3b^3c^2 - 32768a^2b^3c^4 + 131072a^4b^3c) - 24576a^5c - 8192a^2b^4 + 8192a^4b^2 + 172032a^2c^4 + 221184a^3c^3 + 57344a^4c^2 - 57344a^2b^2c^3 + 16384a^3b^2c - 147456a^2b^2c^2 + 24576a^2b^4c) - 8192a^2b^2c^2 - 32768a^2b^3c^3 + 24576a^2b^3c + 49152a^3b^3c) + 49152a^2c^3 + 147456a^3c + 49152a^4 + 2 * \tan(x/2) * (32768a^3b - 32768a^2b^3 + 32768a^2b^3c^2 + 65536a^2b^3c) - 49152a^2b^2 + 147456a^2c^2 - 49152a^2b^2c) * ((-8a^2c^3 - b^4 + 8a^2c^2 - 2b^2c^2 - 6a^2b^2c) / (2 * (16a^2c^4 + b^4c^2 - 8a^2b^2c^3)))^(1/2) * 2i - (2 * \operatorname{atan}((196608a^4 * \tan(x/2)) / (16384a^2c^3 - 32768a^3c + 196608a^4 + 98304a^2b^2 - 65536a^2c^2 + (147456a^5) / c) - (16384a^2b^4) / c - (196608a^3b^2) / c + (32768a^2b^4) / c^2 - (32768a^4b^2) / c^2) - (147456a^5 * \tan(x/2)) / (16384a^2b^4 - 16384a^2c^4 - 196608a^4c - 147456a^5 + 196608a^3b^2 + 65536a^2c^3 + 32768a^3c^2 - 98304a^2b^2c - (32768a^2b^4) / c + (32768a^4b^2) / c) + (32768a^2b^4 * \tan(x/2)) / (16384a^2c^5 + 147456a^5c + 32768a^2b^4 - 32768a^4b^2 - 65536a^2c^4 - 32768
\end{aligned}$$

$$\begin{aligned} & *a^3*c^3 + 196608*a^4*c^2 - 196608*a^3*b^2*c + 98304*a^2*b^2*c^2 - 16384*a* \\ & b^4*c) - (32768*a^4*b^2*\tan(x/2))/(16384*a*c^5 + 147456*a^5*c + 32768*a^2*b \\ & ^4 - 32768*a^4*b^2 - 65536*a^2*c^4 - 32768*a^3*c^3 + 196608*a^4*c^2 - 19660 \\ & 8*a^3*b^2*c + 98304*a^2*b^2*c^2 - 16384*a*b^4*c) + (16384*a*b^4*\tan(x/2))/( \\ & 16384*a*b^4 - 16384*a*c^4 - 196608*a^4*c - 147456*a^5 + 196608*a^3*b^2 + 65 \\ & 536*a^2*c^3 + 32768*a^3*c^2 - 98304*a^2*b^2*c - (32768*a^2*b^4)/c + (32768* \\ & a^4*b^2)/c) + (16384*a*c^3*\tan(x/2))/(16384*a*c^3 - 32768*a^3*c + 196608*a^ \\ & 4 + 98304*a^2*b^2 - 65536*a^2*c^2 + (147456*a^5)/c - (16384*a*b^4)/c - (196 \\ & 608*a^3*b^2)/c + (32768*a^2*b^4)/c^2 - (32768*a^4*b^2)/c^2) - (32768*a^3*c* \\ & \tan(x/2))/(16384*a*c^3 - 32768*a^3*c + 196608*a^4 + 98304*a^2*b^2 - 65536*a \\ & ^2*c^2 + (147456*a^5)/c - (16384*a*b^4)/c - (196608*a^3*b^2)/c + (32768*a^2 \\ & *b^4)/c^2 - (32768*a^4*b^2)/c^2) + (196608*a^3*b^2*\tan(x/2))/(16384*a*b^4 - \\ & 16384*a*c^4 - 196608*a^4*c - 147456*a^5 + 196608*a^3*b^2 + 65536*a^2*c^3 + \\ & 32768*a^3*c^2 - 98304*a^2*b^2*c - (32768*a^2*b^4)/c + (32768*a^4*b^2)/c) + \\ & (98304*a^2*b^2*\tan(x/2))/(16384*a*c^3 - 32768*a^3*c + 196608*a^4 + 98304*a \\ & ^2*b^2 - 65536*a^2*c^2 + (147456*a^5)/c - (16384*a*b^4)/c - (196608*a^3*b^2 \\ & )/c + (32768*a^2*b^4)/c^2 - (32768*a^4*b^2)/c^2) - (65536*a^2*c^2*\tan(x/2)) \\ & /((16384*a*c^3 - 32768*a^3*c + 196608*a^4 + 98304*a^2*b^2 - 65536*a^2*c^2 + \\ & (147456*a^5)/c - (16384*a*b^4)/c - (196608*a^3*b^2)/c + (32768*a^2*b^4)/c^2 \\ & - (32768*a^4*b^2)/c^2)))/c \end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)\*\*2/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Timed out

$$3.11 \quad \int \frac{\cos(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

**Optimal.** Leaf size=35

$$-\frac{2 \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

[Out]  $-2*\operatorname{arctanh}((b+2*c*\sin(x))/(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)}$

**Rubi [A]** time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {3258, 618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[x]/(a + b*\operatorname{Sin}[x] + c*\operatorname{Sin}[x]^2), x]$

[Out]  $(-2*\operatorname{ArcTanh}[(b + 2*c*\operatorname{Sin}[x])/ \operatorname{Sqrt}[b^2 - 4*a*c]])/ \operatorname{Sqrt}[b^2 - 4*a*c]$

**Rule 206**

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/ \operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \operatorname{Lt} Q[b, 0])$

**Rule 618**

$\operatorname{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2)^{-1}), x\_Symbol] :> \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

**Rule 3258**

$\operatorname{Int}[\cos[(d_ + (e_)*(x_))]^{(m_)}*((a_ + (b_)*((f_)*\sin[(d_ + (e_)*(x_))]^{(n_)} + (c_)*((f_)*\sin[(d_ + (e_)*(x_))]^{(n_2_)}))^{(p_)}), x\_Symbol] :> \operatorname{Module}\{g = \operatorname{FreeFactors}[\operatorname{Sin}[d + e*x], x], \operatorname{Dist}[g/e, \operatorname{Subst}[\operatorname{Int}[(1 - g^2*x^2)^{((m - 1)/2)}*(a + b*(f*g*x)^n + c*(f*g*x)^{(2*n)})^p, x], x, \operatorname{Sin}[d + e*x]/g], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x\} \ \&\& \operatorname{EqQ}[n2, 2*n] \ \&\& \operatorname{Integer} Q[(m - 1)/2]$

**Rubi steps**

$$\begin{aligned} \int \frac{\cos(x)}{a+b \sin(x)+c \sin^2(x)} dx &= \operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, \sin(x)\right) \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2c \sin(x)\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] (-2\*ArcTanh[(b + 2\*c\*Sin[x])/Sqrt[b^2 - 4\*a\*c]])/Sqrt[b^2 - 4\*a\*c]

**fricas** [A] time = 0.71, size = 139, normalized size = 3.97

$$\left[ \frac{\log\left(\frac{-2c^2 \cos(x)^2 - 2bc \sin(x) - b^2 + 2ac - 2c^2 + \sqrt{b^2 - 4ac}(2c \sin(x) + b)}{c \cos(x)^2 - b \sin(x) - a - c}\right)}{\sqrt{b^2 - 4ac}}, -\frac{2\sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2c \sin(x) + b)}{b^2 - 4ac}\right)}{b^2 - 4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] [log(-(2\*c^2\*cos(x)^2 - 2\*b\*c\*sin(x) - b^2 + 2\*a\*c - 2\*c^2 + sqrt(b^2 - 4\*a\*c))\*(2\*c\*sin(x) + b))/(c\*cos(x)^2 - b\*sin(x) - a - c))/sqrt(b^2 - 4\*a\*c), -2\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*sin(x) + b)/(b^2 - 4\*a\*c))/(b^2 - 4\*a\*c)]

**giac** [A] time = 0.13, size = 35, normalized size = 1.00

$$\frac{2 \arctan\left(\frac{2c \sin(x) + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] 2\*arctan((2\*c\*sin(x) + b)/sqrt(-b^2 + 4\*a\*c))/sqrt(-b^2 + 4\*a\*c)

**maple** [A] time = 0.21, size = 36, normalized size = 1.03

$$\frac{2 \arctan\left(\frac{b + 2c \sin(x)}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 2/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 15.07, size = 47, normalized size = 1.34

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac - b^2}} + \frac{2c \sin(x)}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + c*sin(x)^2 + b*sin(x)),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*sin(x))/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

**sympy [A]** time = 7.23, size = 99, normalized size = 2.83

$$\left\{ \begin{array}{ll} \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{b} & \text{for } c = 0 \\ -\frac{2}{b + 2c \sin(x)} & \text{for } a = \frac{b^2}{4c} \\ \frac{\log\left(\frac{b}{2c} + \sin(x) - \frac{\sqrt{-4ac + b^2}}{2c}\right)}{\sqrt{-4ac + b^2}} - \frac{\log\left(\frac{b}{2c} + \sin(x) + \frac{\sqrt{-4ac + b^2}}{2c}\right)}{\sqrt{-4ac + b^2}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Piecewise((log(a/b + sin(x))/b, Eq(c, 0)), (-2/(b + 2*c*sin(x)), Eq(a, b**2/(4*c))), (log(b/(2*c) + sin(x) - sqrt(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) - log(b/(2*c) + sin(x) + sqrt(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2), True))
```

### 3.12 $\int \frac{\sec(x)}{a+b \sin(x)+c \sin^2(x)} dx$

**Optimal.** Leaf size=128

$$\frac{(-2ac + b^2 - 2c^2) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{(a-b+c)(a+b+c)\sqrt{b^2-4ac}} - \frac{b \log(a+b \sin(x)+c \sin^2(x))}{2(a-b+c)(a+b+c)} - \frac{\log(1-\sin(x))}{2(a+b+c)} + \frac{\log(\sin(x)+1)}{2(a-b+c)}$$

[Out]  $-1/2*\ln(1-\sin(x))/(a+b+c)+1/2*\ln(1+\sin(x))/(a-b+c)-1/2*b*\ln(a+b*\sin(x)+c*\sin(x)^2)/(a-b+c)/(a+b+c)+(-2*a*c+b^2-2*c^2)*\operatorname{arctanh}((b+2*c*\sin(x))/(-4*a*c+b^2)^{(1/2)))/(a-b+c)/(a+b+c)/(-4*a*c+b^2)^{(1/2)}$

**Rubi [A]** time = 0.17, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {3258, 981, 634, 618, 206, 628, 633, 31}

$$\frac{(-2ac + b^2 - 2c^2) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{(a-b+c)(a+b+c)\sqrt{b^2-4ac}} - \frac{b \log(a+b \sin(x)+c \sin^2(x))}{2(a-b+c)(a+b+c)} - \frac{\log(1-\sin(x))}{2(a+b+c)} + \frac{\log(\sin(x)+1)}{2(a-b+c)}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out]  $((b^2 - 2*a*c - 2*c^2)*\operatorname{ArcTanh}[(b + 2*c*\sin[x])/ \operatorname{Sqrt}[b^2 - 4*a*c]])/((a - b + c)*(a + b + c)*\operatorname{Sqrt}[b^2 - 4*a*c]) - \operatorname{Log}[1 - \sin[x]]/(2*(a + b + c)) + \operatorname{Log}[1 + \sin[x]]/(2*(a - b + c)) - (b*\operatorname{Log}[a + b*\sin[x] + c*\sin[x]^2])/(2*(a - b + c)*(a + b + c))$

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 633

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

#### Rule 634



```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 981

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^p, x_Symbol] := Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && IntegerQ[(m - 1)/2]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{(1-x^2)(a+bx+cx^2)} dx, x, \sin(x) \right) \\ &= -\frac{\text{Subst} \left( \int \frac{-a-c+bx}{1-x^2} dx, x, \sin(x) \right)}{(a-b+c)(a+b+c)} + \frac{\text{Subst} \left( \int \frac{-b^2+ac+c^2-bcx}{a+bx+cx^2} dx, x, \sin(x) \right)}{(a-b+c)(a+b+c)} \\ &= -\frac{\text{Subst} \left( \int \frac{1}{-1-x} dx, x, \sin(x) \right)}{2(a-b+c)} + \frac{\text{Subst} \left( \int \frac{1}{1-x} dx, x, \sin(x) \right)}{2(a+b+c)} - \frac{b \text{Subst} \left( \int \frac{b+bx}{a+bx+cx^2} dx, x, \sin(x) \right)}{2(a-b+c)} \\ &= -\frac{\log(1-\sin(x))}{2(a+b+c)} + \frac{\log(1+\sin(x))}{2(a-b+c)} - \frac{b \log(a+b \sin(x) + c \sin^2(x))}{2(a-b+c)(a+b+c)} + \frac{(b^2 - 2c(a+c)) \tanh^{-1} \left( \frac{b+2c \sin(x)}{\sqrt{b^2-4ac}} \right)}{(a-b+c)(a+b+c)\sqrt{b^2-4ac}} - \frac{\log(1-\sin(x))}{2(a+b+c)} + \frac{\log(1+\sin(x))}{2(a-b+c)} - \frac{b \log(a+b \sin(x) + c \sin^2(x))}{2(a-b+c)(a+b+c)} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 119, normalized size = 0.93

$$\frac{\sqrt{b^2 - 4ac} \left( b \log(a + b \sin(x) + c \sin^2(x)) + (a - b + c) \log(1 - \sin(x)) - (a + b + c) \log(\sin(x) + 1) \right) + (4c(a + b + c) \log(\sin(x) + 1) - (b^2 - 2c(a + c)) \tanh^{-1} \left( \frac{b + 2c \sin(x)}{\sqrt{b^2 - 4ac}} \right))}{2(a - b + c)(a + b + c)\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x] + c*Sin[x]^2), x]
```

```
[Out] -1/2*((-2*b^2 + 4*c*(a + c))*ArcTanh[(b + 2*c*Sin[x])/Sqrt[b^2 - 4*a*c]] + Sqrt[b^2 - 4*a*c]*((a - b + c)*Log[1 - Sin[x]] - (a + b + c)*Log[1 + Sin[x]] + b*Log[a + b*Sin[x] + c*Sin[x]^2]))/((a - b + c)*(a + b + c)*Sqrt[b^2 - 4*a*c])
```

**fricas** [A] time = 1.29, size = 482, normalized size = 3.77

$$\left[ \frac{(b^2 - 2ac - 2c^2)\sqrt{b^2 - 4ac} \log\left(-\frac{2c^2 \cos(x)^2 - 2bc \sin(x) - b^2 + 2ac - 2c^2 + \sqrt{b^2 - 4ac}(2c \sin(x) + b)}{c \cos(x)^2 - b \sin(x) - a - c}\right) + (b^3 - 4abc) \log(-c \cos(x) + b \sin(x) + a + c)}{2(a^2 b^2 - b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] [-1/2\*((b^2 - 2\*a\*c - 2\*c^2)\*sqrt(b^2 - 4\*a\*c)\*log(-(2\*c^2\*cos(x)^2 - 2\*b\*c\*sin(x) - b^2 + 2\*a\*c - 2\*c^2 + sqrt(b^2 - 4\*a\*c)\*(2\*c\*sin(x) + b))/(c\*cos(x)^2 - b\*sin(x) - a - c)) + (b^3 - 4\*a\*b\*c)\*log(-c\*cos(x)^2 + b\*sin(x) + a + c) - (a\*b^2 + b^3 - 4\*a\*c^2 - (4\*a^2 + 4\*a\*b - b^2)\*c)\*log(sin(x) + 1) + (a\*b^2 - b^3 - 4\*a\*c^2 - (4\*a^2 - 4\*a\*b - b^2)\*c)\*log(-sin(x) + 1))/(a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c), 1/2\*(2\*(b^2 - 2\*a\*c - 2\*c^2)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*sin(x) + b)/(b^2 - 4\*a\*c)) - (b^3 - 4\*a\*b\*c)\*log(-c\*cos(x)^2 + b\*sin(x) + a + c) + (a\*b^2 + b^3 - 4\*a\*c^2 - (4\*a^2 + 4\*a\*b - b^2)\*c)\*log(sin(x) + 1) - (a\*b^2 - b^3 - 4\*a\*c^2 - (4\*a^2 - 4\*a\*b - b^2)\*c)\*log(-sin(x) + 1))/(a^2\*b^2 - b^4 - 4\*a\*c^3 - (8\*a^2 - b^2)\*c^2 - 2\*(2\*a^3 - 3\*a\*b^2)\*c)]

**giac** [A] time = 1.23, size = 131, normalized size = 1.02

$$\frac{b \log(c \sin(x)^2 + b \sin(x) + a)}{2(a^2 - b^2 + 2ac + c^2)} - \frac{(b^2 - 2ac - 2c^2) \arctan\left(\frac{2c \sin(x) + b}{\sqrt{-b^2 + 4ac}}\right)}{(a^2 - b^2 + 2ac + c^2)\sqrt{-b^2 + 4ac}} + \frac{\log(\sin(x) + 1)}{2(a - b + c)} - \frac{\log(-\sin(x) + 1)}{2(a + b + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] -1/2\*b\*log(c\*sin(x)^2 + b\*sin(x) + a)/(a^2 - b^2 + 2\*a\*c + c^2) - (b^2 - 2\*a\*c - 2\*c^2)\*arctan((2\*c\*sin(x) + b)/sqrt(-b^2 + 4\*a\*c))/((a^2 - b^2 + 2\*a\*c + c^2)\*sqrt(-b^2 + 4\*a\*c)) + 1/2\*log(sin(x) + 1)/(a - b + c) - 1/2\*log(-sin(x) + 1)/(a + b + c)

**maple** [A] time = 0.29, size = 224, normalized size = 1.75

$$\frac{\ln(-1 + \sin(x))}{2a + 2b + 2c} - \frac{b \ln(a + b \sin(x) + c(\sin^2(x)))}{2(a - b + c)(a + b + c)} + \frac{2 \arctan\left(\frac{b + 2c \sin(x)}{\sqrt{4ca - b^2}}\right) ca}{(a - b + c)(a + b + c)\sqrt{4ca - b^2}} - \frac{\arctan\left(\frac{b + 2c \sin(x)}{\sqrt{4ca - b^2}}\right) b}{(a - b + c)(a + b + c)\sqrt{4ca - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] -1/(2\*a+2\*b+2\*c)\*ln(-1+sin(x))-1/2\*b\*ln(a+b\*sin(x)+c\*sin(x)^2)/(a-b+c)/(a+b+c)+2/(a-b+c)/(a+b+c)/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))\*c\*a-1/(a-b+c)/(a+b+c)/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))\*b^2+2/(a-b+c)/(a+b+c)/(4\*a\*c-b^2)^(1/2)\*arctan((b+2\*c\*sin(x))/(4\*a\*c-b^2)^(1/2))\*c^2+1/(2\*a-2\*b+2\*c)\*ln(1+sin(x))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad [B]** time = 17.35, size = 1001, normalized size = 7.82

$$\frac{\ln(\sin(x)+1)}{2(a-b+c)} - \frac{\ln(\sin(x)-1)}{2(a+b+c)} + \frac{\ln\left(4c^3 \sin(x) + bc^2 + \frac{(a(4bc-2c\sqrt{b^2-4ac})-b^3+b^2\sqrt{b^2-4ac}-2c^2\sqrt{b^2-4ac})}{8ac^3+\sin(x)}\right)}{2(a-b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)\*(a + c\*sin(x)^2 + b\*sin(x))),x)

[Out]  $\log(\sin(x)+1)/(2*(a-b+c)) - \log(\sin(x)-1)/(2*(a+b+c)) + (\log(4*c^3*\sin(x) + b*c^2 + ((a*(4*b*c - 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 + b^2*(b^2 - 4*a*c)^{1/2} - 2*c^2*(b^2 - 4*a*c)^{1/2})*(8*a*c^3 + \sin(x)*(12*b*c^3 - 3*b^3*c + 12*a*b*c^2) + 4*c^4 + 4*a^2*c^2 + 3*b^2*c^2 - ((a*(4*b*c - 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 + b^2*(b^2 - 4*a*c)^{1/2} - 2*c^2*(b^2 - 4*a*c)^{1/2}))*(\sin(x)*(8*a*c^4 + 6*b^4*c + 8*c^5 - 8*a^2*c^3 - 8*a^3*c^2 - 6*b^2*c^3 - 20*a*b^2*c^2 + 2*a^2*b^2*c) + 4*b*c^4 + 4*b^3*c^2 - 28*a^2*b*c^2 - 24*a*b*c^3 + 8*a*b^3*c))/(b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2)) - a*b^2*c)/(b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2)))*(a*(4*b*c - 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 + b^2*(b^2 - 4*a*c)^{1/2} - 2*c^2*(b^2 - 4*a*c)^{1/2}))/ (b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2)) + (\log(4*c^3*\sin(x) + b*c^2 + ((a*(4*b*c + 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 - b^2*(b^2 - 4*a*c)^{1/2} + 2*c^2*(b^2 - 4*a*c)^{1/2})*(8*a*c^3 + \sin(x)*(12*b*c^3 - 3*b^3*c + 12*a*b*c^2) + 4*c^4 + 4*a^2*c^2 + 3*b^2*c^2 - ((a*(4*b*c + 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 - b^2*(b^2 - 4*a*c)^{1/2} + 2*c^2*(b^2 - 4*a*c)^{1/2}))*(\sin(x)*(8*a*c^4 + 6*b^4*c + 8*c^5 - 8*a^2*c^3 - 8*a^3*c^2 - 6*b^2*c^3 - 20*a*b^2*c^2 + 2*a^2*b^2*c) + 4*b*c^4 + 4*b^3*c^2 - 28*a^2*b*c^2 - 24*a*b*c^3 + 8*a*b^3*c))/(b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2)) - a*b^2*c)/(b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2)))*(a*(4*b*c + 2*c*(b^2 - 4*a*c))^{1/2}) - b^3 - b^2*(b^2 - 4*a*c)^{1/2} + 2*c^2*(b^2 - 4*a*c)^{1/2}))/ (b^2*(12*a*c + 2*a^2 - 2*b^2 + 2*c^2) - 4*a*c*(4*a*c + 2*a^2 + 2*c^2))$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Integral(sec(x)/(a + b\*sin(x) + c\*sin(x)\*\*2), x)

$$3.13 \quad \int \frac{\sec^2(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=324

$$\frac{\sqrt{2} bc \left( \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right) + \sqrt{2} bc \left( 1 - \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{(a-b+c)(a+b+c)\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2} + (a-b+c)(a+b+c)\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out] 1/2\*cos(x)/(a+b+c)/(1-sin(x))-1/2\*cos(x)/(a-b+c)/(1+sin(x))-b\*c\*arctan(1/2\*(2\*c+(b-(-4\*a\*c+b^2)^(1/2))\*tan(1/2\*x))\*2^(1/2)/(b^2-2\*c\*(a+c)-b\*(-4\*a\*c+b^2)^(1/2))^(1/2))\*2^(1/2)\*(1+(b^2-2\*c\*(a+c))/b/(-4\*a\*c+b^2)^(1/2))/(a-b+c)/(a+b+c)/(b^2-2\*c\*(a+c)-b\*(-4\*a\*c+b^2)^(1/2))^(1/2)-b\*c\*arctan(1/2\*(2\*c+(b+(-4\*a\*c+b^2)^(1/2))\*tan(1/2\*x))\*2^(1/2)/(b^2-2\*c\*(a+c)+b\*(-4\*a\*c+b^2)^(1/2))^(1/2))\*2^(1/2)\*(1+(-b^2+2\*c\*(a+c))/b/(-4\*a\*c+b^2)^(1/2))/(a-b+c)/(a+b+c)/(b^2-2\*c\*(a+c)+b\*(-4\*a\*c+b^2)^(1/2))^(1/2)

**Rubi [A]** time = 2.27, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {3266, 2648, 3292, 2660, 618, 204}

$$\frac{\sqrt{2} bc \left( \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right) + \sqrt{2} bc \left( 1 - \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{(a-b+c)(a+b+c)\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2} + (a-b+c)(a+b+c)\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] -((Sqrt[2]\*b\*c\*(1 + (b^2 - 2\*c\*(a + c))/(b\*Sqrt[b^2 - 4\*a\*c]))\*ArcTan[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) - b\*Sqrt[b^2 - 4\*a\*c]])]/((a - b + c)\*(a + b + c)\*Sqrt[b^2 - 2\*c\*(a + c) - b\*Sqrt[b^2 - 4\*a\*c]]) - (Sqrt[2]\*b\*c\*(1 - (b^2 - 2\*c\*(a + c))/(b\*Sqrt[b^2 - 4\*a\*c]))\*ArcTan[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*Tan[x/2])/(Sqrt[2]\*Sqrt[b^2 - 2\*c\*(a + c) + b\*Sqrt[b^2 - 4\*a\*c]])]/((a - b + c)\*(a + b + c)\*Sqrt[b^2 - 2\*c\*(a + c) + b\*Sqrt[b^2 - 4\*a\*c]]) + Cos[x]/(2\*(a + b + c)\*(1 - Sin[x])) - Cos[x]/(2\*(a - b + c)\*(1 + Sin[x]))

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 2648

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> -Simp[Cos[c + d\*x]/(d\*(b + a\*Sin[c + d\*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 3266

```
Int[cos[(d_) + (e_)*(x_)]^(m_)*((a_) + (b_)*sin[(d_) + (e_)*(x_)]^(n_) + (c_)*sin[(d_) + (e_)*(x_)]^(n2_))^(p_), x_Symbol] := Int[ExpandTrig[(1 - sin[d + e*x]^2)^(m/2)*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && IntegerQ[m/2] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[n, p]
```

### Rule 3292

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])/((a_) + (b_)*sin[(d_) + (e_)*(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2), x_Symbol] := Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \int \left( -\frac{1}{2(a+b+c)(-1+\sin(x))} + \frac{1}{2(a-b+c)(1+\sin(x))} + \frac{-b^2(1-\sin(x))}{(a-b+c)(a+b+c)} \right) dx \\ &= \frac{\int \frac{1}{1+\sin(x)} dx}{2(a-b+c)} - \frac{\int \frac{1}{-1+\sin(x)} dx}{2(a+b+c)} + \frac{\int \frac{-b^2(1-\frac{c(a+c)}{b^2})-bc \sin(x)}{a+b \sin(x)+c \sin^2(x)} dx}{(a-b+c)(a+b+c)} \\ &= \frac{\cos(x)}{2(a+b+c)(1-\sin(x))} - \frac{\cos(x)}{2(a-b+c)(1+\sin(x))} - \frac{\left( c \left( b + \frac{b^2-2c(a+c)}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{b-\sqrt{b^2-4ac} \sin(x)} dx}{(a-b+c)(a+b+c)} \\ &= \frac{\cos(x)}{2(a+b+c)(1-\sin(x))} - \frac{\cos(x)}{2(a-b+c)(1+\sin(x))} - \frac{\left( 2c \left( b + \frac{b^2-2c(a+c)}{\sqrt{b^2-4ac}} \right) \right) \operatorname{Subst}\left[ \int \frac{1}{b-\sqrt{b^2-4ac} \sin(x)} dx, x, \frac{b-\sqrt{b^2-4ac} \sin(x)}{2} \right]}{(a-b+c)(a+b+c)} \\ &= \frac{\cos(x)}{2(a+b+c)(1-\sin(x))} - \frac{\cos(x)}{2(a-b+c)(1+\sin(x))} + \frac{\left( 4c \left( b + \frac{b^2-2c(a+c)}{\sqrt{b^2-4ac}} \right) \right) \operatorname{Subst}\left[ \int \frac{1}{b-\sqrt{b^2-4ac} \sin(x)} dx, x, \frac{b-\sqrt{b^2-4ac} \sin(x)}{2} \right]}{(a-b+c)(a+b+c)} \\ &= -\frac{\sqrt{2} c \left( b + \frac{b^2-2c(a+c)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{2c + (b - \sqrt{b^2-4ac}) \tan\left(\frac{x}{2}\right)}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right)}{(a-b+c)(a+b+c)\sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} - \frac{\sqrt{2} bc \left( 1 - \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} \right)}{(a-b+c)(a+b+c)} \end{aligned}$$

**Mathematica [C]** time = 1.01, size = 407, normalized size = 1.26

$$\frac{c \left( b\sqrt{4ac - b^2} + 2ic(a+c) - ib^2 \right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) \left( b - i\sqrt{4ac - b^2} \right)}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right) + c \left( b\sqrt{4ac - b^2} - 2ic(a+c) + ib^2 \right) \tan^{-1} \left( \frac{2c + \tan\left(\frac{x}{2}\right) \left( b + i\sqrt{4ac - b^2} \right)}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{2ac - \frac{b^2}{2}} \left( a^2 + 2ac - b^2 + c^2 \right) \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} - \frac{\sqrt{2ac - \frac{b^2}{2}} \left( a^2 + 2ac - b^2 + c^2 \right) \sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}{\sqrt{2ac - \frac{b^2}{2}} \left( a^2 + 2ac - b^2 + c^2 \right) \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(a + b\*SIN[x] + c\*SIN[x]^2),x]

[Out] 
$$-\left(\frac{c\left(-I\right)b^2 + \left(2I\right)c\left(a + c\right) + b\sqrt{-b^2 + 4ac}}{\sqrt{2}\sqrt{b^2 - 2c\left(a + c\right) - Ib\sqrt{-b^2 + 4ac}}}\right)\operatorname{ArcTan}\left[\frac{2c + \left(b - I\sqrt{-b^2 + 4ac}\right)\tan\left[x/2\right]}{\sqrt{2}\sqrt{b^2 - 2c\left(a + c\right) - Ib\sqrt{-b^2 + 4ac}}}\right] - \left(\frac{c\left(Ib^2 - \left(2I\right)c\left(a + c\right) + b\sqrt{-b^2 + 4ac}\right)\operatorname{ArcTan}\left[\frac{2c + \left(b + I\sqrt{-b^2 + 4ac}\right)\tan\left[x/2\right]}{\sqrt{2}\sqrt{b^2 - 2c\left(a + c\right) + Ib\sqrt{-b^2 + 4ac}}}\right]}{\sqrt{-1/2b^2 + 2ac}\left(a^2 - b^2 + 2ac + c^2\right)\sqrt{b^2 - 2c\left(a + c\right) + Ib\sqrt{-b^2 + 4ac}}}\right) + \frac{\sin\left[x/2\right]}{\left(a + b + c\right)\left(\cos\left[x/2\right] - \sin\left[x/2\right]\right)} + \frac{\sin\left[x/2\right]}{\left(a - b + c\right)\left(\cos\left[x/2\right] + \sin\left[x/2\right]\right)}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.38, size = 1934, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x)

[Out] 
$$\begin{aligned} & -6/(a-b+c)/(a+b+c)a/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^3-2/(a-b+c)/(a+b+c)/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^2c-8/(a-b+c)/(a+b+c)a^2/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} c^2+10/(a-b+c)/(a+b+c)a/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^2c-8/(a-b+c)/(a+b+c)a/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^4+2/(a-b+c)/(a+b+c)/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{-2a\tan(1/2x)+(-4ac+b^2)^{1/2}-b}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^2c^2-6/(a-b+c)/(a+b+c)a/(4ac-b^2)/(4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \\ & \arctan\left(\frac{2a\tan(1/2x)+b+(-4ac+b^2)^{1/2}}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right) \\ & \left(\frac{-4ac+b^2}{4ca-2b^2+2b(-4ac+b^2)^{1/2}+4a^2}\right)^{1/2} b^2c+2/(a-b+c)/(a+b+c) \end{aligned}$$

$$\frac{(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot (-4ac+b^2)^{1/2} b^3 - 2/(a-b+c)/(a+b+c)/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot (-4ac+b^2)^{1/2} b^2 c^2 + 8/(a-b+c)/(a+b+c) a^2/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot c^2 - 10/(a-b+c)/(a+b+c) a/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot b^2 c + 8/(a-b+c)/(a+b+c) a/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot c^3 + 2/(a-b+c)/(a+b+c)/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot b^4 - 2/(a-b+c)/(a+b+c)/(4ac-b^2)/(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2} \arctan\left(\frac{2a \tan(1/2x)+b+(-4ac+b^2)^{1/2}}{(4c^2a-2b^2-2b(-4ac+b^2)^{1/2}+4a^2)^{1/2}}\right) \cdot b^2 c^2 - 2/(2a+2b+2c)/(\tan(1/2x)-1) - 2/(2a-2b+2c)/(\tan(1/2x)+1)$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Timed out

**mupad** [B] time = 27.59, size = 37118, normalized size = 114.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2\*(a + c\*sin(x)^2 + b\*sin(x))),x)

[Out] atan(((((-8\*a\*c^7 + b^8 + 24\*a^2\*c^6 + 24\*a^3\*c^5 + 8\*a^4\*c^4 + b^5\*(-(4\*a\*c - b^2)^3)^{1/2} - 2\*b^2\*c^6 + 3\*b^4\*c^4 - 3\*b^6\*c^2 - 18\*a\*b^2\*c^5 + 24\*a\*b^4\*c^3 + 3\*b\*c^4\*(-(4\*a\*c - b^2)^3)^{1/2} - 54\*a^2\*b^2\*c^4 + 33\*a^2\*b^4\*c^2 - 38\*a^3\*b^2\*c^3 - 3\*b^3\*c^2\*(-(4\*a\*c - b^2)^3)^{1/2} - 10\*a\*b^6\*c + 3\*a^2\*b\*c^2\*(-(4\*a\*c - b^2)^3)^{1/2} + 6\*a\*b\*c^3\*(-(4\*a\*c - b^2)^3)^{1/2} - 4\*a\*b^3\*c\*(-(4\*a\*c - b^2)^3)^{1/2}))/((2\*(3\*a^2\*b^8 - b^10 - 3\*a^4\*b^6 + a^6\*b^4 + 16\*a^2\*c^8 + 96\*a^3\*c^7 + 240\*a^4\*c^6 + 320\*a^5\*c^5 + 240\*a^6\*c^4 + 96\*a^7\*c^3 + 16\*a^8\*c^2 + b^4\*c^6 - 3\*b^6\*c^4 + 3\*b^8\*c^2 - 8\*a\*b^2\*c^7 + 30\*a\*b^4\*c^5 - 36\*a\*b^6\*c^3 - 36\*a^3\*b^6\*c + 30\*a^5\*b^4\*c - 8\*a^7\*b^2\*c - 96\*a^2\*b^2\*c^6 + 159\*a^2\*b^4\*c^4 - 82\*a^2\*b^6\*c^2 - 312\*a^3\*b^2\*c^5 + 260\*a^3\*b^4\*c^3 - 448\*a^4\*b^2\*c^4 + 159\*a^4\*b^4\*c^2 - 312\*a^5\*b^2\*c^3 - 96\*a^6\*b^2\*c^2 + 14\*a\*b^8\*c)))^{1/2} \* (((-8\*a\*c^7 + b^8 + 24\*a^2\*c^6 + 24\*a^3\*c^5 + 8\*a^4\*c^4 + b^5\*(-(4\*a\*c - b^2)^3)^{1/2} - 2\*b^2\*c^6 + 3\*b^4\*c^4 - 3\*b^6\*c^2 - 18\*a\*b^2\*c^5 + 24\*a\*b^4\*c^3 + 3\*b\*c^4\*(-(4\*a\*c - b^2)^3)^{1/2} - 54\*a^2\*b^2\*c^4 + 33\*a^2\*b^4\*c^2 - 38\*a^3\*b^2\*c^3 - 3\*b^3\*c^2\*(-(4\*a\*c - b^2)^3)^{1/2} - 10\*a\*b^6\*c + 3\*a^2\*b\*c^2\*(-(4\*a\*c - b^2)^3)^{1/2} + 6\*a\*b\*c^3\*(-(4\*a\*c - b^2)^3)^{1/2} - 4\*a\*b^3\*c\*(-(4\*a\*c - b^2)^3)^{1/2}))/((2\*(3\*a^2\*b^8 - b^10 - 3\*a^4\*b^6 + a^6\*b^4 + 16\*a^2\*c^8 + 96\*a^3\*c^7 + 240\*a^4\*c^6 + 320\*a^5\*c^5 + 240\*a^6\*c^4 + 96\*a^7\*c^3 + 16\*a^8\*c^2 + b^4\*c^6 - 3\*b^6\*c^4 + 3\*b^8\*c^2 - 8\*a\*b^2\*c^7 + 30\*a\*b^4\*c^5 - 36\*a\*b^6\*c^3 - 36\*a^3\*b^6\*c + 30\*a^5\*b^4\*c - 8\*a^7\*b^2\*c - 96\*a^2\*b^2\*c^6 + 159\*a^2\*b^4\*c^4 - 82\*a^2\*b^6\*c^2 - 312\*a^3\*b^2\*c^5 + 260\*a^3\*b^4\*c^3 - 448\*a^4\*b^2\*c^4 + 159\*a^4\*b^4\*c^2 - 312\*a^5\*b^2\*c^3 - 96\*a^6\*b^2\*c^2 + 14\*a\*b^8\*c)))^{1/2} \* (tan(x/2)\*(64\*a\*b^13 - 256\*a^3\*b^11 + 384\*a^5\*b^9 - 256\*a^7\*b^7 + 64\*a^9\*b^5 - 128\*a\*b^3\*c^10 + 576\*a\*b^5\*c^8 - 1024\*a\*b^7\*c^6 + 896\*a\*b^9\*c^4 - 384\*a\*b^11\*c^2 + 512\*a^2\*b\*c^11 - 896\*a





$$\begin{aligned}
& b^2c^4 + 5760a^8b^4c^2 - 9120a^9b^2c^3 - 1856a^{10}b^2c^2) + \tan(x/2) \cdot (32a^8b^{12} + 128a^8c^{12} - 96a^3b^{10} + 96a^5b^8 - 32a^7b^6 + 1088a^2c^{11} + 4096a^3c^{10} + 8960a^4c^9 + 12544a^5c^8 + 11648a^6c^7 + 7168a^7c^6 + 2816a^8c^5 + 640a^9c^4 + 64a^{10}c^3 - 544a^2b^2c^{10} + 992a^2b^4c^8 - 1024a^2b^6c^6 + 640a^2b^8c^4 - 224a^2b^{10}c^2 - 384a^2b^{10}c + 960a^4b^8c - 768a^6b^6c + 192a^8b^4c - 3968a^2b^2c^9 + 6144a^2b^4c^7 - 5120a^2b^6c^5 + 2240a^2b^8c^3 - 12672a^3b^2c^8 + 16032a^3b^4c^6 - 9760a^3b^6c^4 + 2400a^3b^8c^2 - 23168a^4b^2c^7 + 22720a^4b^4c^5 - 8960a^4b^6c^3 - 26560a^5b^2c^6 + 18720a^5b^4c^4 - 4032a^5b^6c^2 - 19584a^6b^2c^5 + 8832a^6b^4c^3 - 9088a^7b^2c^4 + 2144a^7b^4c^2 - 2432a^8b^2c^3 - 288a^9b^2c^2) - 160a^2b^3c^9 + 320a^2b^5c^7 - 320a^2b^7c^5 + 160a^2b^9c^3 + 384a^2b^2c^{10} + 1792a^3b^2c^9 + 96a^3b^9c + 4480a^4b^2c^8 + 6720a^5b^2c^7 - 96a^5b^7c + 6272a^6b^2c^6 + 3584a^7b^2c^5 + 32a^7b^5c + 1152a^8b^2c^4 + 160a^9b^2c^3 - 1504a^2b^3c^8 + 2208a^2b^5c^6 - 1440a^2b^7c^4 + 352a^2b^9c^2 - 5280a^3b^3c^7 + 5280a^3b^5c^5 - 1888a^3b^7c^3 - 9440a^4b^3c^6 + 5824a^4b^5c^4 - 864a^4b^7c^2 - 9440a^5b^3c^5 + 3072a^5b^5c^3 - 5280a^6b^3c^4 + 672a^6b^5c^2 - 1504a^7b^3c^3 - 160a^8b^3c^2 + 32a^2b^3c^{11} - 32a^2b^{11}c) \cdot i + (- (8a^8c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(- (4a^8c - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(- (4a^8c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(- (4a^8c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(- (4a^8c - b^2)^3)^{1/2} + 6a^2b^3c^3(- (4a^8c - b^2)^3)^{1/2} - 4a^2b^3c^3(- (4a^8c - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c))^{1/2} \cdot (\tan(x/2) \cdot (32a^8b^{12} + 128a^8c^{12} - 96a^3b^{10} + 96a^5b^8 - 32a^7b^6 + 1088a^2c^{11} + 4096a^3c^{10} + 8960a^4c^9 + 12544a^5c^8 + 11648a^6c^7 + 7168a^7c^6 + 2816a^8c^5 + 640a^9c^4 + 64a^{10}c^3 - 544a^2b^2c^{10} + 992a^2b^4c^8 - 1024a^2b^6c^6 + 640a^2b^8c^4 - 224a^2b^{10}c^2 - 384a^2b^{10}c + 960a^4b^8c - 768a^6b^6c + 192a^8b^4c - 3968a^2b^2c^9 + 6144a^2b^4c^7 - 5120a^2b^6c^5 + 2240a^2b^8c^3 - 12672a^3b^2c^8 + 16032a^3b^4c^6 - 9760a^3b^6c^4 + 2400a^3b^8c^2 - 23168a^4b^2c^7 + 22720a^4b^4c^5 - 8960a^4b^6c^3 - 26560a^5b^2c^6 + 18720a^5b^4c^4 - 4032a^5b^6c^2 - 19584a^6b^2c^5 + 8832a^6b^4c^3 - 9088a^7b^2c^4 + 2144a^7b^4c^2 - 2432a^8b^2c^3 - 288a^9b^2c^2) - (- (8a^8c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(- (4a^8c - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(- (4a^8c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(- (4a^8c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(- (4a^8c - b^2)^3)^{1/2} + 6a^2b^3c^3(- (4a^8c - b^2)^3)^{1/2} - 4a^2b^3c^3(- (4a^8c - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c))^{1/2} \cdot (\tan(x/2) \cdot (64a^8b^{13} - 256a^3b^{11} + 384a^5b^9 - 256a^7b^7 + 64a^9b^5 - 128a^2b^3c^{10} + 576a^2b^5c^8 - 1024a^2b^7c^6 + 896a^2b^9c^4 - 384a^2b^{11}c^2 + 512a^2b^2c^{11} - 896a^2b^{11}c + 4608a^3b^2c^{10} + 18432a^4b^2c^9 + 3072a^4b^9c + 43008a^5b^2c^8 + 64512a^6b^2c^7 - 3840a^6b^7c + 64512a^7b^2c^6 + 43008a^8b^2c^5 + 2048a^8b^5c + 18432a^9b^2c^4 + 4608a^{10}b^2c^3 - 384a^{10}b^3c + 512a^{11}b^2c^2 - 3456a^2b^3c^9 + 8192a^2b^5c^7 - 8960a^2b^7c^5 + 4608a^2b^9c^3 - 20992a^3b^3c^8 + 34048a^3b^5c^6 - 23808a^3b^7c^4 + 6400a^3b^9c^2 - 60928a^4b^3c^7 + 6
\end{aligned}$$

$$\begin{aligned}
& 7584a^4b^5c^5 - 28160a^4b^7c^3 - 102144a^5b^3c^6 + 73600a^5b^5c^4 - 15872a^5b^7c^2 - 105728a^6b^3c^5 + 45056a^6b^5c^3 - 68096a^7b^3c^4 + 14592a^7b^5c^2 - 26112a^8b^3c^3 - 5248a^9b^3c^2) - ((8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18ab^2c^5 + 24ab^4c^3 + 3b^2c^4(-4ac - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(-4ac - b^2)^3)^{1/2} - 10ab^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 6ab^3c^3(-4ac - b^2)^3)^{1/2} - 4ab^3c^3(-4ac - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8ab^2c^7 + 30ab^4c^5 - 36ab^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14ab^8c))^{1/2} * (\tan(x/2) * (256a^{14}c - 96ab^{14} + 544a^3b^{12} - 1280a^5b^{10} + 1600a^7b^8 - 1120a^9b^6 + 416a^{11}b^4 - 64a^{13}b^2 + 512a^2c^{13} + 5888a^3c^{12} + 30976a^4c^{11} + 98560a^5c^{10} + 211200a^6c^9 + 321024a^7c^8 + 354816a^8c^7 + 287232a^9c^6 + 168960a^{10}c^5 + 70400a^{11}c^4 + 19712a^{12}c^3 + 3328a^{13}c^2 - 128ab^2c^{12} + 736ab^4c^{10} - 1760ab^6c^8 + 2240ab^8c^6 - 1600ab^{10}c^4 + 608ab^{12}c^2 + 1536a^2b^{12}c - 7616a^4b^{10}c + 15360a^6b^8c - 16000a^8b^6c + 8960a^{10}b^4c - 2496a^{12}b^2c - 4416a^2b^2c^{11} + 14080a^2b^4c^9 - 22400a^2b^6c^7 + 19200a^2b^8c^5 - 8512a^2b^{10}c^3 - 35904a^3b^2c^{10} + 84000a^3b^4c^8 - 96000a^3b^6c^6 + 54720a^3b^8c^4 - 13248a^3b^{10}c^2 - 145600a^4b^2c^9 + 256000a^4b^4c^7 - 206720a^4b^6c^5 + 72960a^4b^8c^3 - 360000a^5b^2c^8 + 468160a^5b^4c^6 - 254400a^5b^6c^4 + 48960a^5b^8c^2 - 590976a^6b^2c^7 + 548352a^6b^4c^5 - 184960a^6b^6c^3 - 669312a^7b^2c^6 + 418880a^7b^4c^4 - 76800a^7b^6c^2 - 528768a^8b^2c^5 + 204800a^8b^4c^3 - 288000a^9b^2c^4 + 60000a^9b^4c^2 - 104000a^{10}b^2c^3 - 22848a^{11}b^2c^2) - 32a^2b^{13} + 160a^4b^{11} - 320a^6b^9 + 320a^8b^7 - 160a^{10}b^5 + 32a^{12}b^3 - 32ab^3c^{11} + 160ab^5c^9 - 320ab^7c^7 + 320ab^9c^5 - 160ab^{11}c^3 + 128a^2b^3c^{12} + 1152a^3b^3c^{11} + 288a^3b^{11}c + 4480a^4b^3c^{10} + 9600a^5b^3c^9 - 1600a^5b^9c + 11520a^6b^3c^8 + 5376a^7b^3c^7 + 2880a^7b^7c - 5376a^8b^3c^6 - 11520a^9b^3c^5 - 2400a^9b^5c - 9600a^{10}b^3c^4 - 4480a^{11}b^3c^3 + 928a^{11}b^3c - 1152a^{12}b^3c^2 - 928a^2b^3c^{10} + 2400a^2b^5c^8 - 2880a^2b^7c^6 + 1600a^2b^9c^4 - 288a^2b^{11}c^2 - 5600a^3b^3c^9 + 9600a^3b^5c^7 - 6720a^3b^7c^5 + 1280a^3b^9c^3 - 15200a^4b^3c^8 + 16000a^4b^5c^6 - 4160a^4b^7c^4 - 1280a^4b^9c^2 - 20800a^5b^3c^7 + 8640a^5b^5c^5 + 4160a^5b^7c^3 - 10304a^6b^3c^6 - 8640a^6b^5c^4 + 6720a^6b^7c^2 + 10304a^7b^3c^5 - 16000a^7b^5c^3 + 20800a^8b^3c^4 - 9600a^8b^5c^2 + 15200a^9b^3c^3 + 5600a^{10}b^3c^2 + 32ab^{13}c - 128a^{13}b^3c) + 32a^2b^{12} - 128a^4b^{10} + 192a^6b^8 - 128a^8b^6 + 32a^{10}b^4 + 128a^2c^{12} + 1280a^3c^{11} + 5760a^4c^{10} + 15360a^5c^9 + 26880a^6c^8 + 32256a^7c^7 + 26880a^8c^6 + 15360a^9c^5 + 5760a^{10}c^4 + 1280a^{11}c^3 + 128a^{12}c^2 - 32ab^2c^{11} + 128ab^4c^9 - 192ab^6c^7 + 128ab^8c^5 - 32ab^{10}c^3 - 416a^3b^{10}c + 1408a^5b^8c - 1728a^7b^6c + 896a^9b^4c - 160a^{11}b^2c - 832a^2b^2c^{10} + 1824a^2b^4c^8 - 1792a^2b^6c^6 + 832a^2b^8c^4 - 192a^2b^{10}c^2 - 5664a^3b^2c^9 + 8960a^3b^4c^7 - 6464a^3b^6c^5 + 2304a^3b^8c^3 - 19200a^4b^2c^8 + 22656a^4b^4c^6 - 11904a^4b^6c^4 + 2816a^4b^8c^2 - 38976a^5b^2c^7 + 33792a^5b^4c^5 - 12096a^5b^6c^3 - 51072a^6b^2c^6 + 31168a^6b^4c^4 - 6656a^6b^6c^2 - 44352a^7b^2c^5 + 17664a^7b^4c^3 - 25344a^8b^2c^4 + 5760a^8b^4c^2 - 9120a^9b^2c^3 - 1856a^{10}b^2c^2) - 160ab^3c^9 + 320ab^5c^7 - 320ab^7c^5 + 160ab^9c^3 + 384a^2b^3c^{10} + 1792a^3b^3c^9 + 96a^3b^9c + 4480a^4b^3c^8 + 6720a^5b^3c^7 - 96a^5b^7c + 6272a^6b^3c^6 + 3584a^7b^3c^5 + 32a^7b^5c + 1152a^8b^3c^4 + 160a^9b^3c^3 - 1504a^2b^3c^8 + 2208a^2b^5c^6 - 1440a^2b^7c^4 + 352a^2b^9c^2 - 5280a^3b^3c^7 + 5280a^3b^5c^5 - 5280a^3b^7c^3 + 5280a^3b^9c^1)
\end{aligned}$$

$$\begin{aligned}
& ^5c^5 - 1888a^3b^7c^3 - 9440a^4b^3c^6 + 5824a^4b^5c^4 - 864a^4b^7c^2 - 9440a^5b^3c^5 + 3072a^5b^5c^3 - 5280a^6b^3c^4 + 672a^6b^5c^2 - 1504a^7b^3c^3 - 160a^8b^3c^2 + 32a^8b^5c^1 - 32a^8b^7c^1) \cdot i \\
& ) / ((- (8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(- (4a^3c - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(- (4a^3c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(- (4a^3c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(- (4a^3c - b^2)^3)^{1/2} + 6a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2} - 4a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c))^{1/2} * ((- (8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(- (4a^3c - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(- (4a^3c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(- (4a^3c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(- (4a^3c - b^2)^3)^{1/2} + 6a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2} - 4a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c))^{1/2} * (\tan(x/2) * (64a^2b^{13} - 256a^3b^{11} + 384a^5b^9 - 256a^7b^7 + 64a^9b^5 - 128a^2b^3c^{10} + 576a^2b^5c^8 - 1024a^2b^7c^6 + 896a^2b^9c^4 - 384a^2b^{11}c^2 + 512a^2b^{11}c - 896a^2b^{11}c + 4608a^3b^9c^{10} + 18432a^4b^9c^9 + 3072a^4b^9c^8 + 43008a^5b^9c^8 + 64512a^6b^9c^7 - 3840a^6b^7c^6 + 64512a^7b^9c^6 + 43008a^8b^9c^5 + 2048a^8b^5c^6 + 18432a^9b^9c^4 + 4608a^{10}b^9c^3 - 384a^{10}b^3c^6 + 512a^{11}b^9c^2 - 3456a^2b^3c^9 + 8192a^2b^5c^7 - 8960a^2b^7c^5 + 4608a^2b^9c^3 - 20992a^3b^3c^8 + 34048a^3b^5c^6 - 23808a^3b^7c^4 + 6400a^3b^9c^2 - 60928a^4b^3c^7 + 67584a^4b^5c^5 - 28160a^4b^7c^3 - 102144a^5b^3c^6 + 73600a^5b^5c^4 - 15872a^5b^7c^2 - 105728a^6b^3c^5 + 45056a^6b^5c^3 - 68096a^7b^3c^4 + 14592a^7b^5c^2 - 26112a^8b^3c^3 - 5248a^9b^3c^2) + (- (8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(- (4a^3c - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(- (4a^3c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(- (4a^3c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(- (4a^3c - b^2)^3)^{1/2} + 6a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2} - 4a^2b^3c^3(- (4a^3c - b^2)^3)^{1/2}) / (2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c))^{1/2} * (\tan(x/2) * (256a^{14}c - 96a^2b^{14} + 544a^3b^{12} - 1280a^5b^{10} + 1600a^7b^8 - 1120a^9b^6 + 416a^{11}b^4 - 64a^{13}b^2 + 512a^2c^{13} + 5888a^3c^{12} + 30976a^4c^{11} + 98560a^5c^{10} + 211200a^6c^9 + 321024a^7c^8 + 354816a^8c^7 + 287232a^9c^6 + 168960a^{10}c^5 + 70400a^{11}c^4 + 19712a^{12}c^3 + 3328a^{13}c^2 - 128a^2b^2c^{12} + 736a^2b^4c^{10} - 1760a^2b^6c^8 + 2240a^2b^8c^6 - 1600a^2b^{10}c^4 + 608a^2b^{12}c^2 + 1536a^2b^{12}c - 7616a^4b^{10}c + 15360a^6b^8c - 16000a^8b^6c + 8960a^{10}b^4c - 2496a^{12}b^2c - 4416a^2b^2c^{11} + 14080a^2b^4c^9 - 22400a^2b^6c^7 + 19200a^2b^8c^5 - 8512a^2b^{10}c^3 - 35904a^3b^2c^{10} + 84000a^3b^4c^8 - 96000a^3b^6c^6 + 54720a^3b^8c^4 - 13248a^3b^{10}c^2 - 145600a^4b^2c^9 + 256000a^4b^4c^7 - 206720a^4b^6c^5 + 72960a^4b^8c^3 - 360000a^5b^2c^8 + 4681
\end{aligned}$$

$$\begin{aligned}
& 60*a^5*b^4*c^6 - 254400*a^5*b^6*c^4 + 48960*a^5*b^8*c^2 - 590976*a^6*b^2*c^7 + 548352*a^6*b^4*c^5 - 184960*a^6*b^6*c^3 - 669312*a^7*b^2*c^6 + 418880*a^7*b^4*c^4 - 76800*a^7*b^6*c^2 - 528768*a^8*b^2*c^5 + 204800*a^8*b^4*c^3 - 288000*a^9*b^2*c^4 + 60000*a^9*b^4*c^2 - 104000*a^10*b^2*c^3 - 22848*a^11*b^2*c^2) - 32*a^2*b^13 + 160*a^4*b^11 - 320*a^6*b^9 + 320*a^8*b^7 - 160*a^10*b^5 + 32*a^12*b^3 - 32*a*b^3*c^11 + 160*a*b^5*c^9 - 320*a*b^7*c^7 + 320*a*b^9*c^5 - 160*a*b^11*c^3 + 128*a^2*b*c^12 + 1152*a^3*b*c^11 + 288*a^3*b^11*c + 4480*a^4*b*c^10 + 9600*a^5*b*c^9 - 1600*a^5*b^9*c + 11520*a^6*b*c^8 + 5376*a^7*b*c^7 + 2880*a^7*b^7*c - 5376*a^8*b*c^6 - 11520*a^9*b*c^5 - 2400*a^9*b^5*c - 9600*a^10*b*c^4 - 4480*a^11*b*c^3 + 928*a^11*b^3*c - 1152*a^12*b*c^2 - 928*a^2*b^3*c^10 + 2400*a^2*b^5*c^8 - 2880*a^2*b^7*c^6 + 1600*a^2*b^9*c^4 - 288*a^2*b^11*c^2 - 5600*a^3*b^3*c^9 + 9600*a^3*b^5*c^7 - 6720*a^3*b^7*c^5 + 1280*a^3*b^9*c^3 - 15200*a^4*b^3*c^8 + 16000*a^4*b^5*c^6 - 4160*a^4*b^7*c^4 - 1280*a^4*b^9*c^2 - 20800*a^5*b^3*c^7 + 8640*a^5*b^5*c^5 + 4160*a^5*b^7*c^3 - 10304*a^6*b^3*c^6 - 8640*a^6*b^5*c^4 + 6720*a^6*b^7*c^2 + 10304*a^7*b^3*c^5 - 16000*a^7*b^5*c^3 + 20800*a^8*b^3*c^4 - 9600*a^8*b^5*c^2 + 15200*a^9*b^3*c^3 + 5600*a^10*b^3*c^2 + 32*a*b^13*c - 128*a^13*b*c) + 32*a^2*b^12 - 128*a^4*b^10 + 192*a^6*b^8 - 128*a^8*b^6 + 32*a^10*b^4 + 128*a^2*c^12 + 1280*a^3*c^11 + 5760*a^4*c^10 + 15360*a^5*c^9 + 26880*a^6*c^8 + 32256*a^7*c^7 + 26880*a^8*c^6 + 15360*a^9*c^5 + 5760*a^10*c^4 + 1280*a^11*c^3 + 128*a^12*c^2 - 32*a*b^2*c^11 + 128*a*b^4*c^9 - 192*a*b^6*c^7 + 128*a*b^8*c^5 - 32*a*b^10*c^3 - 416*a^3*b^10*c + 1408*a^5*b^8*c - 1728*a^7*b^6*c + 896*a^9*b^4*c - 160*a^11*b^2*c - 832*a^2*b^2*c^10 + 1824*a^2*b^4*c^8 - 1792*a^2*b^6*c^6 + 832*a^2*b^8*c^4 - 192*a^2*b^10*c^2 - 5664*a^3*b^2*c^9 + 8960*a^3*b^4*c^7 - 6464*a^3*b^6*c^5 + 2304*a^3*b^8*c^3 - 19200*a^4*b^2*c^8 + 22656*a^4*b^4*c^6 - 11904*a^4*b^6*c^4 + 2816*a^4*b^8*c^2 - 38976*a^5*b^2*c^7 + 33792*a^5*b^4*c^5 - 12096*a^5*b^6*c^3 - 51072*a^6*b^2*c^6 + 31168*a^6*b^4*c^4 - 6656*a^6*b^6*c^2 - 44352*a^7*b^2*c^5 + 17664*a^7*b^4*c^3 - 25344*a^8*b^2*c^4 + 5760*a^8*b^4*c^2 - 9120*a^9*b^2*c^3 - 1856*a^10*b^2*c^2) + \tan(x/2) * (32*a*b^12 + 128*a*c^12 - 96*a^3*b^10 + 96*a^5*b^8 - 32*a^7*b^6 + 1088*a^2*c^11 + 4096*a^3*c^10 + 8960*a^4*c^9 + 12544*a^5*c^8 + 11648*a^6*c^7 + 7168*a^7*c^6 + 2816*a^8*c^5 + 640*a^9*c^4 + 64*a^10*c^3 - 544*a*b^2*c^10 + 992*a*b^4*c^8 - 1024*a*b^6*c^6 + 640*a*b^8*c^4 - 224*a*b^10*c^2 - 384*a^2*b^10*c + 960*a^4*b^8*c - 768*a^6*b^6*c + 192*a^8*b^4*c - 3968*a^2*b^2*c^9 + 6144*a^2*b^4*c^7 - 5120*a^2*b^6*c^5 + 2240*a^2*b^8*c^3 - 12672*a^3*b^2*c^8 + 16032*a^3*b^4*c^6 - 9760*a^3*b^6*c^4 + 2400*a^3*b^8*c^2 - 23168*a^4*b^2*c^7 + 22720*a^4*b^4*c^5 - 8960*a^4*b^6*c^3 - 26560*a^5*b^2*c^6 + 18720*a^5*b^4*c^4 - 4032*a^5*b^6*c^2 - 19584*a^6*b^2*c^5 + 8832*a^6*b^4*c^3 - 9088*a^7*b^2*c^4 + 2144*a^7*b^4*c^2 - 2432*a^8*b^2*c^3 - 288*a^9*b^2*c^2) - 160*a*b^3*c^9 + 320*a*b^5*c^7 - 320*a*b^7*c^5 + 160*a*b^9*c^3 + 384*a^2*b*c^10 + 1792*a^3*b*c^9 + 96*a^3*b^9*c + 4480*a^4*b*c^8 + 6720*a^5*b*c^7 - 96*a^5*b^7*c + 6272*a^6*b*c^6 + 3584*a^7*b*c^5 + 32*a^7*b^5*c + 1152*a^8*b*c^4 + 160*a^9*b*c^3 - 1504*a^2*b^3*c^8 + 2208*a^2*b^5*c^6 - 1440*a^2*b^7*c^4 + 352*a^2*b^9*c^2 - 5280*a^3*b^3*c^7 + 5280*a^3*b^5*c^5 - 1888*a^3*b^7*c^3 - 9440*a^4*b^3*c^6 + 5824*a^4*b^5*c^4 - 864*a^4*b^7*c^2 - 9440*a^5*b^3*c^5 + 3072*a^5*b^5*c^3 - 5280*a^6*b^3*c^4 + 672*a^6*b^5*c^2 - 1504*a^7*b^3*c^3 - 160*a^8*b^3*c^2 + 32*a*b*c^11 - 32*a*b^11*c) - 2*\tan(x/2)*(192*a*b^5*c^6 - 192*a*b^3*c^8 - 64*a*b^7*c^4 + 384*a^2*b*c^9 + 960*a^3*b*c^8 + 1280*a^4*b*c^7 + 960*a^5*b*c^6 + 384*a^6*b*c^5 + 64*a^7*b*c^4 - 768*a^2*b^3*c^7 + 384*a^2*b^5*c^5 - 1152*a^3*b^3*c^6 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5 - 192*a^5*b^3*c^4 + 64*a*b*c^10) - ((-8*a*c^7 + b^8 + 24*a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 + b^5)*(- (4*a*c - b^2)^3)^(1/2) - 2*b^2*c^6 + 3*b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 + 3*b*c^4*(- (4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 - 3*b^3*c^2*(- (4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c + 3*a^2*b*c^2*(- (4*a*c - b^2)^3)^(1/2) + 6*a*b*c^3*(- (4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c*(- (4*a*c - b^2)^3)^(1/2))/(2*(3*a^2*b^8 - b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c
\end{aligned}$$

$$\begin{aligned}
& - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^8b^8c))^{1/2}(\tan(x/2)(32a^5b^{12} + 128a^6c^{12} - 96a^3b^{10} + 96a^5b^8 - 32a^7b^6 + 1088a^2c^{11} + 4096a^3c^{10} + 8960a^4c^9 + 12544a^5c^8 + 11648a^6c^7 + 7168a^7c^6 + 2816a^8c^5 + 640a^9c^4 + 64a^{10}c^3 - 544a^2b^2c^{10} + 992a^2b^4c^8 - 1024a^2b^6c^6 + 640a^2b^8c^4 - 224a^2b^{10}c^2 - 384a^2b^{10}c + 960a^4b^8c - 768a^6b^6c + 192a^8b^4c - 3968a^2b^2c^9 + 6144a^2b^4c^7 - 5120a^2b^6c^5 + 2240a^2b^8c^3 - 12672a^3b^2c^8 + 16032a^3b^4c^6 - 9760a^3b^6c^4 + 2400a^3b^8c^2 - 23168a^4b^2c^7 + 22720a^4b^4c^5 - 8960a^4b^6c^3 - 26560a^5b^2c^6 + 18720a^5b^4c^4 - 4032a^5b^6c^2 - 19584a^6b^2c^5 + 8832a^6b^4c^3 - 9088a^7b^2c^4 + 2144a^7b^4c^2 - 2432a^8b^2c^3 - 288a^9b^2c^2) - ((8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(-4ac - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(-4ac - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3(-4ac - b^2)^3)^{1/2} - 4a^2b^3c^2(-4ac - b^2)^3)^{1/2})/(2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^8b^8c))^{1/2}(\tan(x/2)(64a^5b^{13} - 256a^3b^{11} + 384a^5b^9 - 256a^7b^7 + 64a^9b^5 - 128a^2b^3c^{10} + 576a^2b^5c^8 - 1024a^2b^7c^6 + 896a^2b^9c^4 - 384a^2b^{11}c^2 + 512a^2b^2c^{11} - 896a^2b^{11}c + 4608a^3b^2c^{10} + 18432a^4b^2c^9 + 3072a^4b^4c^9 + 43008a^5b^2c^8 + 64512a^6b^2c^7 - 3840a^6b^4c^7 + 64512a^7b^2c^6 + 43008a^8b^2c^5 + 2048a^8b^4c^5 + 18432a^9b^2c^4 + 4608a^{10}b^2c^3 - 384a^{10}b^3c + 512a^{11}b^3c^2 - 3456a^2b^3c^9 + 8192a^2b^5c^7 - 8960a^2b^7c^5 + 4608a^2b^9c^3 - 20992a^3b^3c^8 + 34048a^3b^5c^6 - 23808a^3b^7c^4 + 6400a^3b^9c^2 - 60928a^4b^3c^7 + 67584a^4b^5c^5 - 28160a^4b^7c^3 - 102144a^5b^3c^6 + 73600a^5b^5c^4 - 15872a^5b^7c^2 - 105728a^6b^3c^5 + 45056a^6b^5c^3 - 68096a^7b^3c^4 + 14592a^7b^5c^2 - 26112a^8b^3c^3 - 5248a^9b^3c^2) - ((8a^7c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^2c^4(-4ac - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2(-4ac - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^3(-4ac - b^2)^3)^{1/2} - 4a^2b^3c^2(-4ac - b^2)^3)^{1/2})/(2(3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^8b^8c))^{1/2}(\tan(x/2)(256a^{14}c - 96a^2b^{14} + 544a^3b^{12} - 1280a^5b^{10} + 1600a^7b^8 - 1120a^9b^6 + 416a^{11}b^4 - 64a^{13}b^2 + 512a^2c^{13} + 5888a^3c^{12} + 30976a^4c^{11} + 98560a^5c^{10} + 211200a^6c^9 + 321024a^7c^8 + 354816a^8c^7 + 287232a^9c^6 + 168960a^{10}c^5 + 70400a^{11}c^4 + 19712a^{12}c^3 + 3328a^{13}c^2 - 128a^2b^2c^{12} + 736a^2b^4c^{10} - 1760a^2b^6c^8 + 2240a^2b^8c^6 - 1600a^2b^{10}c^4 + 608a^2b^{12}c^2 + 1536a^2b^{12}c - 7616a^4b^{10}c + 15360a^6b^8c - 16000a^8b^6c + 8960a^{10}b^4c - 2496a^{12}b^2c - 4416a^2b^2c^{11} + 14080a^2b^4c^9 - 22400a^2b^6c^7 + 19200a^2b^8c^5 - 8512a^2b^{10}c^3 - 35904a^3b^2c^{10} + 84000a^3b^4c^8 - 96000a^3b^6c^6 + 54720a^3b^8c^4 - 13248a^3b^{10}c^2 - 145600a^4b^2c^9 + 256000a^4b^4c^7 - 206720a^4b^6c^5 + 72960a^4b^8c^3 - 360000a^5b^2c^8 + 468160a^5b^4c^6 - 254400a^5b^6c^4 + 48960a^5b^8c^2 - 590976a^6b^2c^7 + 548352a^6b^4c^5 - 184960a^6b^6c^3 - 669312a^7b^2c^6 +
\end{aligned}$$

$$\begin{aligned}
& 418880a^7b^4c^4 - 76800a^7b^6c^2 - 528768a^8b^2c^5 + 204800a^8b^4c^3 - 288000a^9b^2c^4 + 60000a^9b^4c^2 - 104000a^{10}b^2c^3 - 22848a^{11}b^2c^2) - 32a^2b^{13} + 160a^4b^{11} - 320a^6b^9 + 320a^8b^7 - 160a^{10}b^5 + 32a^{12}b^3 - 32a^3b^3c^{11} + 160a^5b^5c^9 - 320a^7b^7c^7 + 320a^9b^9c^5 - 160a^{11}b^{11}c^3 + 128a^{12}b^3c^{12} + 1152a^3b^3c^{11} + 288a^3b^{11}c + 4480a^4b^3c^{10} + 9600a^5b^3c^9 - 1600a^5b^9c + 11520a^6b^3c^8 + 5376a^7b^3c^7 + 2880a^7b^7c - 5376a^8b^3c^6 - 11520a^9b^3c^5 - 2400a^9b^5c - 9600a^{10}b^3c^4 - 4480a^{11}b^3c^3 + 928a^{11}b^3c - 1152a^{12}b^3c^2 - 928a^{12}b^3c^{10} + 2400a^{12}b^5c^8 - 2880a^{12}b^7c^6 + 1600a^{12}b^9c^4 - 288a^{12}b^{11}c^2 - 5600a^3b^3c^9 + 9600a^3b^5c^7 - 6720a^3b^7c^5 + 1280a^3b^9c^3 - 15200a^4b^3c^8 + 16000a^4b^5c^6 - 4160a^4b^7c^4 - 1280a^4b^9c^2 - 20800a^5b^3c^7 + 8640a^5b^5c^5 + 4160a^5b^7c^3 - 10304a^6b^3c^6 - 8640a^6b^5c^4 + 6720a^6b^7c^2 + 10304a^7b^3c^5 - 16000a^7b^5c^3 + 20800a^8b^3c^4 - 9600a^8b^5c^2 + 15200a^9b^3c^3 + 5600a^{10}b^3c^2 + 32a^3b^{13}c - 128a^{13}b^3c) + 32a^2b^{12} - 128a^4b^{10} + 192a^6b^8 - 128a^8b^6 + 32a^{10}b^4 + 128a^2c^{12} + 1280a^3c^{11} + 5760a^4c^{10} + 15360a^5c^9 + 26880a^6c^8 + 32256a^7c^7 + 26880a^8c^6 + 15360a^9c^5 + 5760a^{10}c^4 + 1280a^{11}c^3 + 128a^{12}c^2 - 32a^3b^2c^{11} + 128a^5b^4c^9 - 192a^7b^6c^7 + 128a^9b^8c^5 - 32a^{10}b^{10}c^3 - 416a^3b^{10}c + 1408a^5b^8c - 1728a^7b^6c + 896a^9b^4c - 160a^{11}b^2c - 832a^2b^2c^{10} + 1824a^2b^4c^8 - 1792a^2b^6c^6 + 832a^2b^8c^4 - 192a^2b^{10}c^2 - 5664a^3b^2c^9 + 8960a^3b^4c^7 - 6464a^3b^6c^5 + 2304a^3b^8c^3 - 19200a^4b^2c^8 + 22656a^4b^4c^6 - 11904a^4b^6c^4 + 2816a^4b^8c^2 - 38976a^5b^2c^7 + 33792a^5b^4c^5 - 12096a^5b^6c^3 - 51072a^6b^2c^6 + 31168a^6b^4c^4 - 6656a^6b^6c^2 - 44352a^7b^2c^5 + 17664a^7b^4c^3 - 25344a^8b^2c^4 + 5760a^8b^4c^2 - 9120a^9b^2c^3 - 1856a^{10}b^2c^2) - 160a^3b^3c^9 + 320a^5b^5c^7 - 320a^7b^7c^5 + 160a^9b^9c^3 + 384a^2b^3c^{10} + 1792a^3b^3c^9 + 96a^3b^9c + 4480a^4b^3c^8 + 6720a^5b^3c^7 - 96a^5b^7c + 6272a^6b^3c^6 + 3584a^7b^3c^5 + 32a^7b^5c + 1152a^8b^3c^4 + 160a^9b^3c^3 - 1504a^2b^3c^8 + 2208a^2b^5c^6 - 1440a^2b^7c^4 + 352a^2b^9c^2 - 5280a^3b^3c^7 + 5280a^3b^5c^5 - 1888a^3b^7c^3 - 9440a^4b^3c^6 + 5824a^4b^5c^4 - 864a^4b^7c^2 - 9440a^5b^3c^5 + 3072a^5b^5c^3 - 5280a^6b^3c^4 + 672a^6b^5c^2 - 1504a^7b^3c^3 - 160a^8b^3c^2 + 32a^3b^3c^{11} - 32a^3b^{11}c) + 64a^3c^{11} + 448a^2c^{10} + 1344a^3c^9 + 2240a^4c^8 + 2240a^5c^7 + 1344a^6c^6 + 448a^7c^5 + 64a^8c^4 - 256a^3b^2c^9 + 384a^4b^4c^7 - 256a^5b^6c^5 + 64a^6b^8c^3 - 1344a^2b^2c^8 + 1344a^2b^4c^6 - 448a^2b^6c^4 - 2880a^3b^2c^7 + 1728a^3b^4c^5 - 192a^3b^6c^3 - 3200a^4b^2c^6 + 960a^4b^4c^4 - 1920a^5b^2c^5 + 192a^5b^4c^3 - 576a^6b^2c^4 - 64a^7b^2c^3)) * (-(8a^3c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 + b^5 * (-(4a^3c - b^2)^3)^{1/2}) - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 + 3b^3c^4 * (-(4a^3c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 - 3b^3c^2 * (-(4a^3c - b^2)^3)^{1/2} - 10a^2b^6c + 3a^2b^3c^2 * (-(4a^3c - b^2)^3)^{1/2} + 6a^2b^3c^3 * (-(4a^3c - b^2)^3)^{1/2} - 4a^2b^3c^3 * (-(4a^3c - b^2)^3)^{1/2}) / (2 * (3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c)))^{1/2} * 2i + \operatorname{atan}\left(\frac{-(8a^3c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 - b^5 * (-(4a^3c - b^2)^3)^{1/2}) - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 - 3b^3c^4 * (-(4a^3c - b^2)^3)^{1/2} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + 3b^3c^2 * (-(4a^3c - b^2)^3)^{1/2} - 10a^2b^6c - 3a^2b^3c^2 * (-(4a^3c - b^2)^3)^{1/2} - 6a^2b^3c^3 * (-(4a^3c - b^2)^3)^{1/2} + 4a^2b^3c^3 * (-(4a^3c - b^2)^3)^{1/2}}{2 * (3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8}
\end{aligned}$$

$$\begin{aligned}
& *a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8* \\
& a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 312*a^3*b^2 \\
& *c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a^5*b^2*c^ \\
& 3 - 96*a^6*b^2*c^2 + 14*a*b^8*c))^{(1/2)}*((-(8*a*c^7 + b^8 + 24*a^2*c^6 + 2 \\
& 4*a^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2*c^6 + 3*b^4*c^ \\
& 4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a* \\
& b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(3* \\
& a^2*b^8 - b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^ \\
& 6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c \\
& ^4 + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + \\
& 30*a^5*b^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6 \\
& *c^2 - 312*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^ \\
& 2 - 312*a^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14*a*b^8*c))^{(1/2)}*(\tan(x/2)*(64*a* \\
& b^13 - 256*a^3*b^11 + 384*a^5*b^9 - 256*a^7*b^7 + 64*a^9*b^5 - 128*a*b^3*c^ \\
& 10 + 576*a*b^5*c^8 - 1024*a*b^7*c^6 + 896*a*b^9*c^4 - 384*a*b^11*c^2 + 512* \\
& a^2*b*c^11 - 896*a^2*b^11*c + 4608*a^3*b*c^10 + 18432*a^4*b*c^9 + 3072*a^4* \\
& b^9*c + 43008*a^5*b*c^8 + 64512*a^6*b*c^7 - 3840*a^6*b^7*c + 64512*a^7*b*c^ \\
& 6 + 43008*a^8*b*c^5 + 2048*a^8*b^5*c + 18432*a^9*b*c^4 + 4608*a^10*b*c^3 - \\
& 384*a^10*b^3*c + 512*a^11*b*c^2 - 3456*a^2*b^3*c^9 + 8192*a^2*b^5*c^7 - 896 \\
& 0*a^2*b^7*c^5 + 4608*a^2*b^9*c^3 - 20992*a^3*b^3*c^8 + 34048*a^3*b^5*c^6 - \\
& 23808*a^3*b^7*c^4 + 6400*a^3*b^9*c^2 - 60928*a^4*b^3*c^7 + 67584*a^4*b^5*c^ \\
& 5 - 28160*a^4*b^7*c^3 - 102144*a^5*b^3*c^6 + 73600*a^5*b^5*c^4 - 15872*a^5* \\
& b^7*c^2 - 105728*a^6*b^3*c^5 + 45056*a^6*b^5*c^3 - 68096*a^7*b^3*c^4 + 1459 \\
& 2*a^7*b^5*c^2 - 26112*a^8*b^3*c^3 - 5248*a^9*b^3*c^2) + (-(8*a*c^7 + b^8 + \\
& 24*a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2* \\
& c^6 + 3*b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^ \\
& 3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3 \\
& )^{(1/2)} - 6*a*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3 \\
& )^{(1/2)})/(2*(3*a^2*b^8 - b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^ \\
& 7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4 \\
& *c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - \\
& 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^ \\
& 4 - 82*a^2*b^6*c^2 - 312*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + \\
& 159*a^4*b^4*c^2 - 312*a^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14*a*b^8*c))^{(1/2)}*(\tan \\
& (x/2)*(256*a^14*c - 96*a*b^14 + 544*a^3*b^12 - 1280*a^5*b^10 + 1600*a^7*b \\
& ^8 - 1120*a^9*b^6 + 416*a^11*b^4 - 64*a^13*b^2 + 512*a^2*c^13 + 5888*a^3*c^ \\
& 12 + 30976*a^4*c^11 + 98560*a^5*c^10 + 211200*a^6*c^9 + 321024*a^7*c^8 + 35 \\
& 4816*a^8*c^7 + 287232*a^9*c^6 + 168960*a^10*c^5 + 70400*a^11*c^4 + 19712*a^ \\
& 12*c^3 + 3328*a^13*c^2 - 128*a*b^2*c^12 + 736*a*b^4*c^10 - 1760*a*b^6*c^8 + \\
& 2240*a*b^8*c^6 - 1600*a*b^10*c^4 + 608*a*b^12*c^2 + 1536*a^2*b^12*c - 7616 \\
& *a^4*b^10*c + 15360*a^6*b^8*c - 16000*a^8*b^6*c + 8960*a^10*b^4*c - 2496*a^ \\
& 12*b^2*c - 4416*a^2*b^2*c^11 + 14080*a^2*b^4*c^9 - 22400*a^2*b^6*c^7 + 1920 \\
& 0*a^2*b^8*c^5 - 8512*a^2*b^10*c^3 - 35904*a^3*b^2*c^10 + 84000*a^3*b^4*c^8 \\
& - 96000*a^3*b^6*c^6 + 54720*a^3*b^8*c^4 - 13248*a^3*b^10*c^2 - 145600*a^4*b \\
& ^2*c^9 + 256000*a^4*b^4*c^7 - 206720*a^4*b^6*c^5 + 72960*a^4*b^8*c^3 - 3600 \\
& 00*a^5*b^2*c^8 + 468160*a^5*b^4*c^6 - 254400*a^5*b^6*c^4 + 48960*a^5*b^8*c^ \\
& 2 - 590976*a^6*b^2*c^7 + 548352*a^6*b^4*c^5 - 184960*a^6*b^6*c^3 - 669312*a \\
& ^7*b^2*c^6 + 418880*a^7*b^4*c^4 - 76800*a^7*b^6*c^2 - 528768*a^8*b^2*c^5 + \\
& 204800*a^8*b^4*c^3 - 288000*a^9*b^2*c^4 + 60000*a^9*b^4*c^2 - 104000*a^10*b \\
& ^2*c^3 - 22848*a^11*b^2*c^2) - 32*a^2*b^13 + 160*a^4*b^11 - 320*a^6*b^9 + 3 \\
& 20*a^8*b^7 - 160*a^10*b^5 + 32*a^12*b^3 - 32*a*b^3*c^11 + 160*a*b^5*c^9 - 3 \\
& 20*a*b^7*c^7 + 320*a*b^9*c^5 - 160*a*b^11*c^3 + 128*a^2*b*c^12 + 1152*a^3*b \\
& *c^11 + 288*a^3*b^11*c + 4480*a^4*b*c^10 + 9600*a^5*b*c^9 - 1600*a^5*b^9*c \\
& + 11520*a^6*b*c^8 + 5376*a^7*b*c^7 + 2880*a^7*b^7*c - 5376*a^8*b*c^6 - 1152 \\
& 0*a^9*b*c^5 - 2400*a^9*b^5*c - 9600*a^10*b*c^4 - 4480*a^11*b*c^3 + 928*a^11 \\
& *b^3*c - 1152*a^12*b*c^2 - 928*a^2*b^3*c^10 + 2400*a^2*b^5*c^8 - 2880*a^2*b
\end{aligned}$$

$$\begin{aligned}
& ^7c^6 + 1600a^2b^9c^4 - 288a^2b^{11}c^2 - 5600a^3b^3c^9 + 9600a^3b^5c^7 - 6720a^3b^7c^5 + 1280a^3b^9c^3 - 15200a^4b^3c^8 + 16000a^4b^5c^6 - 4160a^4b^7c^4 - 1280a^4b^9c^2 - 20800a^5b^3c^7 + 8640a^5b^5c^5 + 4160a^5b^7c^3 - 10304a^6b^3c^6 - 8640a^6b^5c^4 + 6720a^6b^7c^2 + 10304a^7b^3c^5 - 16000a^7b^5c^3 + 20800a^8b^3c^4 - 9600a^8b^5c^2 + 15200a^9b^3c^3 + 5600a^{10}b^3c^2 + 32a^{13}b^{13}c - 128a^{13}b^9c) + 32a^2b^{12} - 128a^4b^{10} + 192a^6b^8 - 128a^8b^6 + 32a^{10}b^4 + 128a^2c^{12} + 1280a^3c^{11} + 5760a^4c^{10} + 15360a^5c^9 + 26880a^6c^8 + 32256a^7c^7 + 26880a^8c^6 + 15360a^9c^5 + 5760a^{10}c^4 + 1280a^{11}c^3 + 128a^{12}c^2 - 32a^2b^2c^{11} + 128a^2b^4c^9 - 192a^2b^6c^7 + 128a^2b^8c^5 - 32a^2b^{10}c^3 - 416a^3b^{10}c + 1408a^5b^8c - 1728a^7b^6c + 896a^9b^4c - 160a^{11}b^2c - 832a^2b^2c^{10} + 1824a^2b^4c^8 - 1792a^2b^6c^6 + 832a^2b^8c^4 - 192a^2b^{10}c^2 - 5664a^3b^2c^9 + 8960a^3b^4c^7 - 6464a^3b^6c^5 + 2304a^3b^8c^3 - 19200a^4b^2c^8 + 22656a^4b^4c^6 - 11904a^4b^6c^4 + 2816a^4b^8c^2 - 38976a^5b^2c^7 + 33792a^5b^4c^5 - 12096a^5b^6c^3 - 51072a^6b^2c^6 + 31168a^6b^4c^4 - 6656a^6b^6c^2 - 44352a^7b^2c^5 + 17664a^7b^4c^3 - 25344a^8b^2c^4 + 5760a^8b^4c^2 - 9120a^9b^2c^3 - 1856a^{10}b^2c^2) + \tan(x/2) * (32a^2b^{12} + 128a^2c^{12} - 96a^3b^{10} + 96a^5b^8 - 32a^7b^6 + 1088a^2c^{11} + 4096a^3c^{10} + 8960a^4c^9 + 12544a^5c^8 + 11648a^6c^7 + 7168a^7c^6 + 2816a^8c^5 + 640a^9c^4 + 64a^{10}c^3 - 544a^2b^2c^{10} + 992a^2b^4c^8 - 1024a^2b^6c^6 + 640a^2b^8c^4 - 224a^2b^{10}c^2 - 384a^2b^{10}c + 960a^4b^8c - 768a^6b^6c + 192a^8b^4c - 3968a^2b^2c^9 + 6144a^2b^4c^7 - 5120a^2b^6c^5 + 2240a^2b^8c^3 - 12672a^3b^2c^8 + 16032a^3b^4c^6 - 9760a^3b^6c^4 + 2400a^3b^8c^2 - 23168a^4b^2c^7 + 22720a^4b^4c^5 - 8960a^4b^6c^3 - 26560a^5b^2c^6 + 18720a^5b^4c^4 - 4032a^5b^6c^2 - 19584a^6b^2c^5 + 8832a^6b^4c^3 - 9088a^7b^2c^4 + 2144a^7b^4c^2 - 2432a^8b^2c^3 - 288a^9b^2c^2) - 160a^2b^3c^9 + 320a^2b^5c^7 - 320a^2b^7c^5 + 160a^2b^9c^3 + 384a^2b^3c^{10} + 1792a^3b^3c^9 + 96a^3b^9c + 4480a^4b^3c^8 + 6720a^5b^3c^7 - 96a^5b^7c + 6272a^6b^3c^6 + 3584a^7b^3c^5 + 32a^7b^5c + 1152a^8b^3c^4 + 160a^9b^3c^3 - 1504a^2b^3c^8 + 2208a^2b^5c^6 - 1440a^2b^7c^4 + 352a^2b^9c^2 - 5280a^3b^3c^7 + 5280a^3b^5c^5 - 1888a^3b^7c^3 - 9440a^4b^3c^6 + 5824a^4b^5c^4 - 864a^4b^7c^2 - 9440a^5b^3c^5 + 3072a^5b^5c^3 - 5280a^6b^3c^4 + 672a^6b^5c^2 - 1504a^7b^3c^3 - 160a^8b^3c^2 + 32a^2b^3c^{11} - 32a^2b^{11}c) * i + ((-8a^2c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 - b^5 * (-4a^2c - b^2)^3)^{(1/2)} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3 - 3b^2c^4 * (-4a^2c - b^2)^3)^{(1/2)} - 54a^2b^2c^4 + 33a^2b^4c^2 - 38a^3b^2c^3 + 3b^3c^2 * (-4a^2c - b^2)^3)^{(1/2)} - 10a^2b^6c - 3a^2b^2c^2 * (-4a^2c - b^2)^3)^{(1/2)} - 6a^2b^3c^3 * (-4a^2c - b^2)^3)^{(1/2)} + 4a^2b^3c * (-4a^2c - b^2)^3)^{(1/2)}) / (2 * (3a^2b^8 - b^{10} - 3a^4b^6 + a^6b^4 + 16a^2c^8 + 96a^3c^7 + 240a^4c^6 + 320a^5c^5 + 240a^6c^4 + 96a^7c^3 + 16a^8c^2 + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c)))^{(1/2)} * (\tan(x/2) * (32a^2b^{12} + 128a^2c^{12} - 96a^3b^{10} + 96a^5b^8 - 32a^7b^6 + 1088a^2c^{11} + 4096a^3c^{10} + 8960a^4c^9 + 12544a^5c^8 + 11648a^6c^7 + 7168a^7c^6 + 2816a^8c^5 + 640a^9c^4 + 64a^{10}c^3 - 544a^2b^2c^{10} + 992a^2b^4c^8 - 1024a^2b^6c^6 + 640a^2b^8c^4 - 224a^2b^{10}c^2 - 384a^2b^{10}c + 960a^4b^8c - 768a^6b^6c + 192a^8b^4c - 3968a^2b^2c^9 + 6144a^2b^4c^7 - 5120a^2b^6c^5 + 2240a^2b^8c^3 - 12672a^3b^2c^8 + 16032a^3b^4c^6 - 9760a^3b^6c^4 + 2400a^3b^8c^2 - 23168a^4b^2c^7 + 22720a^4b^4c^5 - 8960a^4b^6c^3 - 26560a^5b^2c^6 + 18720a^5b^4c^4 - 4032a^5b^6c^2 - 19584a^6b^2c^5 + 8832a^6b^4c^3 - 9088a^7b^2c^4 + 2144a^7b^4c^2 - 2432a^8b^2c^3 - 288a^9b^2c^2) - (-8a^2c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 - b^5 * (-4a^2c - b^2)^3)^{(1/2)} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5 + 24a^2b^4c^3
\end{aligned}$$



$$\begin{aligned}
& 3 - 3*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38 \\
& *a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c \\
& *(-(4*a*c - b^2)^3)^{(1/2)}/(2*(3*a^2*b^8 - b^{10} - 3*a^4*b^6 + a^6*b^4 + 16* \\
& a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 \\
& + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 \\
& - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 \\
& + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 312*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - \\
& 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14* \\
& a*b^8*c))^{(1/2)}*(\tan(x/2))*(64*a*b^{13} - 256*a^3*b^{11} + 384*a^5*b^9 - 256*a^7 \\
& *b^7 + 64*a^9*b^5 - 128*a*b^3*c^{10} + 576*a*b^5*c^8 - 1024*a*b^7*c^6 + 896* \\
& a*b^9*c^4 - 384*a*b^{11}*c^2 + 512*a^2*b*c^{11} - 896*a^2*b^{11}*c + 4608*a^3*b*c \\
& ^{10} + 18432*a^4*b*c^9 + 3072*a^4*b^9*c + 43008*a^5*b*c^8 + 64512*a^6*b*c^7 \\
& - 3840*a^6*b^7*c + 64512*a^7*b*c^6 + 43008*a^8*b*c^5 + 2048*a^8*b^5*c + 184 \\
& 32*a^9*b*c^4 + 4608*a^{10}*b*c^3 - 384*a^{10}*b^3*c + 512*a^{11}*b*c^2 - 3456*a^2 \\
& *b^3*c^9 + 8192*a^2*b^5*c^7 - 8960*a^2*b^7*c^5 + 4608*a^2*b^9*c^3 - 20992*a \\
& ^3*b^3*c^8 + 34048*a^3*b^5*c^6 - 23808*a^3*b^7*c^4 + 6400*a^3*b^9*c^2 - 609 \\
& 28*a^4*b^3*c^7 + 67584*a^4*b^5*c^5 - 28160*a^4*b^7*c^3 - 102144*a^5*b^3*c^6 \\
& + 73600*a^5*b^5*c^4 - 15872*a^5*b^7*c^2 - 105728*a^6*b^3*c^5 + 45056*a^6*b \\
& ^5*c^3 - 68096*a^7*b^3*c^4 + 14592*a^7*b^5*c^2 - 26112*a^8*b^3*c^3 - 5248*a \\
& ^9*b^3*c^2) - (-(8*a*c^7 + b^8 + 24*a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 - b^5* \\
& (-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2*c^6 + 3*b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 \\
& + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} - 54*a^2*b^2*c^4 + 33*a^2 \\
& *b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c \\
& c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b*c^3*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)}/(2*(3*a^2*b^8 - b^{10} - 3*a^4*b^6 + \\
& a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 \\
& + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7 \\
& + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c \\
& - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 312*a^3*b^2*c^5 + 260 \\
& *a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a^5*b^2*c^3 - 96*a^6 \\
& *b^2*c^2 + 14*a*b^8*c))^{(1/2)}*(\tan(x/2))*(256*a^{14}*c - 96*a*b^{14} + 544*a^3* \\
& b^{12} - 1280*a^5*b^{10} + 1600*a^7*b^8 - 1120*a^9*b^6 + 416*a^{11}*b^4 - 64*a^{13} \\
& *b^2 + 512*a^2*c^{13} + 5888*a^3*c^{12} + 30976*a^4*c^{11} + 98560*a^5*c^{10} + 211 \\
& 200*a^6*c^9 + 321024*a^7*c^8 + 354816*a^8*c^7 + 287232*a^9*c^6 + 168960*a^{10} \\
& *c^5 + 70400*a^{11}*c^4 + 19712*a^{12}*c^3 + 3328*a^{13}*c^2 - 128*a*b^2*c^{12} + \\
& 736*a*b^4*c^{10} - 1760*a*b^6*c^8 + 2240*a*b^8*c^6 - 1600*a*b^{10}*c^4 + 608*a* \\
& b^{12}*c^2 + 1536*a^2*b^{12}*c - 7616*a^4*b^{10}*c + 15360*a^6*b^8*c - 16000*a^8* \\
& b^6*c + 8960*a^{10}*b^4*c - 2496*a^{12}*b^2*c - 4416*a^2*b^2*c^{11} + 14080*a^2*b \\
& ^4*c^9 - 22400*a^2*b^6*c^7 + 19200*a^2*b^8*c^5 - 8512*a^2*b^{10}*c^3 - 35904* \\
& a^3*b^2*c^{10} + 84000*a^3*b^4*c^8 - 96000*a^3*b^6*c^6 + 54720*a^3*b^8*c^4 - \\
& 13248*a^3*b^{10}*c^2 - 145600*a^4*b^2*c^9 + 256000*a^4*b^4*c^7 - 206720*a^4*b \\
& ^6*c^5 + 72960*a^4*b^8*c^3 - 360000*a^5*b^2*c^8 + 468160*a^5*b^4*c^6 - 2544 \\
& 00*a^5*b^6*c^4 + 48960*a^5*b^8*c^2 - 590976*a^6*b^2*c^7 + 548352*a^6*b^4*c^5 \\
& - 184960*a^6*b^6*c^3 - 669312*a^7*b^2*c^6 + 418880*a^7*b^4*c^4 - 76800*a^7 \\
& *b^6*c^2 - 528768*a^8*b^2*c^5 + 204800*a^8*b^4*c^3 - 288000*a^9*b^2*c^4 + \\
& 60000*a^9*b^4*c^2 - 104000*a^{10}*b^2*c^3 - 22848*a^{11}*b^2*c^2) - 32*a^2*b^{13} \\
& + 160*a^4*b^{11} - 320*a^6*b^9 + 320*a^8*b^7 - 160*a^{10}*b^5 + 32*a^{12}*b^3 - \\
& 32*a*b^3*c^{11} + 160*a*b^5*c^9 - 320*a*b^7*c^7 + 320*a*b^9*c^5 - 160*a*b^{11} \\
& c^3 + 128*a^2*b*c^{12} + 1152*a^3*b*c^{11} + 288*a^3*b^{11}*c + 4480*a^4*b*c^{10} + \\
& 9600*a^5*b*c^9 - 1600*a^5*b^9*c + 11520*a^6*b*c^8 + 5376*a^7*b*c^7 + 2880* \\
& a^7*b^7*c - 5376*a^8*b*c^6 - 11520*a^9*b*c^5 - 2400*a^9*b^5*c - 9600*a^{10}*b \\
& *c^4 - 4480*a^{11}*b*c^3 + 928*a^{11}*b^3*c - 1152*a^{12}*b*c^2 - 928*a^2*b^3*c^1 \\
& 0 + 2400*a^2*b^5*c^8 - 2880*a^2*b^7*c^6 + 1600*a^2*b^9*c^4 - 288*a^2*b^{11}*c \\
& ^2 - 5600*a^3*b^3*c^9 + 9600*a^3*b^5*c^7 - 6720*a^3*b^7*c^5 + 1280*a^3*b^9* \\
& c^3 - 15200*a^4*b^3*c^8 + 16000*a^4*b^5*c^6 - 4160*a^4*b^7*c^4 - 1280*a^4*b \\
& ^9*c^2 - 20800*a^5*b^3*c^7 + 8640*a^5*b^5*c^5 + 4160*a^5*b^7*c^3 - 10304*a^6 \\
& *b^3*c^6 - 8640*a^6*b^5*c^4 + 6720*a^6*b^7*c^2 + 10304*a^7*b^3*c^5 - 16000 \\
& *a^7*b^5*c^3 + 20800*a^8*b^3*c^4 - 9600*a^8*b^5*c^2 + 15200*a^9*b^3*c^3 + 5
\end{aligned}$$

$$\begin{aligned}
& 600*a^{10}*b^3*c^2 + 32*a*b^{13}*c - 128*a^{13}*b*c) + 32*a^2*b^{12} - 128*a^4*b^{10} \\
& + 192*a^6*b^8 - 128*a^8*b^6 + 32*a^{10}*b^4 + 128*a^2*c^{12} + 1280*a^3*c^{11} + \\
& 5760*a^4*c^{10} + 15360*a^5*c^9 + 26880*a^6*c^8 + 32256*a^7*c^7 + 26880*a^8* \\
& c^6 + 15360*a^9*c^5 + 5760*a^{10}*c^4 + 1280*a^{11}*c^3 + 128*a^{12}*c^2 - 32*a*b \\
& ^2*c^{11} + 128*a*b^4*c^9 - 192*a*b^6*c^7 + 128*a*b^8*c^5 - 32*a*b^{10}*c^3 - 4 \\
& 16*a^3*b^{10}*c + 1408*a^5*b^8*c - 1728*a^7*b^6*c + 896*a^9*b^4*c - 160*a^{11}* \\
& b^2*c - 832*a^2*b^2*c^{10} + 1824*a^2*b^4*c^8 - 1792*a^2*b^6*c^6 + 832*a^2*b^ \\
& 8*c^4 - 192*a^2*b^{10}*c^2 - 5664*a^3*b^2*c^9 + 8960*a^3*b^4*c^7 - 6464*a^3*b \\
& ^6*c^5 + 2304*a^3*b^8*c^3 - 19200*a^4*b^2*c^8 + 22656*a^4*b^4*c^6 - 11904*a \\
& ^4*b^6*c^4 + 2816*a^4*b^8*c^2 - 38976*a^5*b^2*c^7 + 33792*a^5*b^4*c^5 - 120 \\
& 96*a^5*b^6*c^3 - 51072*a^6*b^2*c^6 + 31168*a^6*b^4*c^4 - 6656*a^6*b^6*c^2 - \\
& 44352*a^7*b^2*c^5 + 17664*a^7*b^4*c^3 - 25344*a^8*b^2*c^4 + 5760*a^8*b^4*c \\
& ^2 - 9120*a^9*b^2*c^3 - 1856*a^{10}*b^2*c^2) - 160*a*b^3*c^9 + 320*a*b^5*c^7 \\
& - 320*a*b^7*c^5 + 160*a*b^9*c^3 + 384*a^2*b*c^{10} + 1792*a^3*b*c^9 + 96*a^3* \\
& b^9*c + 4480*a^4*b*c^8 + 6720*a^5*b*c^7 - 96*a^5*b^7*c + 6272*a^6*b*c^6 + 3 \\
& 584*a^7*b*c^5 + 32*a^7*b^5*c + 1152*a^8*b*c^4 + 160*a^9*b*c^3 - 1504*a^2*b^ \\
& 3*c^8 + 2208*a^2*b^5*c^6 - 1440*a^2*b^7*c^4 + 352*a^2*b^9*c^2 - 5280*a^3*b^ \\
& 3*c^7 + 5280*a^3*b^5*c^5 - 1888*a^3*b^7*c^3 - 9440*a^4*b^3*c^6 + 5824*a^4*b \\
& ^5*c^4 - 864*a^4*b^7*c^2 - 9440*a^5*b^3*c^5 + 3072*a^5*b^5*c^3 - 5280*a^6*b \\
& ^3*c^4 + 672*a^6*b^5*c^2 - 1504*a^7*b^3*c^3 - 160*a^8*b^3*c^2 + 32*a*b*c^{11} \\
& - 32*a*b^{11}*c)*1i)/((-8*a*c^7 + b^8 + 24*a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 \\
& - b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2*c^6 + 3*b^4*c^4 - 3*b^6*c^2 - 18*a* \\
& b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} - 54*a^2*b^2*c^4 \\
& + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10 \\
& *a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b*c^3*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(3*a^2*b^8 - b^{10} - 3*a^ \\
& 4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240 \\
& *a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a* \\
& b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7 \\
& *b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 312*a^3*b^2*c^ \\
& 5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a^5*b^2*c^3 - \\
& 96*a^6*b^2*c^2 + 14*a*b^8*c))^{(1/2)}*((-8*a*c^7 + b^8 + 24*a^2*c^6 + 24*a \\
& ^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2*c^6 + 3*b^4*c^4 - \\
& 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b*c \\
& ^3*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*(3*a^2 \\
& *b^8 - b^{10} - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + \\
& 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 \\
& + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30 \\
& *a^5*b^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^ \\
& 2 - 312*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - \\
& 312*a^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14*a*b^8*c))^{(1/2)}*(tan(x/2)*(64*a*b^1 \\
& 3 - 256*a^3*b^{11} + 384*a^5*b^9 - 256*a^7*b^7 + 64*a^9*b^5 - 128*a*b^3*c^{10} \\
& + 576*a*b^5*c^8 - 1024*a*b^7*c^6 + 896*a*b^9*c^4 - 384*a*b^{11}*c^2 + 512*a^2 \\
& *b*c^{11} - 896*a^2*b^{11}*c + 4608*a^3*b*c^{10} + 18432*a^4*b*c^9 + 3072*a^4*b^9 \\
& *c + 43008*a^5*b*c^8 + 64512*a^6*b*c^7 - 3840*a^6*b^7*c + 64512*a^7*b*c^6 + \\
& 43008*a^8*b*c^5 + 2048*a^8*b^5*c + 18432*a^9*b*c^4 + 4608*a^{10}*b*c^3 - 384 \\
& *a^{10}*b^3*c + 512*a^{11}*b*c^2 - 3456*a^2*b^3*c^9 + 8192*a^2*b^5*c^7 - 8960*a \\
& ^2*b^7*c^5 + 4608*a^2*b^9*c^3 - 20992*a^3*b^3*c^8 + 34048*a^3*b^5*c^6 - 238 \\
& 08*a^3*b^7*c^4 + 6400*a^3*b^9*c^2 - 60928*a^4*b^3*c^7 + 67584*a^4*b^5*c^5 - \\
& 28160*a^4*b^7*c^3 - 102144*a^5*b^3*c^6 + 73600*a^5*b^5*c^4 - 15872*a^5*b^7 \\
& *c^2 - 105728*a^6*b^3*c^5 + 45056*a^6*b^5*c^3 - 68096*a^7*b^3*c^4 + 14592*a \\
& ^7*b^5*c^2 - 26112*a^8*b^3*c^3 - 5248*a^9*b^3*c^2) + (-8*a*c^7 + b^8 + 24* \\
& a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 2*b^2*c^6 \\
& + 3*b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^{( \\
& 1/2)} - 6*a*b*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 4*a*b^3*c*(-(4*a*c - b^2)^3)^{(1
\end{aligned}$$

$$\begin{aligned}
& /2)) / (2 * (3 * a^2 * b^8 - b^{10} - 3 * a^4 * b^6 + a^6 * b^4 + 16 * a^2 * c^8 + 96 * a^3 * c^7 + \\
& 240 * a^4 * c^6 + 320 * a^5 * c^5 + 240 * a^6 * c^4 + 96 * a^7 * c^3 + 16 * a^8 * c^2 + b^4 * c^6 \\
& - 3 * b^6 * c^4 + 3 * b^8 * c^2 - 8 * a * b^2 * c^7 + 30 * a * b^4 * c^5 - 36 * a * b^6 * c^3 - 36 * \\
& a^3 * b^6 * c + 30 * a^5 * b^4 * c - 8 * a^7 * b^2 * c - 96 * a^2 * b^2 * c^6 + 159 * a^2 * b^4 * c^4 - \\
& 82 * a^2 * b^6 * c^2 - 312 * a^3 * b^2 * c^5 + 260 * a^3 * b^4 * c^3 - 448 * a^4 * b^2 * c^4 + 159 \\
& * a^4 * b^4 * c^2 - 312 * a^5 * b^2 * c^3 - 96 * a^6 * b^2 * c^2 + 14 * a * b^8 * c))^{(1/2)} * (\tan(x/2) * \\
& (256 * a^{14} * c - 96 * a * b^{14} + 544 * a^3 * b^{12} - 1280 * a^5 * b^{10} + 1600 * a^7 * b^8 \\
& - 1120 * a^9 * b^6 + 416 * a^{11} * b^4 - 64 * a^{13} * b^2 + 512 * a^2 * c^{13} + 5888 * a^3 * c^{12} \\
& + 30976 * a^4 * c^{11} + 98560 * a^5 * c^{10} + 211200 * a^6 * c^9 + 321024 * a^7 * c^8 + 35481 \\
& 6 * a^8 * c^7 + 287232 * a^9 * c^6 + 168960 * a^{10} * c^5 + 70400 * a^{11} * c^4 + 19712 * a^{12} * \\
& c^3 + 3328 * a^{13} * c^2 - 128 * a * b^2 * c^{12} + 736 * a * b^4 * c^{10} - 1760 * a * b^6 * c^8 + 22 \\
& 40 * a * b^8 * c^6 - 1600 * a * b^{10} * c^4 + 608 * a * b^{12} * c^2 + 1536 * a^2 * b^{12} * c - 7616 * a^4 \\
& * b^{10} * c + 15360 * a^6 * b^8 * c - 16000 * a^8 * b^6 * c + 8960 * a^{10} * b^4 * c - 2496 * a^{12} * \\
& b^2 * c - 4416 * a^2 * b^2 * c^{11} + 14080 * a^2 * b^4 * c^9 - 22400 * a^2 * b^6 * c^7 + 19200 * a^2 \\
& * b^8 * c^5 - 8512 * a^2 * b^{10} * c^3 - 35904 * a^3 * b^2 * c^{10} + 84000 * a^3 * b^4 * c^8 - 9 \\
& 6000 * a^3 * b^6 * c^6 + 54720 * a^3 * b^8 * c^4 - 13248 * a^3 * b^{10} * c^2 - 145600 * a^4 * b^2 * \\
& c^9 + 256000 * a^4 * b^4 * c^7 - 206720 * a^4 * b^6 * c^5 + 72960 * a^4 * b^8 * c^3 - 360000 * \\
& a^5 * b^2 * c^8 + 468160 * a^5 * b^4 * c^6 - 254400 * a^5 * b^6 * c^4 + 48960 * a^5 * b^8 * c^2 - \\
& 590976 * a^6 * b^2 * c^7 + 548352 * a^6 * b^4 * c^5 - 184960 * a^6 * b^6 * c^3 - 669312 * a^7 * \\
& b^2 * c^6 + 418880 * a^7 * b^4 * c^4 - 76800 * a^7 * b^6 * c^2 - 528768 * a^8 * b^2 * c^5 + 204 \\
& 800 * a^8 * b^4 * c^3 - 288000 * a^9 * b^2 * c^4 + 60000 * a^9 * b^4 * c^2 - 104000 * a^{10} * b^2 * \\
& c^3 - 22848 * a^{11} * b^2 * c^2) - 32 * a^2 * b^{13} + 160 * a^4 * b^{11} - 320 * a^6 * b^9 + 320 * \\
& a^8 * b^7 - 160 * a^{10} * b^5 + 32 * a^{12} * b^3 - 32 * a * b^3 * c^{11} + 160 * a * b^5 * c^9 - 320 * \\
& a * b^7 * c^7 + 320 * a * b^9 * c^5 - 160 * a * b^{11} * c^3 + 128 * a^2 * b * c^{12} + 1152 * a^3 * b * c^{11} \\
& + 288 * a^3 * b^{11} * c + 4480 * a^4 * b * c^{10} + 9600 * a^5 * b * c^9 - 1600 * a^5 * b^9 * c + 1 \\
& 1520 * a^6 * b * c^8 + 5376 * a^7 * b * c^7 + 2880 * a^7 * b^7 * c - 5376 * a^8 * b * c^6 - 11520 * a^9 \\
& * b * c^5 - 2400 * a^9 * b^5 * c - 9600 * a^{10} * b * c^4 - 4480 * a^{11} * b * c^3 + 928 * a^{11} * b^3 \\
& * c - 1152 * a^{12} * b * c^2 - 928 * a^2 * b^3 * c^{10} + 2400 * a^2 * b^5 * c^8 - 2880 * a^2 * b^7 * \\
& c^6 + 1600 * a^2 * b^9 * c^4 - 288 * a^2 * b^{11} * c^2 - 5600 * a^3 * b^3 * c^9 + 9600 * a^3 * b^5 \\
& * c^7 - 6720 * a^3 * b^7 * c^5 + 1280 * a^3 * b^9 * c^3 - 15200 * a^4 * b^3 * c^8 + 16000 * a^4 * \\
& b^5 * c^6 - 4160 * a^4 * b^7 * c^4 - 1280 * a^4 * b^9 * c^2 - 20800 * a^5 * b^3 * c^7 + 8640 * a^5 \\
& * b^5 * c^5 + 4160 * a^5 * b^7 * c^3 - 10304 * a^6 * b^3 * c^6 - 8640 * a^6 * b^5 * c^4 + 6720 * \\
& a^6 * b^7 * c^2 + 10304 * a^7 * b^3 * c^5 - 16000 * a^7 * b^5 * c^3 + 20800 * a^8 * b^3 * c^4 - 9 \\
& 600 * a^8 * b^5 * c^2 + 15200 * a^9 * b^3 * c^3 + 5600 * a^{10} * b^3 * c^2 + 32 * a * b^{13} * c - 128 \\
& * a^{13} * b * c) + 32 * a^2 * b^{12} - 128 * a^4 * b^{10} + 192 * a^6 * b^8 - 128 * a^8 * b^6 + 32 * a^{10} \\
& * b^4 + 128 * a^2 * c^{12} + 1280 * a^3 * c^{11} + 5760 * a^4 * c^{10} + 15360 * a^5 * c^9 + 268 \\
& 80 * a^6 * c^8 + 32256 * a^7 * c^7 + 26880 * a^8 * c^6 + 15360 * a^9 * c^5 + 5760 * a^{10} * c^4 \\
& + 1280 * a^{11} * c^3 + 128 * a^{12} * c^2 - 32 * a * b^2 * c^{11} + 128 * a * b^4 * c^9 - 192 * a * b^6 * \\
& c^7 + 128 * a * b^8 * c^5 - 32 * a * b^{10} * c^3 - 416 * a^3 * b^{10} * c + 1408 * a^5 * b^8 * c - 172 \\
& 8 * a^7 * b^6 * c + 896 * a^9 * b^4 * c - 160 * a^{11} * b^2 * c - 832 * a^2 * b^2 * c^{10} + 1824 * a^2 * \\
& b^4 * c^8 - 1792 * a^2 * b^6 * c^6 + 832 * a^2 * b^8 * c^4 - 192 * a^2 * b^{10} * c^2 - 5664 * a^3 * \\
& b^2 * c^9 + 8960 * a^3 * b^4 * c^7 - 6464 * a^3 * b^6 * c^5 + 2304 * a^3 * b^8 * c^3 - 19200 * a^4 \\
& * b^2 * c^8 + 22656 * a^4 * b^4 * c^6 - 11904 * a^4 * b^6 * c^4 + 2816 * a^4 * b^8 * c^2 - 3897 \\
& 6 * a^5 * b^2 * c^7 + 33792 * a^5 * b^4 * c^5 - 12096 * a^5 * b^6 * c^3 - 51072 * a^6 * b^2 * c^6 + \\
& 31168 * a^6 * b^4 * c^4 - 6656 * a^6 * b^6 * c^2 - 44352 * a^7 * b^2 * c^5 + 17664 * a^7 * b^4 * c^3 \\
& - 25344 * a^8 * b^2 * c^4 + 5760 * a^8 * b^4 * c^2 - 9120 * a^9 * b^2 * c^3 - 1856 * a^{10} * b^2 \\
& * c^2) + \tan(x/2) * (32 * a * b^{12} + 128 * a * c^{12} - 96 * a^3 * b^{10} + 96 * a^5 * b^8 - 32 * a^7 \\
& * b^6 + 1088 * a^2 * c^{11} + 4096 * a^3 * c^{10} + 8960 * a^4 * c^9 + 12544 * a^5 * c^8 + 116 \\
& 48 * a^6 * c^7 + 7168 * a^7 * c^6 + 2816 * a^8 * c^5 + 640 * a^9 * c^4 + 64 * a^{10} * c^3 - 544 * \\
& a * b^2 * c^{10} + 992 * a * b^4 * c^8 - 1024 * a * b^6 * c^6 + 640 * a * b^8 * c^4 - 224 * a * b^{10} * c^2 \\
& - 384 * a^2 * b^{10} * c + 960 * a^4 * b^8 * c - 768 * a^6 * b^6 * c + 192 * a^8 * b^4 * c - 3968 * a^2 \\
& * b^2 * c^9 + 6144 * a^2 * b^4 * c^7 - 5120 * a^2 * b^6 * c^5 + 2240 * a^2 * b^8 * c^3 - 12672 \\
& * a^3 * b^2 * c^8 + 16032 * a^3 * b^4 * c^6 - 9760 * a^3 * b^6 * c^4 + 2400 * a^3 * b^8 * c^2 - 23 \\
& 168 * a^4 * b^2 * c^7 + 22720 * a^4 * b^4 * c^5 - 8960 * a^4 * b^6 * c^3 - 26560 * a^5 * b^2 * c^6 \\
& + 18720 * a^5 * b^4 * c^4 - 4032 * a^5 * b^6 * c^2 - 19584 * a^6 * b^2 * c^5 + 8832 * a^6 * b^4 * c^3 \\
& - 9088 * a^7 * b^2 * c^4 + 2144 * a^7 * b^4 * c^2 - 2432 * a^8 * b^2 * c^3 - 288 * a^9 * b^2 * c^2) - \\
& 160 * a * b^3 * c^9 + 320 * a * b^5 * c^7 - 320 * a * b^7 * c^5 + 160 * a * b^9 * c^3 + 384 * a^2 \\
& * b * c^{10} + 1792 * a^3 * b * c^9 + 96 * a^3 * b^9 * c + 4480 * a^4 * b * c^8 + 6720 * a^5 * b * c^7 \\
& - 96 * a^5 * b^7 * c + 6272 * a^6 * b * c^6 + 3584 * a^7 * b * c^5 + 32 * a^7 * b^5 * c + 1152 * a^8
\end{aligned}$$

$$\begin{aligned}
& *b*c^4 + 160*a^9*b*c^3 - 1504*a^2*b^3*c^8 + 2208*a^2*b^5*c^6 - 1440*a^2*b^7 \\
& *c^4 + 352*a^2*b^9*c^2 - 5280*a^3*b^3*c^7 + 5280*a^3*b^5*c^5 - 1888*a^3*b^7 \\
& *c^3 - 9440*a^4*b^3*c^6 + 5824*a^4*b^5*c^4 - 864*a^4*b^7*c^2 - 9440*a^5*b^3 \\
& *c^5 + 3072*a^5*b^5*c^3 - 5280*a^6*b^3*c^4 + 672*a^6*b^5*c^2 - 1504*a^7*b^3 \\
& *c^3 - 160*a^8*b^3*c^2 + 32*a*b*c^11 - 32*a*b^11*c) - 2*\tan(x/2)*(192*a*b^5 \\
& *c^6 - 192*a*b^3*c^8 - 64*a*b^7*c^4 + 384*a^2*b*c^9 + 960*a^3*b*c^8 + 1280* \\
& a^4*b*c^7 + 960*a^5*b*c^6 + 384*a^6*b*c^5 + 64*a^7*b*c^4 - 768*a^2*b^3*c^7 \\
& + 384*a^2*b^5*c^5 - 1152*a^3*b^3*c^6 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5 - \\
& 192*a^5*b^3*c^4 + 64*a*b*c^10) - ((-8*a*c^7 + b^8 + 24*a^2*c^6 + 24*a^3*c^5 \\
& + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) - 2*b^2*c^6 + 3*b^4*c^4 - 3*b^6 \\
& *c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^(1/2) - 54* \\
& a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - b^2)^3 \\
& )^(1/2) - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^3*(-( \\
& 4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(2*(3*a^2*b^8 - \\
& b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a \\
& ^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^ \\
& 8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b \\
& ^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 31 \\
& 2*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a \\
& ^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14*a*b^8*c)))^(1/2)*(tan(x/2)*(32*a*b^12 + 12 \\
& 8*a*c^12 - 96*a^3*b^10 + 96*a^5*b^8 - 32*a^7*b^6 + 1088*a^2*c^11 + 4096*a^3 \\
& *c^10 + 8960*a^4*c^9 + 12544*a^5*c^8 + 11648*a^6*c^7 + 7168*a^7*c^6 + 2816* \\
& a^8*c^5 + 640*a^9*c^4 + 64*a^10*c^3 - 544*a*b^2*c^10 + 992*a*b^4*c^8 - 1024 \\
& *a*b^6*c^6 + 640*a*b^8*c^4 - 224*a*b^10*c^2 - 384*a^2*b^10*c + 960*a^4*b^8* \\
& c - 768*a^6*b^6*c + 192*a^8*b^4*c - 3968*a^2*b^2*c^9 + 6144*a^2*b^4*c^7 - 5 \\
& 120*a^2*b^6*c^5 + 2240*a^2*b^8*c^3 - 12672*a^3*b^2*c^8 + 16032*a^3*b^4*c^6 \\
& - 9760*a^3*b^6*c^4 + 2400*a^3*b^8*c^2 - 23168*a^4*b^2*c^7 + 22720*a^4*b^4*c \\
& ^5 - 8960*a^4*b^6*c^3 - 26560*a^5*b^2*c^6 + 18720*a^5*b^4*c^4 - 4032*a^5*b^ \\
& 6*c^2 - 19584*a^6*b^2*c^5 + 8832*a^6*b^4*c^3 - 9088*a^7*b^2*c^4 + 2144*a^7* \\
& b^4*c^2 - 2432*a^8*b^2*c^3 - 288*a^9*b^2*c^2) - ((-8*a*c^7 + b^8 + 24*a^2*c \\
& ^6 + 24*a^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) - 2*b^2*c^6 + 3* \\
& b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2) \\
& ^3)^(1/2) - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(- \\
& (4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) \\
& - 6*a*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/ \\
& (2*(3*a^2*b^8 - b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240* \\
& a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3 \\
& *b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7 + 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b \\
& ^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c - 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a \\
& ^2*b^6*c^2 - 312*a^3*b^2*c^5 + 260*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4* \\
& b^4*c^2 - 312*a^5*b^2*c^3 - 96*a^6*b^2*c^2 + 14*a*b^8*c)))^(1/2)*(tan(x/2)* \\
& (64*a*b^13 - 256*a^3*b^11 + 384*a^5*b^9 - 256*a^7*b^7 + 64*a^9*b^5 - 128*a* \\
& b^3*c^10 + 576*a*b^5*c^8 - 1024*a*b^7*c^6 + 896*a*b^9*c^4 - 384*a*b^11*c^2 \\
& + 512*a^2*b*c^11 - 896*a^2*b^11*c + 4608*a^3*b*c^10 + 18432*a^4*b*c^9 + 307 \\
& 2*a^4*b^9*c + 43008*a^5*b*c^8 + 64512*a^6*b*c^7 - 3840*a^6*b^7*c + 64512*a^ \\
& 7*b*c^6 + 43008*a^8*b*c^5 + 2048*a^8*b^5*c + 18432*a^9*b*c^4 + 4608*a^10*b* \\
& c^3 - 384*a^10*b^3*c + 512*a^11*b*c^2 - 3456*a^2*b^3*c^9 + 8192*a^2*b^5*c^7 \\
& - 8960*a^2*b^7*c^5 + 4608*a^2*b^9*c^3 - 20992*a^3*b^3*c^8 + 34048*a^3*b^5* \\
& c^6 - 23808*a^3*b^7*c^4 + 6400*a^3*b^9*c^2 - 60928*a^4*b^3*c^7 + 67584*a^4* \\
& b^5*c^5 - 28160*a^4*b^7*c^3 - 102144*a^5*b^3*c^6 + 73600*a^5*b^5*c^4 - 1587 \\
& 2*a^5*b^7*c^2 - 105728*a^6*b^3*c^5 + 45056*a^6*b^5*c^3 - 68096*a^7*b^3*c^4 \\
& + 14592*a^7*b^5*c^2 - 26112*a^8*b^3*c^3 - 5248*a^9*b^3*c^2) - ((-8*a*c^7 + \\
& b^8 + 24*a^2*c^6 + 24*a^3*c^5 + 8*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) - \\
& 2*b^2*c^6 + 3*b^4*c^4 - 3*b^6*c^2 - 18*a*b^2*c^5 + 24*a*b^4*c^3 - 3*b*c^4*( \\
& -(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^4 + 33*a^2*b^4*c^2 - 38*a^3*b^2*c^3 \\
& + 3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c - 3*a^2*b*c^2*(-(4*a*c - \\
& b^2)^3)^(1/2) - 6*a*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c*(-(4*a*c - b \\
& ^2)^3)^(1/2))/(2*(3*a^2*b^8 - b^10 - 3*a^4*b^6 + a^6*b^4 + 16*a^2*c^8 + 96* \\
& a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^4 + 96*a^7*c^3 + 16*a^8*c^2
\end{aligned}$$

$$\begin{aligned}
& + b^4c^6 - 3b^6c^4 + 3b^8c^2 - 8a^2b^2c^7 + 30a^2b^4c^5 - 36a^2b^6c^3 - 36a^3b^6c + 30a^5b^4c - 8a^7b^2c - 96a^2b^2c^6 + 159a^2b^4c^4 - 82a^2b^6c^2 - 312a^3b^2c^5 + 260a^3b^4c^3 - 448a^4b^2c^4 + 159a^4b^4c^2 - 312a^5b^2c^3 - 96a^6b^2c^2 + 14a^2b^8c) \Big)^{(1/2)} \\
& \cdot (\tan(x/2) \cdot (256a^{14}c - 96a^2b^{14} + 544a^3b^{12} - 1280a^5b^{10} + 1600a^7b^8 - 1120a^9b^6 + 416a^{11}b^4 - 64a^{13}b^2 + 512a^2c^{13} + 5888a^3c^{12} + 30976a^4c^{11} + 98560a^5c^{10} + 211200a^6c^9 + 321024a^7c^8 + 354816a^8c^7 + 287232a^9c^6 + 168960a^{10}c^5 + 70400a^{11}c^4 + 19712a^{12}c^3 + 3328a^{13}c^2 - 128a^2b^2c^{12} + 736a^2b^4c^{10} - 1760a^2b^6c^8 + 2240a^2b^8c^6 - 1600a^2b^{10}c^4 + 608a^2b^{12}c^2 + 1536a^2b^{12}c - 7616a^4b^{10}c + 15360a^6b^8c - 16000a^8b^6c + 8960a^{10}b^4c - 2496a^{12}b^2c - 4416a^2b^2c^{11} + 14080a^2b^4c^9 - 22400a^2b^6c^7 + 19200a^2b^8c^5 - 8512a^2b^{10}c^3 - 35904a^3b^2c^{10} + 84000a^3b^4c^8 - 96000a^3b^6c^6 + 54720a^3b^8c^4 - 13248a^3b^{10}c^2 - 145600a^4b^2c^9 + 256000a^4b^4c^7 - 206720a^4b^6c^5 + 72960a^4b^8c^3 - 360000a^5b^2c^8 + 468160a^5b^4c^6 - 254400a^5b^6c^4 + 48960a^5b^8c^2 - 590976a^6b^2c^7 + 548352a^6b^4c^5 - 184960a^6b^6c^3 - 669312a^7b^2c^6 + 418880a^7b^4c^4 - 76800a^7b^6c^2 - 528768a^8b^2c^5 + 204800a^8b^4c^3 - 288000a^9b^2c^4 + 60000a^9b^4c^2 - 104000a^{10}b^2c^3 - 22848a^{11}b^2c^2) - 32a^2b^{13} + 160a^4b^{11} - 320a^6b^9 + 320a^8b^7 - 160a^{10}b^5 + 32a^{12}b^3 - 32a^2b^3c^{11} + 160a^2b^5c^9 - 320a^2b^7c^7 + 320a^2b^9c^5 - 160a^2b^{11}c^3 + 128a^2b^3c^{12} + 1152a^3b^3c^{11} + 288a^3b^{11}c + 4480a^4b^3c^{10} + 9600a^5b^3c^9 - 1600a^5b^9c + 11520a^6b^3c^8 + 5376a^7b^3c^7 + 2880a^7b^7c - 5376a^8b^3c^6 - 11520a^9b^3c^5 - 2400a^9b^5c - 9600a^{10}b^3c^4 - 4480a^{11}b^3c^3 + 928a^{11}b^3c - 1152a^{12}b^3c^2 - 928a^2b^3c^{10} + 2400a^2b^5c^8 - 2880a^2b^7c^6 + 1600a^2b^9c^4 - 288a^2b^{11}c^2 - 5600a^3b^3c^9 + 9600a^3b^5c^7 - 6720a^3b^7c^5 + 1280a^3b^9c^3 - 15200a^4b^3c^8 + 16000a^4b^5c^6 - 4160a^4b^7c^4 - 1280a^4b^9c^2 - 20800a^5b^3c^7 + 8640a^5b^5c^5 + 4160a^5b^7c^3 - 10304a^6b^3c^6 - 8640a^6b^5c^4 + 6720a^6b^7c^2 + 10304a^7b^3c^5 - 16000a^7b^5c^3 + 20800a^8b^3c^4 - 9600a^8b^5c^2 + 15200a^9b^3c^3 + 5600a^{10}b^3c^2 + 32a^2b^{13}c - 128a^{13}b^3c) + 32a^2b^{12} - 128a^4b^{10} + 192a^6b^8 - 128a^8b^6 + 32a^{10}b^4 + 128a^2c^{12} + 1280a^3c^{11} + 5760a^4c^{10} + 15360a^5c^9 + 26880a^6c^8 + 32256a^7c^7 + 26880a^8c^6 + 15360a^9c^5 + 5760a^{10}c^4 + 1280a^{11}c^3 + 128a^{12}c^2 - 32a^2b^2c^{11} + 128a^2b^4c^9 - 192a^2b^6c^7 + 128a^2b^8c^5 - 32a^2b^{10}c^3 - 416a^3b^{10}c + 1408a^5b^8c - 1728a^7b^6c + 896a^9b^4c - 160a^{11}b^2c - 832a^2b^2c^{10} + 1824a^2b^4c^8 - 1792a^2b^6c^6 + 832a^2b^8c^4 - 192a^2b^{10}c^2 - 5664a^3b^2c^9 + 8960a^3b^4c^7 - 6464a^3b^6c^5 + 2304a^3b^8c^3 - 19200a^4b^2c^8 + 22656a^4b^4c^6 - 11904a^4b^6c^4 + 2816a^4b^8c^2 - 38976a^5b^2c^7 + 33792a^5b^4c^5 - 12096a^5b^6c^3 - 51072a^6b^2c^6 + 31168a^6b^4c^4 - 6656a^6b^6c^2 - 44352a^7b^2c^5 + 17664a^7b^4c^3 - 25344a^8b^2c^4 + 5760a^8b^4c^2 - 9120a^9b^2c^3 - 1856a^{10}b^2c^2) - 160a^2b^3c^9 + 320a^2b^5c^7 - 320a^2b^7c^5 + 160a^2b^9c^3 + 384a^2b^3c^{10} + 1792a^3b^3c^9 + 96a^3b^9c + 4480a^4b^3c^8 + 6720a^5b^3c^7 - 96a^5b^7c + 6272a^6b^3c^6 + 3584a^7b^3c^5 + 32a^7b^5c + 1152a^8b^3c^4 + 160a^9b^3c^3 - 1504a^2b^3c^8 + 2208a^2b^5c^6 - 1440a^2b^7c^4 + 352a^2b^9c^2 - 5280a^3b^3c^7 + 5280a^3b^5c^5 - 1888a^3b^7c^3 - 9440a^4b^3c^6 + 5824a^4b^5c^4 - 864a^4b^7c^2 - 9440a^5b^3c^5 + 3072a^5b^5c^3 - 5280a^6b^3c^4 + 672a^6b^5c^2 - 1504a^7b^3c^3 - 160a^8b^3c^2 + 32a^2b^3c^{11} - 32a^2b^{11}c) + 64a^2c^{11} + 448a^2c^{10} + 1344a^3c^9 + 2240a^4c^8 + 2240a^5c^7 + 1344a^6c^6 + 448a^7c^5 + 64a^8c^4 - 256a^2b^2c^9 + 384a^2b^4c^7 - 256a^2b^6c^5 + 64a^2b^8c^3 - 1344a^2b^2c^8 + 1344a^2b^4c^6 - 448a^2b^6c^4 - 2880a^3b^2c^7 + 1728a^3b^4c^5 - 192a^3b^6c^3 - 3200a^4b^2c^6 + 960a^4b^4c^4 - 1920a^5b^2c^5 + 192a^5b^4c^3 - 576a^6b^2c^4 - 64a^7b^2c^3) \cdot (- (8a^2c^7 + b^8 + 24a^2c^6 + 24a^3c^5 + 8a^4c^4 - b^5c - (4a^2c - b^2)^3)^{(1/2)} - 2b^2c^6 + 3b^4c^4 - 3b^6c^2 - 18a^2b^2c^5
\end{aligned}$$

```

+ 24*a*b^4*c^3 - 3*b*c^4*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^4 + 33*a^
2*b^4*c^2 - 38*a^3*b^2*c^3 + 3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*
c - 3*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^3*(-(4*a*c - b^2)^3)^(1/
2) + 4*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(2*(3*a^2*b^8 - b^10 - 3*a^4*b^6 +
a^6*b^4 + 16*a^2*c^8 + 96*a^3*c^7 + 240*a^4*c^6 + 320*a^5*c^5 + 240*a^6*c^
4 + 96*a^7*c^3 + 16*a^8*c^2 + b^4*c^6 - 3*b^6*c^4 + 3*b^8*c^2 - 8*a*b^2*c^7
+ 30*a*b^4*c^5 - 36*a*b^6*c^3 - 36*a^3*b^6*c + 30*a^5*b^4*c - 8*a^7*b^2*c
- 96*a^2*b^2*c^6 + 159*a^2*b^4*c^4 - 82*a^2*b^6*c^2 - 312*a^3*b^2*c^5 + 260
*a^3*b^4*c^3 - 448*a^4*b^2*c^4 + 159*a^4*b^4*c^2 - 312*a^5*b^2*c^3 - 96*a^6
*b^2*c^2 + 14*a*b^8*c))^(1/2)*2i + ((2*b)/(2*a*c + a^2 - b^2 + c^2) - (2*t
an(x/2)*(a + c))/(2*a*c + a^2 - b^2 + c^2))/(tan(x/2)^2 - 1)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)\*\*2/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Integral(sec(x)\*\*2/(a + b\*sin(x) + c\*sin(x)\*\*2), x)

$$3.14 \quad \int \frac{\sec^3(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

**Optimal.** Leaf size=206

$$\frac{b(b^2 - 2c(a+c)) \log(a+b \sin(x)+c \sin^2(x))}{2(a^2 + 2ac - b^2 + c^2)^2} - \frac{(-2b^2c(2a+c) + 2c^2(a+c)^2 + b^4) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (a^2 + 2ac - b^2 + c^2)^2} (a+c)$$

[Out] -1/4\*(a+2\*b+3\*c)\*ln(1-sin(x))/(a+b+c)^2+1/4\*(a-2\*b+3\*c)\*ln(1+sin(x))/(a-b+c)^2+1/2\*b\*(b^2-2\*c\*(a+c))\*ln(a+b\*sin(x)+c\*sin(x)^2)/(a^2+2\*a\*c-b^2+c^2)^2-1/2\*sec(x)^2\*(b-(a+c)\*sin(x))/(a-b+c)/(a+b+c)-(b^4+2\*c^2\*(a+c)^2-2\*b^2\*c\*(2\*a+c))\*arctanh((b+2\*c\*sin(x))/(-4\*a\*c+b^2)^(1/2))/(a^2+2\*a\*c-b^2+c^2)^2/(-4\*a\*c+b^2)^(1/2)

**Rubi [A]** time = 0.50, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3258, 976, 1074, 634, 618, 206, 628, 633, 31}

$$\frac{b(b^2 - 2c(a+c)) \log(a+b \sin(x)+c \sin^2(x))}{2(a^2 + 2ac - b^2 + c^2)^2} - \frac{(-2b^2c(2a+c) + 2c^2(a+c)^2 + b^4) \tanh^{-1}\left(\frac{b+2c \sin(x)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (a^2 + 2ac - b^2 + c^2)^2} (a+c)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^3/(a + b\*Sin[x] + c\*Sin[x]^2),x]

[Out] -(((b^4 + 2\*c^2\*(a+c)^2 - 2\*b^2\*c\*(2\*a+c))\*ArcTanh[(b + 2\*c\*Sin[x])/Sqrt[b^2 - 4\*a\*c]])/(Sqrt[b^2 - 4\*a\*c]\*(a^2 - b^2 + 2\*a\*c + c^2)^2) - ((a + 2\*b + 3\*c)\*Log[1 - Sin[x]]/(4\*(a + b + c)^2) + ((a - 2\*b + 3\*c)\*Log[1 + Sin[x]]/(4\*(a - b + c)^2) + (b\*(b^2 - 2\*c\*(a+c))\*Log[a + b\*Sin[x] + c\*Sin[x]^2])/(2\*(a^2 - b^2 + 2\*a\*c + c^2)^2) - (Sec[x]^2\*(b - (a+c)\*Sin[x]))/(2\*(a - b + c)\*(a + b + c))

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^-1, x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^-1, x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 633**

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c

\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 976

Int[((a\_.) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e + c\*(2\*c^2\*d - c\*(2\*a\*f))\*x)\*(a + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((-4\*a\*c)\*(a\*c\*e^2 + (c\*d - a\*f)^2)\*(p + 1)), x] - Dist[1/((-4\*a\*c)\*(a\*c\*e^2 + (c\*d - a\*f)^2)\*(p + 1)), Int[(a + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (-(a\*e))\*(c\*e))\*(p + 1) - (2\*c^2\*d - c\*(2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(-2\*a\*c^2\*e)\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e)\*(p + q + 2) - (2\*c^2\*d - c\*(2\*a\*f))\*(-(c\*e\*(2\*p + q + 4)))]\*x + c\*f\*(2\*c^2\*d - c\*(2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[a\*c\*e^2 + (c\*d - a\*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1074

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (f\_.)\*(x\_)^2), x\_Symbol] := With[{q = c^2\*d^2 + b^2\*d\*f - 2\*a\*c\*d\*f + a^2\*f^2}, Dist[1/q, Int[(A\*c^2\*d - a\*c\*C\*d + A\*b^2\*f - a\*b\*B\*f - a\*A\*c\*f + a^2\*C\*f + c\*(B\*c\*d - b\*C\*d + A\*b\*f - a\*B\*f)\*x)/(a + b\*x + c\*x^2), x], x] + Dist[1/q, Int[(c\*C\*d^2 + b\*B\*d\*f - A\*c\*d\*f - a\*C\*d\*f + a\*A\*f^2 - f\*(B\*c\*d - b\*C\*d + A\*b\*f - a\*B\*f)\*x)/(d + f\*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3258

Int[cos[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n\_.) + (c\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n2\_.))^p, x\_Symbol] := Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)^((m - 1)/2)\*(a + b\*(f\*g\*x)^n + c\*(f\*g\*x)^(2\*n))^p, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

### Rubi steps



$$\begin{aligned}
\int \frac{\sec^3(x)}{a + b \sin(x) + c \sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{(1-x^2)^2 (a+bx+cx^2)} dx, x, \sin(x) \right) \\
&= -\frac{\sec^2(x)(b-(a+c)\sin(x))}{2(a-b+c)(a+b+c)} + \frac{\text{Subst} \left( \int \frac{2(a^2-2b^2+3ac+2c^2)+2b(a-c)x+2c(a+c)x^2}{(1-x^2)(a+bx+cx^2)} dx, x \right)}{4(a-b+c)(a+b+c)} \\
&= -\frac{\sec^2(x)(b-(a+c)\sin(x))}{2(a-b+c)(a+b+c)} + \frac{\text{Subst} \left( \int \frac{-2b^2(a-c)+2ac(a+c)+2c^2(a+c)+2a(a^2-2b^2+3ac)}{(1-x^2)(a+bx+cx^2)} dx, x \right)}{4(a-b+c)(a+b+c)} \\
&= -\frac{\sec^2(x)(b-(a+c)\sin(x))}{2(a-b+c)(a+b+c)} - \frac{(a-2b+3c) \text{Subst} \left( \int \frac{1}{-1-x} dx, x, \sin(x) \right)}{4(a-b+c)^2} + \frac{(a-2b+3c) \text{Subst} \left( \int \frac{1}{-1+x} dx, x, \sin(x) \right)}{4(a-b+c)^2} \\
&= -\frac{(a+2b+3c) \log(1-\sin(x))}{4(a+b+c)^2} + \frac{(a-2b+3c) \log(1+\sin(x))}{4(a-b+c)^2} + \frac{b(b^2-2c(a+c))}{4(a-b+c)(a+b+c)^2} \\
&= -\frac{(b^4+2c^2(a+c)^2-2b^2c(2a+c)) \tanh^{-1} \left( \frac{b+2c \sin(x)}{\sqrt{b^2-4ac}} \right)}{(a-b+c)^2(a+b+c)^2 \sqrt{b^2-4ac}} - \frac{(a+2b+3c) \log(1-\sin(x))}{4(a+b+c)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.77, size = 202, normalized size = 0.98

$$\frac{1}{4} \left( \frac{2b(b^2-2c(a+c)) \log(a+b \sin(x) + c \sin^2(x))}{(a^2+2ac-b^2+c^2)^2} - \frac{4(-2b^2c(2a+c) + 2c^2(a+c)^2 + b^4) \tanh^{-1} \left( \frac{b+2c \sin(x)}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac} (a^2+2ac-b^2+c^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^3/(a + b\*Sin[x] + c\*Sin[x]^2), x]

[Out]  $((-4*(b^4 + 2*c^2*(a + c)^2 - 2*b^2*c*(2*a + c))*\text{ArcTanh}[(b + 2*c*\text{Sin}[x])/ \sqrt{b^2 - 4*a*c}]) / (\text{Sqrt}[b^2 - 4*a*c]*(a^2 - b^2 + 2*a*c + c^2)^2) - ((a + 2*b + 3*c)*\text{Log}[1 - \text{Sin}[x]]) / (a + b + c)^2 + ((a - 2*b + 3*c)*\text{Log}[1 + \text{Sin}[x]]) / (a - b + c)^2 + (2*b*(b^2 - 2*c*(a + c))*\text{Log}[a + b*\text{Sin}[x] + c*\text{Sin}[x]^2]) / (a^2 - b^2 + 2*a*c + c^2)^2 - 1/((a + b + c)*(-1 + \text{Sin}[x])) - 1/((a - b + c)*(1 + \text{Sin}[x]))) / 4$

**fricas [B]** time = 9.17, size = 1244, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^3/(a+b\*sin(x)+c\*sin(x)^2), x, algorithm="fricas")

[Out]  $[-1/4*(2*a^2*b^3 - 2*b^5 - 8*a*b*c^3 - 2*(b^4 - 4*a*b^2*c + 4*a*c^3 + 2*c^4 + 2*(a^2 - b^2)*c^2)*\text{sqrt}(b^2 - 4*a*c)*\cos(x)^2*\log(-(2*c^2*\cos(x)^2 - 2*b*c*\sin(x) - b^2 + 2*a*c - 2*c^2 + \text{sqrt}(b^2 - 4*a*c)*(2*c*\sin(x) + b)) / (c*\cos(x)^2 - b*\sin(x) - a - c)) - 2*(b^5 - 6*a*b^3*c + 8*a*b*c^3 + 2*(4*a^2*b - b^3)*c^2)*\cos(x)^2*\log(-c*\cos(x)^2 + b*\sin(x) + a + c) - (a^3*b^2 - 3*a*b^4 - 4 - 2*b^5 - 12*a*c^4 - (28*a^2 + 16*a*b - 3*b^2)*c^3 - (20*a^3 + 16*a^2*b - 11*a*b^2 - 4*b^3)*c^2 - (4*a^4 - 17*a^2*b^2 - 12*a*b^3 + b^4)*c)*\cos(x)^2*\log(\sin(x) + 1) + (a^3*b^2 - 3*a*b^4 + 2*b^5 - 12*a*c^4 - (28*a^2 - 16*a*b - 3*b^2)*c^3 - (20*a^3 - 16*a^2*b - 11*a*b^2 + 4*b^3)*c^2 - (4*a^4 - 17*a^2*b^2 + 12*a*b^3 + b^4)*c)*\cos(x)^2*\log(-\sin(x) + 1) - 2*(8*a^2*b - b^3)*c^2 - 4*(2*a^3*b - 3*a*b^3)*c - 2*(a^3*b^2 - a*b^4 - 4*a*c^4 - (12*a^2 - b^2)*c^3 - (12*a^3 - 7*a*b^2)*c^2 - (4*a^4 - 7*a^2*b^2 + b^4)*c)*\sin(x)] / ((a^4*b$



**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^3/(a+b\*sin(x)+c\*sin(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 35.31, size = 2743, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^3\*(a + c\*sin(x)^2 + b\*sin(x))),x)

[Out] 
$$\log(\sin(x) + 1) * (1 / (4 * (a - b + c))) - (b / 4 - c / 2) / (a - b + c)^2 - (b / (2 * (2 * a * c + a^2 - b^2 + c^2))) - (\sin(x) * (a + c)) / (2 * (2 * a * c + a^2 - b^2 + c^2)) / \cos(x)^2 - \log(\sin(x) - 1) * ((b / 4 + c / 2) / (a + b + c)^2 + 1 / (4 * (a + b + c))) + (\log((c^4 * (4 * a * c + a^2 - 4 * b^2 + 3 * c^2)) / (4 * (2 * a * c + a^2 - b^2 + c^2)^2) - (((c * (a * b^4 + 28 * a * c^4 + 4 * a^4 * c - 5 * b^4 * c + 8 * c^5 - a^3 * b^2 + 36 * a^2 * c^3 + 20 * a^3 * c^2 + 5 * b^2 * c^3 - 3 * a * b^2 * c^2 - 9 * a^2 * b^2 * c)) / (2 * (2 * a * c + a^2 - b^2 + c^2))) + (b * c * \sin(x) * (36 * a * c^3 + 4 * a^3 * c + 3 * b^4 + 16 * c^4 - a^2 * b^2 + 24 * a^2 * c^2 - 13 * b^2 * c^2 - 18 * a * b^2 * c)) / (2 * a * c + a^2 - b^2 + c^2) - (2 * c * ((b^4 * (b^2 - 4 * a * c)^{(1/2)}) / 2 - b^5 / 2 + c^4 * (b^2 - 4 * a * c)^{(1/2)} + b^3 * c^2 + 2 * a * c^3 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a^2 * b * c^2 + a^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - b^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a * b * c^3 + 3 * a * b^3 * c - 2 * a * b^2 * c * (b^2 - 4 * a * c)^{(1/2})) * (3 * b^4 * \sin(x) + 4 * c^4 * \sin(x) + 4 * a * b^3 + 2 * b * c^3 + 2 * b^3 * c + 4 * a * c^3 * \sin(x) - 4 * a^3 * c * \sin(x) + a^2 * b^2 * \sin(x) - 4 * a^2 * c^2 * \sin(x) - 3 * b^2 * c^2 * \sin(x) - 12 * a * b * c^2 - 14 * a^2 * b * c - 10 * a * b^2 * c * \sin(x))) / ((4 * a * c - b^2) * (2 * a * c + a^2 - b^2 + c^2)^2)) * ((b^4 * (b^2 - 4 * a * c)^{(1/2)}) / 2 - b^5 / 2 + c^4 * (b^2 - 4 * a * c)^{(1/2)} + b^3 * c^2 + 2 * a * c^3 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a^2 * b * c^2 + a^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - b^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a * b * c^3 + 3 * a * b^3 * c - 2 * a * b^2 * c * (b^2 - 4 * a * c)^{(1/2}))) / ((4 * a * c - b^2) * (2 * a * c + a^2 - b^2 + c^2)^2) - (b * c * (2 * a * b^4 - 20 * a * c^4 + 3 * a^4 * c - 6 * b^4 * c + 7 * c^5 - a^3 * b^2 - 26 * a^2 * c^3 + 4 * a^3 * c^2 + 23 * a * b^2 * c^2 - 6 * a^2 * b^2 * c)) / (4 * (2 * a * c + a^2 - b^2 + c^2)^2) + (c * \sin(x) * (64 * a * c^5 + 26 * c^6 + a^2 * b^4 + 52 * a^2 * c^4 + 16 * a^3 * c^3 + 2 * a^4 * c^2 - 18 * b^2 * c^4 + 9 * b^4 * c^2 - 32 * a * b^2 * c^3 - 4 * a^3 * b^2 * c - 2 * a^2 * b^2 * c^2 - 2 * a * b^4 * c)) / (4 * (2 * a * c + a^2 - b^2 + c^2)^2)) * ((b^4 * (b^2 - 4 * a * c)^{(1/2)}) / 2 - b^5 / 2 + c^4 * (b^2 - 4 * a * c)^{(1/2)} + b^3 * c^2 + 2 * a * c^3 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a^2 * b * c^2 + a^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - b^2 * c^2 * (b^2 - 4 * a * c)^{(1/2)} - 4 * a * b * c^3 + 3 * a * b^3 * c - 2 * a * b^2 * c * (b^2 - 4 * a * c)^{(1/2}))) / ((4 * a * c - b^2) * (2 * a * c + a^2 - b^2 + c^2)^2) - (b * c^5 * \sin(x)) / (2 * a * c + a^2 - b^2 + c^2)^2 * (b^3 * (3 * a * c + c^2) - b^2 * (c^2 * (b^2 - 4 * a * c)^{(1/2)} + 2 * a * c * (b^2 - 4 * a * c)^{(1/2})) - b * (4 * a * c^3 + 4 * a^2 * c^2) - b^5 / 2 + (b^4 * (b^2 - 4 * a * c)^{(1/2)}) / 2 + c^4 * (b^2 - 4 * a * c)^{(1/2)} + 2 * a * c^3 * (b^2 - 4 * a * c)^{(1/2)} + a^2 * c^2 * (b^2 - 4 * a * c)^{(1/2}))) / (4 * a * c^5 + 4 * a^5 * c - b^6 + 2 * a^2 * b^4 - a^4 * b^2 + 16 * a^2 * c^4 + 24 * a^3 * c^3 + 16 * a^4 * c^2 - b^2 * c^4 + 2 * b^4 * c^2 - 12 * a * b^2 * c^3 - 12 * a^3 * b^2 * c - 22 * a^2 * b^2 * c^2 + 8 * a * b^4 * c) - (\log((c^4 * (4 * a * c + a^2 - 4 * b^2 + 3 * c^2)) / (4 * (2 * a * c + a^2 - b^2 + c^2)^2) - (((b * c * (2 * a * b^4 - 20 * a * c^4 + 3 * a^4 * c - 6 * b^4 * c + 7 * c^5 - a^3 * b^2 - 26 * a^2 * c^3 + 4 * a^3 * c^2 + 23 * a * b^2 * c^2 - 6 * a^2 * b^2 * c)) / (4 * (2 * a * c + a^2 - b^2 + c^2)^2) + (((c * (a * b^4 + 28 * a * c^4 + 4 * a^4 * c - 5 * b^4 * c + 8 * c^5 - a^3 * b^2 + 36 * a^2 * c^3 + 20 * a^3 * c^2 + 5 * b^2 * c^3 - 3 * a * b^2 * c^2 - 9 * a^2 * b^2 * c)) / (2 * (2 * a * c + a^2 - b^2 + c^2))) + (b * c * \sin(x) * (36 * a * c^3 + 4 * a^3 * c + 3 * b^4 + 16 * c^4 - a^2 * b^2 + 24 * a^2 * c^2 - 13 * b^2 * c^2 - 18 * a * b^2 * c)) / (2 * a * c + a^2 - b^2 + c^2) + (2 * c * (b^5 / 2 + (b^4 * (b^2 - 4 * a * c)^{(1/2)}) / 2 + c^4 * (b^2 -$$

$$\begin{aligned}
& 4ac^{1/2} - b^3c^2 + 2a^3c(b^2 - 4ac)^{1/2} + 4a^2bc^2 + a^2c^2(b^2 - 4ac)^{1/2} - b^2c^2(b^2 - 4ac)^{1/2} + 4ab^3c^3 - 3ab^3c \\
& - 2ab^2c(b^2 - 4ac)^{1/2})(3b^4\sin(x) + 4c^4\sin(x) + 4ab^3 + 2b^3c^3 + 2b^3c + 4a^3c^3\sin(x) - 4a^3c\sin(x) + a^2b^2\sin(x) - 4a^2c^2\sin(x) \\
& - 3b^2c^2\sin(x) - 12ab^2c^2 - 14a^2bc - 10ab^2c\sin(x)))/((4ac - b^2)(2ac + a^2 - b^2 + c^2)^2)(b^{5/2} + (b^4(b^2 - 4ac)^{1/2}))/2 + c^4(b^2 - 4ac)^{1/2} - b^3c^2 + 2a^3c(b^2 - 4ac)^{1/2} + 4a^2bc^2 + a^2c^2(b^2 - 4ac)^{1/2} - b^2c^2(b^2 - 4ac)^{1/2} + 4ab^3c^3 - 3ab^3c - 2ab^2c(b^2 - 4ac)^{1/2}))/((4ac - b^2)(2ac + a^2 - b^2 + c^2)^2) - (c\sin(x))(64a^5c^5 + 26c^6 + a^2b^4 + 52a^2c^4 + 16a^3c^3 + 2a^4c^2 - 18b^2c^4 + 9b^4c^2 - 32ab^2c^3 - 4a^3b^2c - 2a^2b^2c^2 - 2ab^4c))/((4(2ac + a^2 - b^2 + c^2)^2)(b^{5/2} + (b^4(b^2 - 4ac)^{1/2}))/2 + c^4(b^2 - 4ac)^{1/2} - b^3c^2 + 2a^3c(b^2 - 4ac)^{1/2} + 4a^2bc^2 + a^2c^2(b^2 - 4ac)^{1/2} - b^2c^2(b^2 - 4ac)^{1/2} + 4ab^3c^3 - 3ab^3c - 2ab^2c(b^2 - 4ac)^{1/2}))/((4ac - b^2)(2ac + a^2 - b^2 + c^2)^2) - (bc^5\sin(x))/(2ac + a^2 - b^2 + c^2)^2)(b(4a^3c^3 + 4a^2c^2) - b^3(3ac + c^2) - b^2(c^2(b^2 - 4ac)^{1/2} + 2ac(b^2 - 4ac)^{1/2})) + b^{5/2} + (b^4(b^2 - 4ac)^{1/2}))/2 + c^4(b^2 - 4ac)^{1/2} + 2a^3c(b^2 - 4ac)^{1/2} + a^2c^2(b^2 - 4ac)^{1/2}))/((4a^5c^5 + 4a^5c - b^6 + 2a^2b^4 - a^4b^2 + 16a^2c^4 + 24a^3c^3 + 16a^4c^2 - b^2c^4 + 2b^4c^2 - 12ab^2c^3 - 12a^3b^2c - 22a^2b^2c^2 + 8ab^4c)
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{a + b \sin(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)\*\*3/(a+b\*sin(x)+c\*sin(x)\*\*2),x)

[Out] Integral(sec(x)\*\*3/(a + b\*sin(x) + c\*sin(x)\*\*2), x)

$$3.15 \quad \int \frac{\cos(x)}{-6 + \sin(x) + \sin^2(x)} dx$$

Optimal. Leaf size=21

$$\frac{1}{5} \log(2 - \sin(x)) - \frac{1}{5} \log(\sin(x) + 3)$$

[Out] 1/5\*ln(2-sin(x))-1/5\*ln(3+sin(x))

**Rubi [A]** time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {3258, 616, 31}

$$\frac{1}{5} \log(2 - \sin(x)) - \frac{1}{5} \log(\sin(x) + 3)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(-6 + Sin[x] + Sin[x]^2), x]

[Out] Log[2 - Sin[x]]/5 - Log[3 + Sin[x]]/5

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

Rule 3258

Int[cos[(d\_.) + (e\_.)\*(x\_)]^(m\_.)\*((a\_.) + (b\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n\_.) + (c\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)])^(n2\_.))^(p\_.), x\_Symbol] :> Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)^(m - 1)/2\*(a + b\*(f\*g\*x)^n + c\*(f\*g\*x)^(2\*n))^p, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{-6 + \sin(x) + \sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{-6 + x + x^2} dx, x, \sin(x) \right) \\ &= \frac{1}{5} \text{Subst} \left( \int \frac{1}{-2 + x} dx, x, \sin(x) \right) - \frac{1}{5} \text{Subst} \left( \int \frac{1}{3 + x} dx, x, \sin(x) \right) \\ &= \frac{1}{5} \log(2 - \sin(x)) - \frac{1}{5} \log(3 + \sin(x)) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 0.71

$$-\frac{2}{5} \tanh^{-1} \left( \frac{1}{5} (2 \sin(x) + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(-6 + Sin[x] + Sin[x]^2),x]

[Out] (-2\*ArcTanh[(1 + 2\*Sin[x])/5])/5

**fricas** [A] time = 0.45, size = 17, normalized size = 0.81

$$-\frac{1}{5} \log(\sin(x) + 3) + \frac{1}{5} \log\left(-\frac{1}{2} \sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-6+sin(x)+sin(x)^2),x, algorithm="fricas")

[Out] -1/5\*log(sin(x) + 3) + 1/5\*log(-1/2\*sin(x) + 1)

**giac** [A] time = 0.19, size = 17, normalized size = 0.81

$$-\frac{1}{5} \log(\sin(x) + 3) + \frac{1}{5} \log(-\sin(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-6+sin(x)+sin(x)^2),x, algorithm="giac")

[Out] -1/5\*log(sin(x) + 3) + 1/5\*log(-sin(x) + 2)

**maple** [A] time = 0.20, size = 16, normalized size = 0.76

$$-\frac{\ln(3 + \sin(x))}{5} + \frac{\ln(\sin(x) - 2)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(-6+sin(x)+sin(x)^2),x)

[Out] -1/5\*ln(3+sin(x))+1/5\*ln(sin(x)-2)

**maxima** [A] time = 0.32, size = 15, normalized size = 0.71

$$-\frac{1}{5} \log(\sin(x) + 3) + \frac{1}{5} \log(\sin(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-6+sin(x)+sin(x)^2),x, algorithm="maxima")

[Out] -1/5\*log(sin(x) + 3) + 1/5\*log(sin(x) - 2)

**mupad** [B] time = 0.15, size = 9, normalized size = 0.43

$$-\frac{2 \operatorname{atanh}\left(\frac{2 \sin(x)}{5} + \frac{1}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(sin(x) + sin(x)^2 - 6),x)

[Out] -(2\*atanh((2\*sin(x))/5 + 1/5))/5

**sympy** [A] time = 0.45, size = 15, normalized size = 0.71

$$\frac{\log(\sin(x) - 2)}{5} - \frac{\log(\sin(x) + 3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-6+sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2)/5 - log(sin(x) + 3)/5
```

$$3.16 \quad \int \frac{\cos(x)}{2-3\sin(x)+\sin^2(x)} dx$$

Optimal. Leaf size=17

$$\log(2 - \sin(x)) - \log(1 - \sin(x))$$

[Out]  $-\ln(1-\sin(x))+\ln(2-\sin(x))$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3258, 616, 31}

$$\log(2 - \sin(x)) - \log(1 - \sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(2 - 3\*Sin[x] + Sin[x]^2),x]

[Out] -Log[1 - Sin[x]] + Log[2 - Sin[x]]

Rule 31

Int[((a\_) + (b\_)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

Rule 3258

Int[cos[(d\_.) + (e\_.)\*(x\_)]<sup>(m\_.)</sup>((a\_.) + (b\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]<sup>(n\_.)</sup> + (c\_.)\*((f\_.)\*sin[(d\_.) + (e\_.)\*(x\_)]<sup>(n2\_.)</sup>)<sup>(p\_.)</sup>, x\_Symbol] := Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)<sup>((m - 1)/2)</sup>(a + b\*(f\*g\*x)<sup>n</sup> + c\*(f\*g\*x)<sup>(2\*n)</sup>)<sup>p</sup>, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{2-3\sin(x)+\sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{2-3x+x^2} dx, x, \sin(x) \right) \\ &= \text{Subst} \left( \int \frac{1}{-2+x} dx, x, \sin(x) \right) - \text{Subst} \left( \int \frac{1}{-1+x} dx, x, \sin(x) \right) \\ &= -\log(1 - \sin(x)) + \log(2 - \sin(x)) \end{aligned}$$

Mathematica [A] time = 0.06, size = 26, normalized size = 1.53

$$\log(2 - \sin(x)) - 2 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(2 - 3\*Sin[x] + Sin[x]^2),x]



[Out]  $-2*\text{Log}[\text{Cos}[x/2] - \text{Sin}[x/2]] + \text{Log}[2 - \text{Sin}[x]]$

**fricas** [A] time = 0.44, size = 17, normalized size = 1.00

$$\log\left(-\frac{1}{2}\sin(x) + 1\right) - \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")`

[Out]  $\log(-1/2*\sin(x) + 1) - \log(-\sin(x) + 1)$

**giac** [A] time = 0.15, size = 17, normalized size = 1.00

$$\log(-\sin(x) + 2) - \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")`

[Out]  $\log(-\sin(x) + 2) - \log(-\sin(x) + 1)$

**maple** [A] time = 0.21, size = 14, normalized size = 0.82

$$-\ln(-1 + \sin(x)) + \ln(\sin(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(2-3*sin(x)+sin(x)^2),x)`

[Out]  $-\ln(-1+\sin(x))+\ln(\sin(x)-2)$

**maxima** [A] time = 0.34, size = 13, normalized size = 0.76

$$-\log(\sin(x) - 1) + \log(\sin(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")`

[Out]  $-\log(\sin(x) - 1) + \log(\sin(x) - 2)$

**mupad** [B] time = 15.14, size = 9, normalized size = 0.53

$$-2 \operatorname{atanh}(2 \sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(sin(x)^2 - 3*sin(x) + 2),x)`

[Out]  $-2*\operatorname{atanh}(2*\sin(x) - 3)$

**sympy** [A] time = 0.42, size = 12, normalized size = 0.71

$$\log(\sin(x) - 2) - \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(2-3*sin(x)+sin(x)**2),x)`

[Out]  $\log(\sin(x) - 2) - \log(\sin(x) - 1)$

$$3.17 \quad \int \frac{\cos(x)}{-5+4\sin(x)+\sin^2(x)} dx$$

Optimal. Leaf size=21

$$\frac{1}{6} \log(1 - \sin(x)) - \frac{1}{6} \log(\sin(x) + 5)$$

[Out] 1/6\*ln(1-sin(x))-1/6\*ln(5+sin(x))

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3258, 616, 31}

$$\frac{1}{6} \log(1 - \sin(x)) - \frac{1}{6} \log(\sin(x) + 5)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(-5 + 4\*Sin[x] + Sin[x]^2),x]

[Out] Log[1 - Sin[x]]/6 - Log[5 + Sin[x]]/6

Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

Rule 3258

Int[cos[(d\_) + (e\_)\*(x\_)]^(m\_)\*((a\_) + (b\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)]^(n\_) + (c\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)]^(n2\_))^(p\_)), x\_Symbol] := Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)^(m/2)\*(a + b\*(f\*g\*x)^n + c\*(f\*g\*x)^(2\*n))^p, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{-5+4\sin(x)+\sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{-5+4x+x^2} dx, x, \sin(x) \right) \\ &= \frac{1}{6} \text{Subst} \left( \int \frac{1}{-1+x} dx, x, \sin(x) \right) - \frac{1}{6} \text{Subst} \left( \int \frac{1}{5+x} dx, x, \sin(x) \right) \\ &= \frac{1}{6} \log(1 - \sin(x)) - \frac{1}{6} \log(5 + \sin(x)) \end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 1.43

$$\frac{1}{6} \left( 2 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) - \log(\sin(x) + 5) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(-5 + 4\*Sin[x] + Sin[x]^2),x]

[Out] (2\*Log[Cos[x/2] - Sin[x/2]] - Log[5 + Sin[x]])/6

**fricas** [A] time = 0.45, size = 17, normalized size = 0.81

$$-\frac{1}{6} \log(\sin(x) + 5) + \frac{1}{6} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-5+4\*sin(x)+sin(x)^2),x, algorithm="fricas")

[Out] -1/6\*log(sin(x) + 5) + 1/6\*log(-sin(x) + 1)

**giac** [A] time = 0.14, size = 17, normalized size = 0.81

$$-\frac{1}{6} \log(\sin(x) + 5) + \frac{1}{6} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-5+4\*sin(x)+sin(x)^2),x, algorithm="giac")

[Out] -1/6\*log(sin(x) + 5) + 1/6\*log(-sin(x) + 1)

**maple** [A] time = 0.22, size = 16, normalized size = 0.76

$$\frac{\ln(-1 + \sin(x))}{6} - \frac{\ln(5 + \sin(x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(-5+4\*sin(x)+sin(x)^2),x)

[Out] 1/6\*ln(-1+sin(x))-1/6\*ln(5+sin(x))

**maxima** [A] time = 0.33, size = 15, normalized size = 0.71

$$-\frac{1}{6} \log(\sin(x) + 5) + \frac{1}{6} \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-5+4\*sin(x)+sin(x)^2),x, algorithm="maxima")

[Out] -1/6\*log(sin(x) + 5) + 1/6\*log(sin(x) - 1)

**mupad** [B] time = 0.12, size = 9, normalized size = 0.43

$$-\frac{\operatorname{atanh}\left(\frac{\sin(x)}{3} + \frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(4\*sin(x) + sin(x)^2 - 5),x)

[Out] -atanh(sin(x)/3 + 2/3)/3

**sympy** [A] time = 0.46, size = 15, normalized size = 0.71

$$\frac{\log(\sin(x) - 1)}{6} - \frac{\log(\sin(x) + 5)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-5+4\*sin(x)+sin(x)\*\*2),x)

[Out] log(sin(x) - 1)/6 - log(sin(x) + 5)/6

$$3.18 \quad \int \frac{\cos(x)}{10-6\sin(x)+\sin^2(x)} dx$$

Optimal. Leaf size=9

$$-\tan^{-1}(3 - \sin(x))$$

[Out] arctan(-3+sin(x))

**Rubi [A]** time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3258, 618, 204}

$$-\tan^{-1}(3 - \sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(10 - 6\*Sin[x] + Sin[x]^2),x]

[Out] -ArcTan[3 - Sin[x]]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 3258

Int[cos[(d\_) + (e\_)\*(x\_)]^(m\_)\*((a\_) + (b\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)])^(n\_) + (c\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)])^(n2\_))^(p\_), x\_Symbol] := Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)^((m - 1)/2)\*(a + b\*(f\*g\*x)^n + c\*(f\*g\*x)^(2\*n))^p, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{10-6\sin(x)+\sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{10-6x+x^2} dx, x, \sin(x) \right) \\ &= - \left( 2 \text{Subst} \left( \int \frac{1}{-4-x^2} dx, x, -6+2\sin(x) \right) \right) \\ &= -\tan^{-1}(3 - \sin(x)) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 9, normalized size = 1.00

$$-\tan^{-1}(3 - \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(10 - 6\*Sin[x] + Sin[x]^2),x]

[Out] -ArcTan[3 - Sin[x]]

**fricas** [A] time = 0.44, size = 5, normalized size = 0.56

$$\arctan(\sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(10-6\*sin(x)+sin(x)^2),x, algorithm="fricas")

[Out] arctan(sin(x) - 3)

**giac** [A] time = 0.12, size = 5, normalized size = 0.56

$$\arctan(\sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(10-6\*sin(x)+sin(x)^2),x, algorithm="giac")

[Out] arctan(sin(x) - 3)

**maple** [A] time = 0.22, size = 6, normalized size = 0.67

$$\arctan(-3 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(10-6\*sin(x)+sin(x)^2),x)

[Out] arctan(-3+sin(x))

**maxima** [A] time = 0.46, size = 5, normalized size = 0.56

$$\arctan(\sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(10-6\*sin(x)+sin(x)^2),x, algorithm="maxima")

[Out] arctan(sin(x) - 3)

**mupad** [B] time = 0.12, size = 5, normalized size = 0.56

$$\operatorname{atan}(\sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(sin(x)^2 - 6\*sin(x) + 10),x)

[Out] atan(sin(x) - 3)

**sympy** [A] time = 0.50, size = 5, normalized size = 0.56

$$\operatorname{atan}(\sin(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(10-6\*sin(x)+sin(x)\*\*2),x)

[Out] atan(sin(x) - 3)

$$3.19 \quad \int \frac{\cos(x)}{2+2\sin(x)+\sin^2(x)} dx$$

Optimal. Leaf size=5

$$\tan^{-1}(\sin(x) + 1)$$

[Out] arctan(1+sin(x))

Rubi [A] time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3258, 617, 204}

$$\tan^{-1}(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(2 + 2\*Sin[x] + Sin[x]^2),x]

[Out] ArcTan[1 + Sin[x]]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 3258

Int[cos[(d\_) + (e\_)\*(x\_)]^(m\_)\*((a\_) + (b\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)])^(n\_) + (c\_)\*((f\_)\*sin[(d\_) + (e\_)\*(x\_)])^(n2\_))^(p\_), x\_Symbol] := Module[{g = FreeFactors[Sin[d + e\*x], x]}, Dist[g/e, Subst[Int[(1 - g^2\*x^2)^((m - 1)/2)\*(a + b\*(f\*g\*x)^n + c\*(f\*g\*x)^(2\*n))^p, x], x, Sin[d + e\*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2\*n] && IntegerQ[(m - 1)/2]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{2+2\sin(x)+\sin^2(x)} dx &= \text{Subst} \left( \int \frac{1}{2+2x+x^2} dx, x, \sin(x) \right) \\ &= -\text{Subst} \left( \int \frac{1}{-1-x^2} dx, x, 1+\sin(x) \right) \\ &= \tan^{-1}(1+\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$\tan^{-1}(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(2 + 2\*Sin[x] + Sin[x]^2),x]

[Out] ArcTan[1 + Sin[x]]

**fricas** [A] time = 0.43, size = 5, normalized size = 1.00

$\arctan(\sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2+2\*sin(x)+sin(x)^2),x, algorithm="fricas")

[Out] arctan(sin(x) + 1)

**giac** [A] time = 0.14, size = 5, normalized size = 1.00

$\arctan(\sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2+2\*sin(x)+sin(x)^2),x, algorithm="giac")

[Out] arctan(sin(x) + 1)

**maple** [A] time = 0.20, size = 6, normalized size = 1.20

$\arctan(1 + \sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(2+2\*sin(x)+sin(x)^2),x)

[Out] arctan(1+sin(x))

**maxima** [A] time = 0.43, size = 5, normalized size = 1.00

$\arctan(\sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2+2\*sin(x)+sin(x)^2),x, algorithm="maxima")

[Out] arctan(sin(x) + 1)

**mupad** [B] time = 14.93, size = 5, normalized size = 1.00

$\operatorname{atan}(\sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(2\*sin(x) + sin(x)^2 + 2),x)

[Out] atan(sin(x) + 1)

**sympy** [A] time = 0.45, size = 5, normalized size = 1.00

$\operatorname{atan}(\sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2+2\*sin(x)+sin(x)\*\*2),x)

[Out] atan(sin(x) + 1)





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3, ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```



```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```